Inflation and Unemployment in the Long Run Revisited

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Abstract

We construct a continuous-time, monetary model with frictional goods and labor markets to revisit the long-run relationship between inflation and unemployment. The novelty relative to the literature (e.g., Berentsen et al., 2011) is the possibility given to consumers to search sequentially among different sellers to fulfill idiosyncratic consumption shocks. The value of consumers' outside options and firms' market power are endogenous and depend on the inflation rate. The long-run Phillips curve is generically U-shaped, i.e., at low inflation rates, an increase in anticipated inflation reduces the unemployment rate whereas at high inflation rates it raises it.

JEL Classification: D82, D83, E40, E50

Keywords: Unemployment, money, inflation, search, bargaining

1 Introduction

The relationship between inflation and unemployment – the Phillips curve – is a cornerstone of macroeconomic models. The textbook view is that in the long run the Phillips curve is vertical, i.e., by virtue of the classical dichotomy, inflation does not affect the equilibrium or natural unemployment rate.[1] However, it has long been recognized that the long-run Phillips curve might actually slope upward (e.g., Friedman, 1977), since anticipated inflation is a distortionary tax that penalizes monetary exchange and market activities. Inflation reduces consumers' payment capacity, which has an adverse effect on firms' profits and their incentives to hire workers. This idea has been formalized by Berentsen et al. (2011) – BMW thereafter – within a framework that combines the Mortensen and Pissarides (1994) model of unemployment – MP thereafter – with the model of frictional goods markets with monetary exchange due to Lagos and Wright (2005).

While BMW offers an elegant description of the goods and labor markets as two frictional markets that open sequentially, they introduce an overlooked asymmetry regarding the treatment of agents' outside options in each market.[2] In the labor market, a worker who is negotiating her employment contract with a firm has the option to keep searching for an alternative firm. In the goods market, in every period, consumers want to consume and they are matched with at most one producer. Hence, consumers do not have the option to search for an alternative producer, either within a period or across periods, than the one they are matched with to satisfy their current desire for consumption. Put differently, matched consumers face no opportunity cost from accepting a trade since their decision to trade does not affect their future trading opportunities. The lack of meaningful consumer outside options is not a triviality. It inhibits competitive pressures in the goods market, thereby undermining a core rationale for using search and bargaining models to formalize decentralized markets, namely, that they converge to a perfect competition outcome at the limit when trading frictions vanish. It also has important implications for the slope of the long-run Phillips curve, the relationship between real wages and inflation, and the optimality of the Friedman rule.

Our paper has two contributions: one is methodological and the other is theoretical. In terms of the methodology, we construct a continuous-time monetary model of goods and labor markets with search frictions that provides a symmetric treatment of workers' and consumers' outside options and the determination of prices and wages. Specifically, we disentangle preference shocks from random matching shocks in the goods market, as illustrated in Figure 1. The matching in the goods market will be represented by $\alpha \in \mathbb{R}_+$, the rate at which consumers find sellers. The market will be said to be frictionless when $\alpha \to +\infty$. Preference shocks will be represented by $\lambda \in \mathbb{R}_+$, the rate at which a buyer who has just consumed receives a new preference shock and wants to consume again. BMW is the limit $\lambda = +\infty$, i.e., buyers want to consume all the time. As a result of this formalization, the value of consumers' outside options is endogenous (it depends on α,

[1] This view has been embraced by canonical models of equilibrium unemployment, e.g., Lucas and Prescott (1974) and Mortensen and Pissarides (1994).

[2] Models with both frictional goods and labor markets also include Lehmann and Van der Linden (2010), and Petrosky-Nadeau and Wasmer (2015). See the literature review for additional references.

λ, and inflation) and it affects prices and wages, markups and markdowns. We will show that the economy under $\lambda < +\infty$ differs fundamentally from the one with $\lambda = +\infty$ in terms of its positive (e.g., slope of the Phillips curve) and normative (e.g., optimality of the Friedman rule) implications.

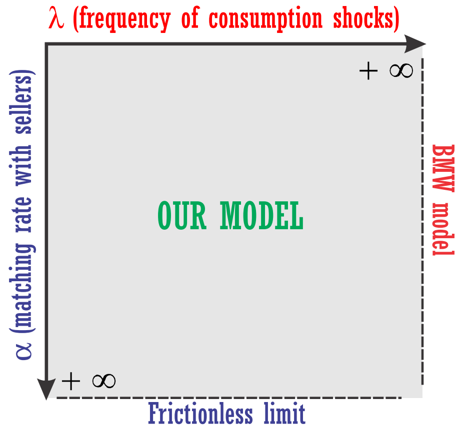

Figure 1: Disentangling search frictions and preference shocks

Our theoretical contribution establishes the generic nonmonotonicity of the long-run Phillips curve. At low inflation rates, there is a negative relationship between inflation and unemployment whereas the correlation is positive at high inflation rates. This nonmonotonicity arises from two opposite effects of inflation on equilibrium unemployment. First, there is a negative effect on consumers' real balances that tends to increase unemployment. This effect is the one identified in BMW. Second, there is a market power effect according to which inflation reduces the value of consumers' outside option, which consists in searching across different sellers. This search activity in monetary economies requires agents to hold real money balances at a cost that increases with inflation. This second effect, which reduces unemployment, dominates at low inflation rates whereas the first effect dominates at high inflation rates.

Figure 2 illustrates this finding by plotting the theoretical long-run Phillips curve. There is an inflation rate above the Friedman rule, denoted π^*, that minimizes unemployment. Below that inflation rate, the Phillips curve is downward-sloping, which generates a trade-off for policymakers whose dual mandate is to keep inflation and unemployment low. Above π^*, the long-run Phillips curve is upward-sloping, in which case there is no trade-off between inflation and unemployment. These results rationalize the choice of a positive inflation target between the Friedman rule and π^*. In a calibrated version of our model, π^* is close to 0, but can range as high as 3% when the search horizon goes to infinity ($\lambda \to 0$).

In addition to showing that the Friedman rule fails to minimize unemployment, we also establish that it fails to maximize social welfare when workers and consumers have high bargaining powers. The logic goes as follows. The constrained-efficient allocations are implemented with the Friedman rule and a Hosios condition in each market. When $\lambda < +\infty$, a deviation from the Friedman rule has a first-order, positive effect on firm entry and labor market tightness. Hence, it is optimal to raise the money growth rate above the Friedman rule when entry is inefficiently low, e.g., due to high worker's and consumer's bargaining powers.

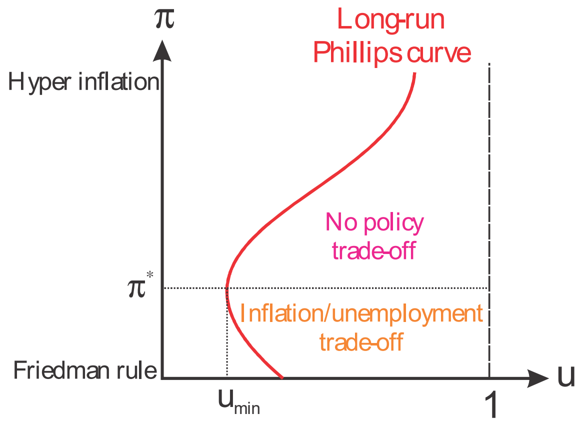

Figure 2: The U-shaped long-run Phillips curve

Maybe surprisingly, this logic breaks down in the BMW model, when $\lambda = +\infty$. Indeed, a deviation from the Friedman rule only has a second-order effect on entry and welfare and, under ex post bargaining, the Friedman rule is always optimal.

Our second positive insight is related to the limit of equilibrium outcomes as trading frictions vanish. A central result in the literature on decentralized market with search and bargaining is the convergence of equilibrium allocations to a Walrasian or perfect-competition outcome as trading frictions vanish (Gale, 1986a,b, 1987). We show that the result on frictionless limits holds in our two-market economies for all $\lambda \in (0, +\infty)$. It means that markups and markdowns are driven to zero as the process of matching buyers and sellers becomes infinitely efficient. All rents in all markets vanish. In contrast, positive rents persist in the BMW model even when trading frictions vanish (see Table 1).

	BMW, $\lambda = +\infty$	Our model, $\lambda \in (0, +\infty)$
Slope of Phillips curve	positive	negative at low inflation rates
Limit at $\alpha \to +\infty$	rents & markups>0	rents & markup=0
Friedman rule	always optimal	suboptimal if low firms' bargaining power

Table 1: Comparison to BMW

Even though we emphasize the role of consumer search in the determination of firms' market power, in our baseline model, on the equilibrium path, consumers do not exercise their option to search once matched with a firm. Search is a threat that disciplines the demand of the firms during their negotiation with consumers. In an extension, we consider horizontally differentiated products to induce search on the equilibrium path. We show that our main insight is robust, i.e. the long-run Phillips curve is downward sloping at low inflation rates.

Finally, in the last section of the paper, we generalize the description of monetary policy by introducing liquid government bonds and by allowing the policymaker to choose both a short-term nominal interest rate and a trend for the money growth rate. Liquid bonds can bear interest due to limited participation in the bonds market. More specifically, there are sophisticated agents who can make payments with either liquid government bonds or money and there are unsophisticated agents who can only hold fiat money. When the interest-rate spread between illiquid and liquid bonds becomes positive, unemployment decreases. If the inflation rate is sufficiently large, as the short-term interest rate approaches zero – the liquidity trap – then unemployment increases. Hence, the relationship between unemployment and the short-term interest rate can be non-monotone. When agents have homogenous preferences, the unemployment rate is minimum when the short-term nominal interest rate is set to zero and long-run inflation is above the Friedman rule, i.e., the economy is in a liquidity trap but is away from the Friedman rule.

1.1 Literature review

Our model builds on Berentsen et al. (2011) that introduces a frictional labor market into the Lagos and Wright (2005) model.[3] Other models with frictional goods and labor markets include Petrosky-Nadeau and Wasmer (2015, 2017) and Michaillat and Saez (2015). In the former, there is no money, i.e., agents pay with transferable utility. In the latter, money is introduced via a money-in-the-utility-function specification. In contrast, we formalize explicitly the role of money and the associated liquidity constraints that are critical for the formation of the terms of trade. In addition, we endogenize consumers' outside options and firms' market power in a dynamic setting.

There is a literature on search and inflation under menu costs that shows that inflation erodes firms' market power, e.g., Benabou (1988) and Diamond (1993). In those models, the economy is cashless, i.e., money has no transaction role. Inflation reduces market power by preventing firms from maintaining their price at a monopoly level as in Diamond (1971). In contrast, in our model, prices are perfectly flexible and are determined through bilateral bargaining. Moreover, we formalize a monetary economy where consumers need to carry real money balances while searching for a producer. Therefore, contrary to Benabou (1988) and Diamond (1993), inflation makes it more costly for consumers to search, which reduces the value of their outside option and raises firms' market power.

Similar results are obtained in traditional New Keynesian models with sticky prices. Such models can feature a positive correlation between macroeconomic activity and inflation (see Walsh, 2017). For instance, in King and Wolman (1996), firms adjust prices infrequently as the inflation rate rises, resulting in a lower markup and a higher output. Devereux and Yetman (2002) show that, when the frequency of price adjustments is endogenized, the correlation between inflation and output becomes non-linear, and non-monotone. In their calibration, long-run inflation has a positive effect on output when the inflation rate is below 2%, and the effect becomes negative for higher inflation rates. Our theory also predicts that output is hump-shaped in the long-run inflation rate, but the main mechanism works through consumers' outside option of search and does not require sticky prices.

[3] Other models of unemployment and inflation based on the Mortensen-Pissarides framework include Shi (1998), Cooley and Quadrini (2004) and Lehmann (2012). A related approach is provided by Williamson (2015). Versions of the model with money and credit include Bethune et al. (2015) and Branch et al. (2016). A continuous-time version was constructed by Rocheteau and Rodriguez-Lopez (2014).

In our model the long-run Phillips curve slopes downward at low inflation rates. There are alternative explanations for why the long-run Phillips curve could be downward sloping. In Rocheteau et al. (2007) where unemployment emerges due to indivisible labor, the long-run Phillips curve can be downward sloping, depending on the complementarity or substitutability between leisure and consumption.[4] Another mechanism to obtain a negative relationship between unemployment and inflation is through a Tobin effect according to which the rate of return on capital decreases, e.g., as in Rocheteau and Rodriguez-Lopez (2014).

There are alternative approaches to model imperfect competition in monetary economies. Head and Kumar (2005); Wang (2016); Wang et al. (2020) introduce noisy search, as in Burdett and Judd (1983), into the Lagos-Wright model.[5] By changing the information structure, the pricing outcome can vary from monopoly pricing to marginal cost pricing. A crucial difference is that, in our model, the search process is sequential and monetary policies affect the inter-temporal outside option of searching for alternative sellers by raising the cost of holding money. In the Burdett-Judd model, search is simultaneous and thus monetary policies cannot affect the outside option of search. This difference leads to different predictions regarding the normative and positive impact of monetary policies.

The consumer search literature has a long tradition of modeling consumers' outside options explicitly, i.e., the opportunity cost of trading includes the forgone benefit of searching for other sellers, e.g, McCall (1970), Wolinsky (1986), and Anderson and Renault (1999). These models generate consumer search in equilibrium by assuming that products are horizontally differentiated. We follow this tradition in Section 5.

Our extension with liquid bonds where only a fraction of consumers have access to bonds markets is related to Alvarez et al. (2001), Alvarez et al. (2002), and Williamson (2006). In equilibrium, bonds pay a liquidity premium and the real rate of return depends on monetary policy, as in Krishnamurthy and Vissing-Jorgensen (2012). We distinguishes two types of equilibria — with or without a liquidity trap. Williamson (2012) and Rocheteau et al. (2018) also illustrate how a liquidity trap can arise in a New Monetarist model.

1.2 Empirical evidence

A key challenge in assessing the long-run effects of monetary policy is to identify monetary policy shocks. There are two approaches to this problem (Blanchard, 2018). One is to study recessions caused by intentional disinflations. These disinflations are large monetary shocks driven by changes in policies and are not reactions to other shocks, see, for example, Ball (2009, 2014) and Blanchard et al. (2015). Blanchard (2018) concludes that, according to cross country macroeconomic evidence, disinflation has a persistent positive effect on the unemployment rate and a negative effect on output.

Another approach, pioneered by King and Watson (1994), is to use a vector autoregression (VAR) methodology with identified monetary policy shocks. According to this literature, the sign of the slope of the long-run

[4] Dong (2011) adopts the notion of competitive search equilibrium in this model to study the relationship between inflation and unemployment.

[5] Julien et al. (2008), Galenianos and Kircher (2009) and Bethune et al. (2020) also consider imperfect competitions by introducing multilateral meetings into search models.

Philips curve is sensitive to the choice of assumptions. Using bivariate structural VARs, King and Watson (1994) find evidence of a negative long-run trade-off between inflation and unemployment in the US postwar, under the identifying assumption that business cycles are entirely due to demand shocks. But King and Watson (1997) show that the results are sensitive to the choice of identifying assumptions. More recently, Benati (2015) adopts both classical and Bayesian structural VARs, and shows that the US data is compatible with both positively and negatively sloped Phillips curves. The estimated long-run impact of a 1% increase in inflation on the unemployment rate ranges from -0.56% to 0.15% at a 90% confidence interval. Ascari et al. (2022) introduce stochastic trends into Bayesian VAR. They find that there is a threshold level of inflation below which potential output is independent of inflation, and above which potential output and inflation are negatively correlated. The threshold level of inflation is slightly below 4%. This finding is consistent with our theory which predicts that output rises with inflation when inflation is low, and otherwise falls.

In terms of cross-country evidence, Bullard and Keating (1995) study a large sample of postwar economies using a structural VAR approach and find that inflation raised output in low-inflation countries, and either did not affect or reduced output in countries with higher inflation rate. A meta-study of VAR studies by De Grauwe and Costa Storti (2004) finds that the average effect of a 1% interest rate shock on output is -0.15% after five years. But the standard deviation of the distribution of estimates is 0.27%, so the Philips curve can have a positive or a negative slope. De Grauwe and Costa Storti conclude that the differences in results across different studies are mainly due to the differences in econometric methodologies.

The evidence on inflation and firms' market power is mixed. For instance, Chirinko and Fazzari (2000) and Neiss (2001) find a positive relationship between inflation and firm-level markups while Banerjee and Russell (2001, 2005) find a negative relationship. The topic has gained renewed interest following the 2021-22 inflation surge and discussion of "greed-flation" as a source of rising prices.[6] Glover et al. (2023) document that firm-level markups in the US increased by 3.4 percentage points in 2021 while inflation increased by around 3 percentage points, and Hansen et al. (2023) find that increases in corporate profits contributed up to 45% of the Euro area inflation.

2 Environment

The benchmark environment builds on Choi and Rocheteau (2021).[7]

Time, agents, commodities Time is continuous and indexed by $t \in \mathbb{R}_+$. The economy is composed of three types of infinitely-lived agents: a unit measure of workers, a measure ω of consumers, and an endogenous measure of firms, n. There are two perishable goods, $y \in \mathbb{R}_+$ and $c \in \mathbb{R}$. Good c can be consumed and produced by all agents and it is traded competitively and continuously over time. Good y

[6] See, e.g., DePillis (2022).

[7] It can be interpreted as a continuous-time version of Lagos and Wright (2005) and Rocheteau and Wright (2005), except that centralized and decentralized markets do not alternate in discrete time but instead coexist in continuous time. For earlier versions, see Craig and Rocheteau (2008), Rocheteau and Rodriguez-Lopez (2014), and Rocheteau et al. (2018).

is produced by firms and valued by consumers only. Good c is taken as the numéraire. The flows of goods and payments between agents are summarized in Figure 3. The worker produces y units of goods for the consumer (labelled B as for buyer) who makes a payment p (expressed in the numeraire) to the firm, who compensates the worker with a wage w.

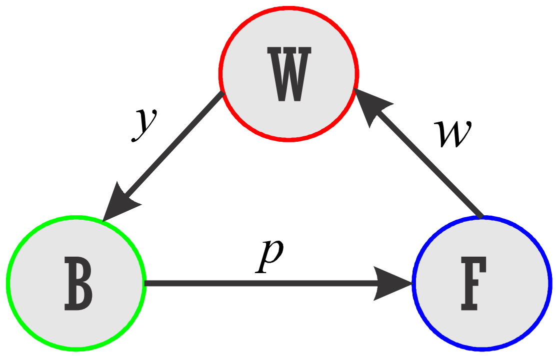

Figure 3: Agents: worker (W), firm (F), consumer (B)

Preferences Preferences of consumers, workers, and firms are represented by the following utility functions:

$$\mathcal{U}^b = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t) + \sum_{n=1}^{+\infty} e^{-\rho T_n} \varepsilon_{T_n} \upsilon\left[y(T_n)\right]\right] \tag{1}$$

$$\mathcal{U}^w = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t)\right] \tag{2}$$

$$\mathcal{U}^f = \mathbb{E}\left[\int_0^\infty e^{-\rho t} dC(t)\right], \tag{3}$$

where $C(t)$ measures the cumulative net consumption of the numéraire good.[8] Negative consumption of the numéraire good is interpreted as production. So preferences of all agents are linear in the numéraire.[9] The second term on the right side of (1) represents the consumption of the y good produced by firms for consumers. In (1) the idiosyncratic stochastic process, $\{T_n\}$, indicates the times at which the consumer gets to consume good y. The utility function, $\upsilon(y)$, is such that $\upsilon(0) = 0$, $\upsilon' > 0$, and $\upsilon'' < 0$.

At any point in time, consumers can be in one of two states, idle or active, captured by $\varepsilon_t \in \{0, 1\}$. An idle consumer has no desire to consume ($\varepsilon_t = 0$). The desire to consume arrives at Poisson rate $\lambda > 0$, in which case the consumer becomes active ($\varepsilon_t = 1$). This desire is fulfilled after the consumption of any quantity $y > 0$ (e.g., consumers are satiated) or it disappears at Poisson rate $\gamma \geq 0$. In both events, the active consumer becomes idle. As an example, assume consumers wish to consume ice cream at some Poisson rate λ. Once they have consumed it, or if they have waited too long, they no longer desire ice cream for a while. (In the

[8] A similar cumulative consumption process is assumed in the continuous-time models of OTC trades of Duffie et al. (2005). If consumption (or production) of the numéraire happens in flows, then $C(t)$ admits a density, $dC(t) = c(t)dt$. If the buyer consumes or produces a discrete quantity of the numéraire good at some instant t, then $C(t^+) - C(t^-) \neq 0$.

[9] In the formulation of this environment, we separate agents according to their role (consumer, worker, firm) relative to the consumption or production of good y. It would be equivalent to consider a household composed of a unit measure of workers and a measure ω of buyers with a single integrated budget constraint that incorporates firms' profits, as in, e.g., Shi (1998).

calibrated version of the model, agents buy different goods characterized by different λ's.) The assumption, $\lambda < +\infty$, according to which consumers remain temporarily idle (satiated) following consumption, $y_t > 0$, generates an opportunity cost from accepting a trade. This cost, together with bargaining shares, determines sellers' market power. In Berentsen et al. (2011), $\lambda = +\infty$, in which case consumers are always active and accepting a trade has no opportunity cost. The measure of active consumers is denoted ω_1 and the measure of idle consumers is ω_0.

Technology Each worker-firm pair produces both types of output: a constant flow, $x > 0$, of the numéraire good and endogenous quantities, y, of the decentralized-market good in bilateral meetings with consumers. The production of y units of the decentralized-market good requires $\varphi(y)$ units of numéraire.[10] It is such that $\varphi(0) = \varphi'(0) = 0$, $\varphi'(y) > 0$, and $\varphi''(y) > 0$. We denote y^* such that $\varphi'(y^*) = \upsilon'(y^*)$.

Frictions in the goods market We denote $q \equiv n/\omega_1$ the market tightness in the decentralized goods market. It is defined as the ratio of the measure of firms to the measure of active consumers. The matching rate of a consumer in the decentralized goods market is $\alpha \equiv \alpha(q)$ where $\alpha(0) = 0$, $\alpha' > 0$, $\alpha'(0) = +\infty$, and $\alpha'' < 0$. The matching rate of a firm is $\alpha^s \equiv \alpha(q)/q$.

Payments We distinguish two types of pairwise meetings according to the method of payment. A fraction χ^d of meetings are such that the consumer can produce the numéraire to pay for his consumption. We interpret these meetings as credit meetings. With complement probability, χ^m, the consumer cannot produce the numéraire in the meeting and is not trusted to repay his debt in the future. In such meetings, the consumer pays with fiat money, an intrinsically useless object that is perfectly storable and durable. The quantity of money at time t is denoted M_t. The constant money growth rate is $\pi \equiv \dot{M}_t/M_t$ and new money is injected in the economy through lump-sum transfers (or taxes if $\pi < 0$) to buyers. The price of money in terms of the numéraire is denoted ϕ_t and the lump-sum transfer is denoted $\tau_t = \phi_t\dot{M}_t$.

In pairwise meetings in the goods market, the quantities produced and consumed, and payments, are determined according to the proportional solution of Kalai (1977) where the share of the surplus received by sellers is $\mu \in [0, 1]$. The reasons for using the Kalai solution are explained in Aruoba et al. (2007), and Hu and Rocheteau (2020) who also provide strategic foundations based on a variant of the Rubinstein game.[11]

Frictions in the labor market Labor market tightness is defined as the ratio of vacancies per unemployed worker, $\theta \equiv \nu/u$. The job finding rate of a worker is $f(\theta)$ with $f(0) = 0$, $f' > 0$, $f'(0) = +\infty$, and $f'' < 0$. The vacancy filling rate is $f(\theta)/\theta$. The flow cost of opening a vacancy is $k > 0$. We restrict our attention

[10] We also worked out a version where the variable cost is in terms of workers' labor or disutility. It does not affect the allocations or results, except for the expression of the worker's compensation. See Appendix B.

[11] The monotonicity property of the Kalai solution guarantees that it implements efficient quantities at the Friedman rule (Aruoba et al., 2007). This result is instrumental for some of our proofs.

to contracts where workers are paid a constant wage, w, that is negotiated between the worker and the firm according to the Nash/Kalai solution. Unemployed workers receive a flow income b.

3 Equilibrium

3.1 Goods market

We denote $V^b(a)$ the value function of an active consumer with a units of real balances and $W^b(a)$ the value function of an idle consumer. Given the linearity of preferences with respect to the numéraire good, both value functions are linear with $V^b(a) = a + V^b$ and $W^b(a) = a + W^b$. We denote $Z \equiv V^b - W^b$ the loss in terms of lifetime expected utility from transitioning from being active to being idle. It is the opportunity cost from accepting a trade.

The outcome of the negotiation between a consumer and a firm is a pair, (p, y), where p is the payment by the consumer expressed in the numéraire and y is the output produced by the firm. The surplus of the firm is the difference between the firm revenue expressed in terms of the numéraire and the variable cost to produce y, $p - \varphi(y)$. The firm does not incur an opportunity cost since producing for its current consumer does not affect its ability to serve its future consumers. The surplus of the consumer is the difference between the utility from consuming y net of the payment and the opportunity cost of accepting a trade, $\upsilon(y) - p - Z$. There are gains from trade if

$$\max_y \{\upsilon(y) - \varphi(y) : \varphi(y) \leq a\} > Z. \tag{4}$$

For a monetary trade to be incentive feasible, the payment, which is bounded above by a_t, must at least cover the firm's variable cost, $\varphi(y)$. Moreover, the utility of consumption net of the variable cost must be greater than the consumer's opportunity cost.

A necessary condition for (4) to hold is that $Z \leq \upsilon(y^*) - \varphi(y^*)$. So, the opportunity cost of the consumer is bounded above by the first-best surplus. If $a \geq \varphi(y^*)$, the condition is also sufficient. If $a < \varphi(y^*)$, then the liquidity constraint binds and (4) can be reexpressed as $\upsilon \circ \varphi^{-1}(a) - a > Z$. The existence of gains from trade requires that the consumer holds enough real balances.

The terms of trade, which are determined according to the Kalai proportional solution, solve

$$\max_{p,y} \{p - \varphi(y)\} \quad \text{s.t. } p - \varphi(y) = \mu [\upsilon(y) - \varphi(y) - Z] \quad \text{and} \quad p \leq a. \tag{5}$$

The firm chooses (p, y) to maximize its profits subject to the condition that: (i) the profits are a fraction μ of the whole gains from trade; and (ii) the consumer's payment does not exceed her real balances. Since the firm's and the consumer's surpluses are proportional to each other, the problem can be reduced to the maximization of the joint profits subject to a liquidity constraint:

$$\max_{p,y} \{\upsilon(y) - \varphi(y)\} \quad \text{s.t. } p = \varphi(y) + \mu [\upsilon(y) - \varphi(y) - Z] \leq a. \tag{6}$$

If the liquidity constraint does not bind, then $y = y^*$ and $p = (1-\mu)\varphi(y^*) + \mu\upsilon(y^*) - \mu Z$. Otherwise, $p = a$ and $a = (1-\mu)\varphi(y) + \mu\upsilon(y) - \mu Z$. The solution to the bargaining problem, (6), is represented graphically in Figure 4 in the utility space (U^b, U^s), where U^b is the net utility of the buyer and U^s are the profits of the seller. The bargaining outcome is obtained at the intersection of the downward-sloping Pareto frontier and the upward-sloping proportional sharing rule. We denote $y\,(a,Z)$ the outcome of the negotiation. It is nondecreasing in a and Z. In a credit trade, $y = y^*$.

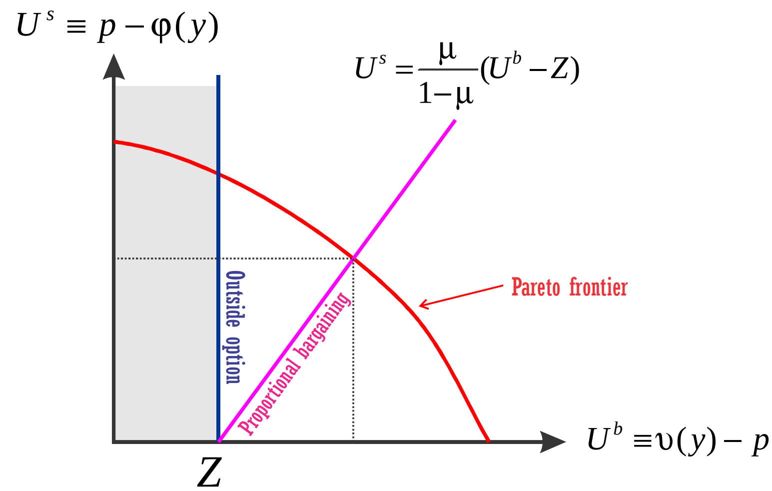

Figure 4: Bargaining in the goods market

In order to establish the connection between Z and firms' market power, we define the markup associated with a transaction as

$$MKUP \equiv \frac{p - \varphi(y)}{\varphi(y)} = \frac{\mu\,[\upsilon(y) - \varphi(y) - Z]}{\varphi(y)}. \tag{7}$$

The markup is the difference, in percentage terms, between the payment and the variable production cost.[12] For given y, the markup increases with μ but decreases with Z. Moreover, when $y = y\,(a,Z)$, the markup tends to 0 as Z tends to its upper bound, $\upsilon(y^*) - \varphi(y^*)$, irrespective of μ.

We define the match surpluses in monetary and credit matches as

$$S^m(a,Z) \equiv \upsilon[y\,(a,Z)] - \varphi\,[y\,(a,Z)] - Z \tag{8}$$

$$S^d(Z) \equiv \upsilon(y^*) - \varphi\,(y^*) - Z. \tag{9}$$

The surplus in monetary matches given by (8) is increasing in a, and decreasing in Z. The surplus in credit matches given by (9) is decreasing in Z.

We now turn to the value functions of the consumer. The HJB equation for $V^b \equiv V^b(0)$ in a steady state (i.e. $\dot{V}^b = 0$) is

$$\rho V^b = \max_{a \geq 0} \left\{ -ia + \tau + \alpha(1-\mu)\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] - \gamma Z \right\}, \tag{10}$$

where $i \equiv \rho + \pi$ can be interpreted, by virtue of the Fisher equation, as the nominal interest rate on an illiquid bond. An active buyer incurs the cost of holding real balances, ia, receives a lump-sum transfer, τ,

[12] Our measure of the markup is the average price of producing y goods, p/y, over average costs, $\varphi(y)/y$. This is related but different from defining the markup as price over marginal cost, $\varphi'(y)$, and more directly comparable to measures of gross sales margins that we use in the calibration.

enters a match with a firm at Poisson rate α, and receives $1-\mu$ share of the trade surplus. With probability χ^m, the match is monetary and the surplus is S^m. With complement probability, χ^d, the consumer can pay with the numéraire and the surplus is S^d. According to the last term on the right side of (10), if the preference shock is reversed and the consumer no longer wants to consume, at Poisson rate γ, he incurs a lifetime utility loss of $Z \equiv V^b - W^b$.

From the right side of (10), the optimal real balances are given by

$$a \in \arg\max_{\hat{a}\geq 0} \left\{-i\hat{a} + \alpha(1-\mu)\chi^m S^m(\hat{a}, Z)\right\}. \tag{11}$$

They maximize the expected consumer surplus in monetary matches net of the cost of holding real balances. From the first-order condition:

$$a = (1-\mu)\varphi(y) + \mu v(y) - \mu Z \;\; \text{where} \;\; \frac{\alpha(1-\mu)\chi^m\left[v'(y) - \varphi'(y)\right]}{\mu v'(y) + (1-\mu)\varphi'(y)} = i, \tag{12}$$

if $-ia + \alpha(1-\mu)\chi^m S^m(a, Z) \geq 0$. Otherwise, $a = 0$. From (12), the output in monetary matches does not depend on the value of the consumers' outside options but the payment does. As Z increases, consumers purchase the same amount of goods but reduce their payment.

The HJB equation for the value function of an idle buyer, W^b, is:

$$\rho W^b = \tau + \lambda Z. \tag{13}$$

The idle buyer receives a preference shock with Poisson arrival rate λ, in which case he becomes active and enjoys a lifetime utility gain of $Z \equiv V^b - W^b$.

Substituting (13) from (10), the outside option of the consumer, Z, solves:

$$(\rho + \lambda + \gamma) Z = \max_{a\geq 0} \left\{-ia + \alpha(1-\mu)\left[\chi^m S^m(a, Z) + \chi^d S^d(Z)\right]\right\}. \tag{14}$$

The left side of (14) can be interpreted as the opportunity cost of consumer search – the effective discount rate multiplied by the value of consumers' outside options – while the right side is the expected return from the search activity. The effective discount rate is composed of the rate of time preference, ρ, the rate at which idle consumers become active, and the rate at which active consumers become idle. The right side is decreasing in Z, is positive when $Z = 0$ provided that $\chi^d > 0$ and it approaches 0 as $Z \to v(y^*) - \varphi(y^*)$. Hence, as shown in Figure 5, there is a unique $Z \in (0, v(y^*) - \varphi(y^*))$ solution to (14). As the nominal interest rate increases, the curve representing the right side of (14) moves downward and Z decreases. If the cost of holding real balances increases, searching for better opportunities in the goods market becomes more costly, and the value of consumers' outside options decreases.

Finally, we compute the measure of idle and active buyers. The steady-state measure of active consumers solves $(\gamma + \alpha)\omega_1 = \lambda(\omega - \omega_1)$, i.e.,

$$\omega_1 = \frac{\lambda}{\gamma + \alpha(q) + \lambda}\omega. \tag{15}$$

The measure of active consumers decreases with tightness in the goods market, q.

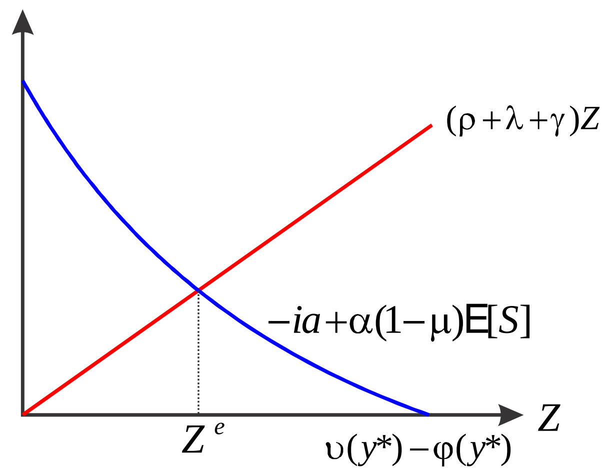

Figure 5: Determination of the value of consumers' outside options

3.2 Labor market

The HJB equation for the value of a match composed of a worker and a firm is J that solves:

$$(\rho+\delta)\,J = \alpha^s \mu \left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x - \rho U, \tag{16}$$

where δ is the job destruction rate, $\alpha^s \equiv \alpha(q)/q$ is the arrival rate of consumers, and U is the value function of an unemployed worker. The firm receives a share μ of the expected surplus generated by the trade. The second term on the right side is the flow of numéraire good produced by the worker-firm pair. The last term is the reservation wage of an unemployed worker. It solves

$$\rho U = b + f(\theta)\beta J, \tag{17}$$

where b is the income when unemployed, β is the worker's bargaining share, and $f(\theta)$ is the job finding rate.

The HJB equation for an employed worker is:

$$\rho E = w - \delta \beta J. \tag{18}$$

On the right side, the worker receives a wage, w, and, at rate δ, the match with the firm is destroyed, which generates a capital loss equal to βJ. We subtract ρU from both sides and use that $E - U = \beta J$ to obtain:

$$w = (\rho+\delta)\,\beta J + \rho U. \tag{19}$$

The first term on the right side is the fraction β of the value of the match that the worker can capture. The last term correspond to the reservation wage of the worker, which is the flow value of being unemployed. We substitute $(\rho+\delta)\,\beta J$ by its expression given by (16) to simplify the wage equation as follows:

$$w = \beta \left\{\alpha^s \mu \left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x\right\} + (1-\beta)\rho U. \tag{20}$$

The free-entry of firms in the labor market implies

$$k = \frac{f(\theta)}{\theta}(1-\beta)J. \tag{21}$$

The flow cost of posting a vacancy is equal to the vacancy filling rate multiplied by the value of a filled job, $(1-\beta)J$. Substituting $f(\theta)$ from (21) into (17), $\rho U = b + \beta k\theta/(1-\beta)$. We substitute this expression into (16) and replace J from (21) to obtain the equilibrium condition for market tightness at the steady state:

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k\theta. \tag{22}$$

There is a unique θ solution to (22). Relative to the Mortensen-Pissarides textbook model, the novelty is the term $\alpha^s\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right]$ that represents the sales in the frictional goods market. It is increasing in a and decreasing in q and Z. Firms are more profitable when consumers have a higher payment capacity but they are less profitable when consumers have better outside options.

We use $\rho U = b + \beta k\theta/(1-\beta)$ to rewrite the wage in (20) as

$$w = \beta\left\{\alpha^s\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x\right\} + (1-\beta)b + \beta k\theta. \tag{23}$$

Relative to both the MP and BMW models, the value of the consumers' outside option in the goods market affects the wage. As Z increases, the market power of the firm in the goods market decreases, which reduces wages. It will also have implications for how inflation affects wages. As a preview of the comparative statics, a higher i reduces consumers' payment capacity, a, which tends to lower firms' profits and wages. But a higher i also reduces Z, which affects firms' profits in the opposite direction.

We define the wage markdown in a symmetric fashion as the markup, that is:

$$MKDOWN \equiv \frac{\hat{x} - w}{\hat{x}}$$

where $\hat{x}$ is the net expected revenue generated by a worker, $\hat{x} = \mathbb{E}[p - \varphi(y)] + x$. It is also equal to

$$\hat{x} = \alpha^s\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x. \tag{24}$$

which allows us to rewrite the wage markdown as

$$MKDOWN \equiv \frac{(1-\beta)\left\{\alpha^s\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k\theta}{\alpha^s\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x} \tag{25}$$

We can see that the markdown depends on bargaining powers in labor and goods markets as well as consumers' outside options in the goods market.

The measures of employed and unemployed workers, at the steady state, are:

$$n = \frac{f(\theta)}{\delta + f(\theta)}, \quad u = \frac{\delta}{\delta + f(\theta)}. \tag{26}$$

Equation (26), which is analogous to the Beveridge curve, gives a positive relationship between employment and market tightness.

Finally, our model will have implications for the sensitity of stocks and market capitalization to changes in inflation or the nominal interest. We denote the value of a firm as $\Pi \equiv (1-\beta)J$. From (21),

$$\Pi = \frac{k\theta}{f(\theta)}. \tag{27}$$

So there is a one-to-one relationship between the value of a firm and labor market tightness. Market capitalization is $K \equiv n\Pi$. It is equal to

$$K = \frac{k\theta}{\delta + f(\theta)}. \tag{28}$$

There is a one-to-one relationship between K and θ.

3.3 Definition of equilibrium

In order to simplify the definition of an equilibrium, we derive a relationship between the market tightness of the goods market, q, and the market tightness of the labor market, θ. From the definition $q\omega_1 \equiv n$, (15), and (26), the relationship between q and θ is given by:

$$\frac{\lambda\omega q}{\gamma + \lambda + \alpha(q)} = \frac{f(\theta)}{\delta + f(\theta)}. \tag{29}$$

The implicit solution, $q = Q(\theta)$, from (29) is an increasing function of θ with $Q(0) = 0$ and $Q'(\theta) > 0$. Using $Q(\theta)$, we rewrite the matching rates in the goods market as

$$\alpha^s(\theta) \equiv \frac{\alpha\left[Q(\theta)\right]}{Q(\theta)}, \quad \alpha^b(\theta) \equiv \alpha\left[Q(\theta)\right].$$

The rate at which firms sell their output decreases with θ. Indeed, as θ increases, the number of active firms increases, which raises tightness in the goods market and reduces firms' matching rate with consumers.

An equilibrium can be reduced to a 4-tuple, (θ, a, Z, w), solution to:

$$\begin{aligned}
(\rho + \delta)\frac{k\theta}{f(\theta)} &= (1-\beta)\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a, Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k\theta && (30)\\
a &\in \arg\max_{\hat{a}\geq 0}\left\{-i\hat{a} + \alpha^b(\theta)(1-\mu)\chi^m S^m(\hat{a}, Z)\right\} && (31)\\
(\rho + \lambda + \gamma) Z &= -ia + \alpha^b(\theta)(1-\mu)\left[\chi^m S^m(a, Z) + \chi^d S^d(Z)\right] && (32)\\
w &= \beta\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a, Z) + \chi^d S^d(Z)\right] + x\right\} + (1-\beta)b + \beta k\theta. && (33)
\end{aligned}$$

Proposition 1 *If $\chi^d > 0$, then there exists an active steady-state equilibrium.*

The logic for the existence claim goes as follows. From (32) we express the value of the consumer's outside option, Z, as a function of labor market tightness, θ. It is an increasing function: as labor market tightness increases, more firms are created, and hence the measure of producers per consumer in the goods market increases, which improves consumers' outside options. From (31), we express consumers' real balances, a, as a function of θ. We obtain an increasing function, because as θ increases, the average time for an active consumer to find a producer decreases, which in turn reduces the average holding cost of real balances. We then substitute $Z(\theta)$ and $a(\theta)$ into (30) to obtain a single equilibrium condition in θ. We use the continuity properties of this equation and its values at $\theta = 0$ and $\theta = +\infty$ to establish that a positive solution exists. A sufficient condition for the existence of an active equilibrium is that a positive measure of transactions is conducted with credit, $\chi^d > 0$. Indeed, if θ becomes very small, the expected revenue of firms becomes very large because they can serve a large measure of consumers per firm. Hence, irrespective of b or x, there is always a sufficiently low θ so that firms' profits are positive and entry is profitable. Before we get to our main result, we consider some special cases.

3.4 Pure credit economy

Suppose that all meetings in the decentralized goods market are credit meetings, $\chi^d = 1$.[13] An equilibrium can then be reduced to a triple (θ, Z, w) solution to

$$\begin{aligned}
(\rho+\delta)\frac{k\theta}{f(\theta)} &= (1-\beta)\left\{\alpha^s(\theta)\mu\left[\upsilon(y^*)-\varphi(y^*)-Z\right]+x-b\right\}-\beta k\theta && (34)\\
(\rho+\lambda+\gamma)Z &= \alpha^b(\theta)(1-\mu)\left[\upsilon(y^*)-\varphi(y^*)-Z\right] && (35)\\
w &= \beta\left\{\alpha^s(\theta)\mu\left[\upsilon(y^*)-\varphi(y^*)-Z\right]+x\right\}+(1-\beta)b+\beta k\theta. && (36)
\end{aligned}$$

If firms have no bargaining power in the goods market, $\mu = 0$, θ is determined from (34), which is identical to the equilibrium condition of the MP model. In particular, it is independent of Z. The other polar case is when firms have all the bargaining power in the goods market, $\mu = 1$. From (35), $Z = 0$ and from (34) θ is uniquely determined by

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\left[\upsilon(y^*)-\varphi(y^*)\right]+x-b\right\}-\beta k\theta.$$

The revenue of the firm depends negatively on θ through a competition/congestion effect in the goods market. As a result, the frictions in the goods market, as captured by α^s, have an impact on the labor market, i.e., as α^s decreases, θ goes down. As frictions vanish, $\alpha^s, \alpha^b \to +\infty$, and $Z \to \upsilon(y^*) - \varphi(y^*)$.

Next, consider the interior case where the firm's bargaining power is $\mu \in (0, 1)$. From (34), θ is a decreasing function of Z because as the value of consumers' outside option increases, the profits of the firm decrease. From (35), we can solve for Z in closed form and obtain:

$$Z = \frac{\alpha^b(\theta)(1-\mu)}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}\left[\upsilon(y^*)-\varphi(y^*)\right]. \tag{37}$$

The value of consumers' outside option increases with θ and decreases with μ. We can substitute the expression for Z into (34) to reduce an equilibrium to a single equation in θ:

$$(\rho+\delta)\frac{k\theta}{f(\theta)}+\beta k\theta = (1-\beta)\left\{\frac{\alpha^s(\theta)\mu(\rho+\lambda+\gamma)}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}\left[\upsilon(y^*)-\varphi(y^*)\right]+x-b\right\}. \tag{38}$$

It is easy to check that in a pure credit economy an equilibrium exists and is unique. The following proposition shows how the determinants of market power in the goods market, λ and γ, affect labor market outcomes.

Proposition 2 ***(Unemployment and consumer search in a pure credit economy)*** *Suppose $\chi^d = 1$ and $\mu \in (0, 1)$. As λ or γ increases, the value of consumers' outside options (Z) decreases, labor market tightness (θ) increases, wages (w) increase, and unemployment (u) decreases.*

[13] For a review of models with frictional goods and labor markets where trades in the goods market are not subject to liquidity constraints, see Petrosky-Nadeau and Wasmer (2017).

If λ increases, i.e., consumers do not stay idle long, or γ increases, i.e., the desire to consume vanishes quickly, then the opportunity cost of accepting a trade decreases, which makes the search for an alternative producer less profitable and raises producers' market power. As a result, market tightness in the labor market increases, $\partial\theta/\partial\lambda > 0$ and $\partial\theta/\partial\gamma > 0$, wages increase, $\partial w/\partial\lambda > 0$ and $\partial w/\partial\gamma > 0$, and unemployment decreases, $\partial u/\partial\lambda < 0$ and $\partial u/\partial\gamma < 0$.

3.5 Pure currency economy without consumer search

The economy in BMW is a pure currency economy, $\chi^m = 1$, in which there is no opportunity cost for the consumer to complete a trade, $\lambda = +\infty$. From (15), $\omega_1 = \omega$, all buyers are active at all points in time. From (29) market tightness in the goods market is

$$q = \frac{f(\theta)}{\omega\left[\delta + f(\theta)\right]}. \tag{39}$$

From (32) $Z = 0$ and an equilibrium can be reduced to a pair (θ, y) solution to:

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\mu\left[\upsilon(y) - \varphi(y)\right] + x - b\right\} - \beta k\theta \tag{40}$$

$$\frac{\upsilon'(y) - \varphi'(y)}{\mu\upsilon'(y) + (1-\mu)\varphi'(y)} = \frac{i}{(1-\mu)\alpha^b(\theta)}. \tag{41}$$

Given (θ, y), the wage is given by

$$w = \beta\left\{\alpha^s(\theta)\mu\left[\upsilon(y) - \varphi(y)\right] + x\right\} + (1-\beta)b + \beta k\theta. \tag{42}$$

Suppose $x < b$. Equation (40) gives a positive relationship between θ and y with $\theta = 0$ when $y = 0$ and $\theta = \bar{\theta} > 0$ when $y = y^*$. Equation (41) gives a positive relationship between y and θ with $y = 0$ if $\theta < \underline{\theta}$ where $\underline{\theta}$ solves $\alpha^b(\underline{\theta}) = [i\mu/(1-\mu)]$ and $y \to y^*$ as $\theta \to +\infty$. There always exists a non-active equilibrium with $y = \theta = 0$ and, generically, if an active equilibrium exists, then the number of active equilibria is even.[14] In order to describe the effect of anticipated inflation on unemployment, we focus on the equilibrium with the highest labor market tightness.

Proposition 3 ***(Long-run Phillips curve in the absence of consumer search)*** *Assume $x < b$ and $\lambda = +\infty$ and focus on the equilibrium with the highest market tightness. An increase in π leads to a decrease in market tightness (θ), a decrease in wages (w), and an increase in unemployment (u).*

We only provide a graphical proof (see Figure 6) as the result is similar to the one in BMW. At the high equilibrium, the MD curve representing (41) in the (θ, y) space intersects the JC curve representing (40) by above. As i increases, the MD curve (41) shifts downward. Hence, y and θ decrease. So the model predicts a positive relationship between unemployment and inflation in the long run, i.e., the long-run Phillips curve is upward sloping.

[14] These results mirror those of Rocheteau and Wright (2005) who describe a pure currency economy with free entry of sellers.

If we reintroduce credit trades, $\chi^d \in (0,1)$, then an active equilibrium always exists even though the equilibrium might not be monetary. To see this, note that (40) with credit trades becomes

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\chi^m\mu\left[\upsilon(y)-\varphi(y)\right]+\alpha^s(\theta)\chi^d\mu\left[\upsilon(y^*)-\varphi(y^*)\right]+x-b\right\}-\beta k\theta.$$

As θ goes to 0, $\alpha^s(\theta)$ goes to $+\infty$, so that firm's revenue is larger than b, which guarantees that an active equilibrium always exists. Moreover, if $\chi^d \to 1$, then the JC curve becomes vertical. Also, the MD curve becomes flat, which implies the equilibrium is unique. But at the limit $\chi^d \to 1$, money is not valued.

When $y=0$ (real balances $a=0$), the JC curve intersect the horizontal axis at a positive θ. There are still credit trades so that firms are willing to participate in the market.

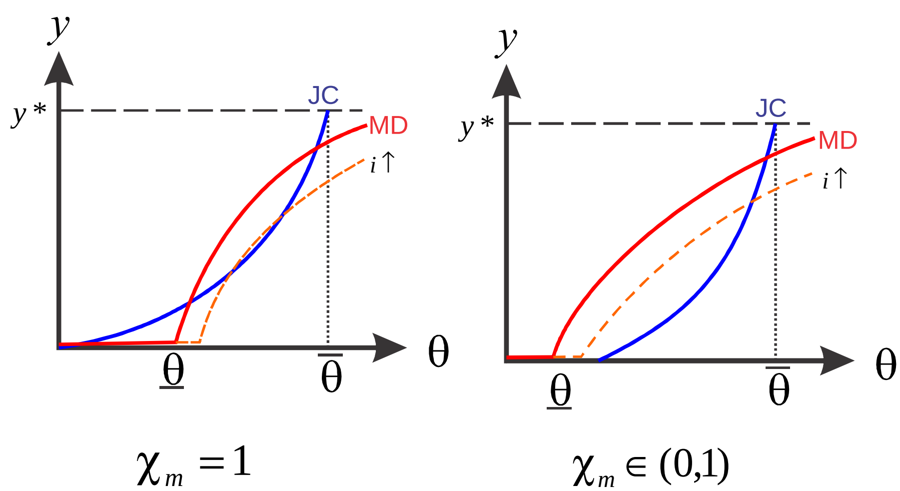

Figure 6: Left panel: Pure monetary economy. Right panel: Economy with money and credit.

3.6 The U-shaped long-run Phillips curve

We now turn to an economy with both money and credit, $\chi^m \in (0,1)$. We study increases in the inflation rate of different magnitudes: (1) a small increase in the neighborhood of the Friedman rule, $i=0$; (2) a large increase from $\pi=-\rho$ to $\pi=+\infty$ that reduces the value of money to zero.

Proposition 4 ***(The non-monotone long-run Phillips curve)*** *Suppose* $\chi^m \in (0,1)$.

1. *A small increase in* i*, starting from* $i=0^+$*, leads to: an increase in labor market tightness (*θ*); a decrease in the unemployment rate (*u*); an increase in wages (*w*); and an increase in stock prices (*Π*) and market capitalization (*K*).*

2. *A large increase in* i *from* $i=0^+$ *to* $i=+\infty$ *leads to: a decrease in labor market tightness (*θ*); an increase of the unemployment rate (*u*); a decrease in wages (*w*); and a decrease in stock prices (*Π*) and market capitalization (*K*).*

An increase in π has two effects on the labor market. First, it reduces consumers' real balances, which tightens liquidity constraints and reduces the amount of goods firms can sell to consumers. This effect tends to reduce market tightness, raise unemployment and reduce wages. There is a second effect according to which an increase in i raises the cost for consumers to search for an alternative producer since they have to carry real money balances until they find a new opportunity to trade. When i is close to 0, y is close to y^*, and the first effect - *the real balance effect of inflation* - is negligible. Only the second effect on consumers' outside options - *the market power effect of inflation* - matters, i.e., inflation raises firms' market power by making it more costly for consumers to use their option to search.

We illustrate the two effects graphically in Figure 7 that represents the outcome of the bargaining in pairwise meetings in the goods market. As π increases, a decreases, and the Pareto frontier of the bargaining problem shifts downward. It reduces the profits of the firm, U^s. But as π increases, Z decreases, thereby shifting to the left the origine of the rent sharing line. It increases U^s. If $\pi = -\rho$, the Pareto frontier is linear when it intersects the rent sharing rule, and that linear part does not shift as a decreases. Hence, only the shift of Z matters for the allocations in pairwise meetings.

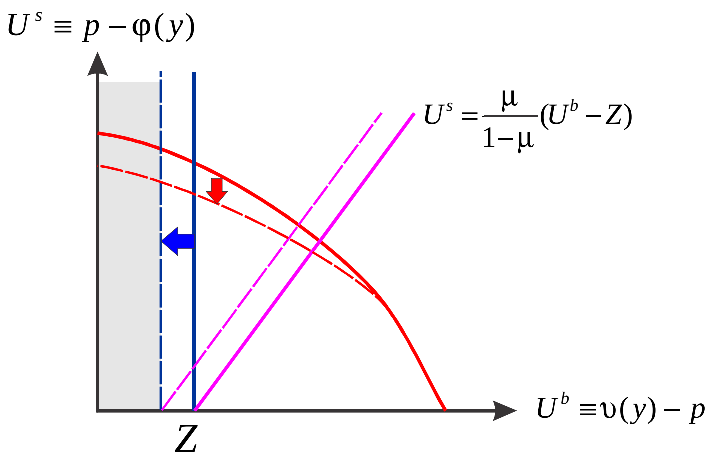

Figure 7: Effects of inflation on the bargaining outcome

When i is sufficiently large, the two effects described above are first order. We can show that when i is so high that consumers do not hold real balances, i.e., all trades are conducted with credit, then firms are worse-off compared to the equilibrium at the Friedman, i.e., θ is lower when $i = +\infty$ relative to $i = 0$. This guarantees that the relation between θ and i is nonmonotone: it is first increasing and it eventually decreases for sufficiently large values of i.

Since real wages increase with firms' profits (we show in the proof of Proposition 4 that w and θ comove as i changes), the relationship between w and i is also nonmonotone. A small, anticipated inflation pushes real wages up while a large inflation rate depresses real wages. Hence, the unemployment minimizing level of i maximizes real wages.

Finally, we discuss how the market power effect of inflation depends on λ. As $\lambda \to +\infty$, the market power effect always vanishes, but the magnitude of this effect can be nonmonotone in λ for $\lambda < +\infty$. For intuition, consider the special case where agents are infinitely patient, $\rho = 0$, and the job separation rate vanishes, $\delta \to 0$. As δ vanishes, $n \to 1$, $u \to 0$, and $q \to 1/\omega$, but θ is still endogenous and determined by the free entry condition. At this limit, by (30), the market tightness, θ, is linear in the surplus generated by the firm-work pair, which in turn is linear in Z. Therefore,

$$\frac{\partial \theta}{\partial i}|_{i=0} = \frac{(1-\beta)\alpha^s(1/\omega)\mu}{\beta k}\left(-\frac{\partial Z}{\partial i}|_{i=0}\right). \tag{43}$$

Therefore, the change in θ is determined solely by the market power effect and is proportional to the change in the outside option. The impact of inflation on the outside option, Z, can be derived using (32), i.e.

$$\frac{\partial Z}{\partial i}|_{i=0} = \frac{-a}{\lambda + \gamma + \alpha^b(1/\omega)(1-\mu)}. \tag{44}$$

According to (44), $\partial Z/\partial i$ is proportional to $-a$. It is because an infinitesimal increase in i reduces Z via raising the cost of money holding, by an amount that is equal to a. As λ rises, a rises because the outside option, Z, falls and thus buyers must carry more money for payment. Also as λ rises, the denominator in (44) rises because Z is smaller and becomes less sensitive to changes in i. Altogether, $\partial\theta/\partial i$ and $\partial Z/\partial i$ can be non-monotone in λ — in the appendix we show that $\partial\theta/\partial i$ is hump-shaped in λ, provided that μ is large, and is decreasing in λ otherwise. This example suggests that the relationship between the market power effect of inflation and λ is subtle and is nonmonotone in general.

3.7 Frictionless limits

By introducing consumer search and by formalizing consumers' outside options, we generalized the model of decentralized markets of Rubinstein and Wolinsky (1985) to an economy with two frictional markets and liquidity constraints. We now show that the results of Gale (1986a,b, 1987) regarding frictionless limits also apply to our economy. More precisely, we characterize the equilibrium outcomes when search frictions in the goods market vanish, when search frictions in labor market vanish, and when frictions vanish in both markets simultaneously. We interpret vanishing frictions in terms of matching technologies that become infinitely efficient at pairing buyers and sellers.

Before we state our results for the generic case, $\lambda \in (0, +\infty)$, it is useful to consider as a reference point the BMW economy where consumers are always active, $\lambda = +\infty$. We write the matching function in the goods market as $\alpha(q) = A\bar{\alpha}(q)$, where $A > 0$, and we take the limit as $A \to +\infty$. From (41), for all $\theta > 0$, as $A \to +\infty$, $y \to y^*$. Consumers do not economize on their real balances because they know they can find a seller almost instantly. Hence, the surpluses in pairwise meetings, $v(y^*) - \varphi(y^*)$, are maximum, i.e., there are rents to be bargained over even when frictions vanish. We represent the outcome of the bargaining in the left panel of Figure 8. The Pareto frontier is the fartherst away from the origin, i.e., rents are maximum, and the rent sharing line intersects the Pareto frontier in its linear part, i.e., $y = y^*$. Following Makowski and Ostroy (2001), we define perfect competition as a situation where rents are zero. By that criterion, the equilibrium outcome of BMW does not converge to a perfect competition outcome as frictions vanish. Another way to establish the lack of convergence to perfect competition is by computing the markup at the limit, which is positive and equal to $MKUP = \mu\left[\upsilon(y^*)/\varphi(y^*) - 1\right] > 0$. From (39),

$$\alpha^s(\theta) = A\frac{\alpha\left[q(\theta)\right]}{q(\theta)} \text{ where } q(\theta) = \frac{f(\theta)}{\omega\left[\delta + f(\theta)\right]}.$$

It follows from (40) that $\theta \to +\infty$ as $A \to +\infty$. Labor market tightness becomes unbounded because consumers are always active and can find sellers from whom to buy at infinite speed. As a result, the consumer base of each firm grows unbounded.

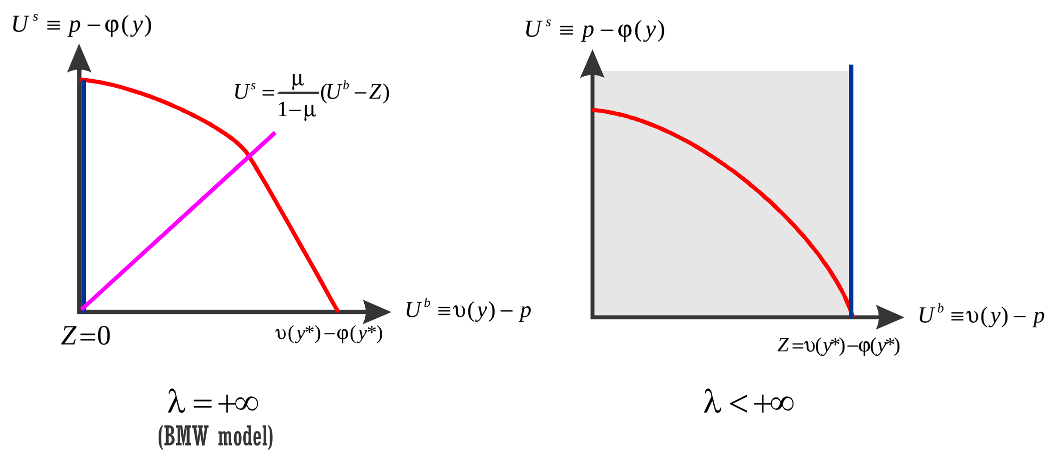

Figure 8: Outcome of the bargaining when the goods market becomes frictionless ($A \to +\infty$). Left panel: BMW economy ($\lambda = +\infty$). Right panel: Economy with consumers' outside options ($\lambda < +\infty$).

We now contrast these results to the ones in an economy where consumers have meaningful outside options in both the goods and labor markets.

Proposition 5 ***(Frictionless limits)*** *Suppose $\lambda \in (0, +\infty)$, $\alpha(q) = A\bar{\alpha}(q)$ and $f(\theta) \equiv B\overline{f}(\theta)$.*

1. ***Limit as the goods market becomes frictionless.*** *Consider the limit as $A \to +\infty$. Then, $Z \to \upsilon(y^*) - \varphi(y^*)$ and $MKUP \to 0$. If $x > b$, then $q \to +\infty$, and $\theta \to \theta^g_\infty$ where $\theta^g_\infty > 0$ is the unique solution to*

$$(\rho + \delta)\frac{k\theta^g_\infty}{f(\theta^g_\infty)} + \beta k\theta^g_\infty = (1-\beta)(x - b). \tag{45}$$

   If $x \le b$, then $\theta \to 0$.

2. ***Limit as the labor market becomes frictionless.*** *Consider the limit as $B \to +\infty$. Then, $(\theta, a, Z) \to \left(\theta^\ell_\infty, a^\ell_\infty, Z^\ell_\infty\right)$ where $\left(\theta^\ell_\infty, a^\ell_\infty, Z^\ell_\infty\right)$ is the solution of*

$$\beta k\theta^\ell_\infty = (1-\beta)\left\{\alpha^s(\theta^\ell_\infty)\mu\left[\chi^m S^m(a^\ell_\infty, Z^\ell_\infty) + \chi^d S^d(Z^\ell_\infty)\right] + x - b\right\} \tag{46}$$

$$a^\ell_\infty \in \arg\max_{a\ge 0}\left\{-ia + \alpha^b(\theta^\ell_\infty)(1-\mu)\chi^m S^m(a, Z^\ell_\infty)\right\} \tag{47}$$

$$(\rho + \lambda + \gamma) Z^\ell_\infty = \max_{a\ge 0}\left\{-ia + \alpha^b(\theta^\ell_\infty)(1-\mu)\left[\chi^m S^m(a, Z^\ell_\infty) + \chi^d S^d(Z^\ell_\infty)\right]\right\}. \tag{48}$$

Moreover, $w \to \hat{x}^{\ell}_{\infty}$, where

$$\hat{x}^{\ell}_{\infty} = \alpha^s(\theta^{\ell}_{\infty})\mu\left[\chi^m S^m(a^{\ell}_{\infty}, Z^{\ell}_{\infty}) + \chi^d S^d(Z^{\ell}_{\infty})\right] + x, \tag{49}$$

and $MKDOWN \to 0$.

3. ***Limit as both markets become frictionless**. Consider the limit as $A \to +\infty$ and $B \to +\infty$. Then, $Z \to \upsilon(y^*) - \varphi(y^*)$ and $w \to x$. If $x > b$, then $\theta \to \theta_{\infty}$ where*

$$\theta_{\infty} = \frac{(1-\beta)(x-b)}{\beta k}. \tag{50}$$

If $x \leq b$, then $\theta \to 0$.

As the frictions in the goods market vanish (Part 1 of Proposition 5), the value of consumers' outside options exhausts the gains from trade, $Z \to \upsilon(y^*) - \varphi(y^*)$. As a result, rents in pairwise meetings, $S^m(a, Z)$ and $S^d(Z)$, go to zero. This finding is illustrated in the right panel of Figure 8 that shows that the outcome of the bargaining is pinned down by the consumer's outside option. It is a definition of perfect competition in the goods market (e.g., Makowski and Ostroy, 2001). The quantities are efficient, $y \to y^*$, and the payment just covers the production cost, $p \to \varphi(y^*)$, i.e., the average markup goes to zero. Provided $x > b$, market tightness is positive and bounded at the limit but it is independent of monetary policy, i.e., the long-run Phillips curve becomes vertical.

The results above, which hold for all $\lambda < +\infty$, are in sharp contrast with the ones of the BMW economy described earlier. While in the BMW economy rents and markups remain positive at the frictionless limit, they all disappear when consumers incur an opportunity cost from trading. Moreover, the consumer base remains bounded when $\lambda < +\infty$ because as consumers trade faster, the measure of active consumers shrinks.

One implication of these results is that the order according to which we take the limits, $\lambda \to +\infty$ and $A \to +\infty$, matters. If we take the limit, $\lambda \to +\infty$, first, we obtain the BMW economy. As $A \to +\infty$, the economy grows unbounded but remains imperfectly competitive. In contrast, if we take the limit, $A \to +\infty$, first then the equilibrium outcome converges to a perfect-competition outcome that is independent of λ. So, as λ goes to $+\infty$, allocations remain competitive and labor market tightness stays bounded.

Our results can be related to Lauermann (2013) who provides a necessary and sufficient condition for decentralized market equilibria under quasi-linear preferences to converge to a Walrasian outcome as the rate of time preference approaches zero. He shows that the convergence occurs if and only if the economy features *competitive pressure* in the sense that buyers can secure a positive trade surplus in at least some meetings. In our model, competitive pressures only exist if $Z > 0$, which requires both that consumers obtain a positive surplus in meetings, $\mu < 1$, and they have outside options, $\lambda < +\infty$. As A increases, consumers can realize their outside options faster, which exacerbates the competitive pressures. In contrast, if $Z = 0$, there are no competitive pressure irrespective of A.

As the frictions in the labor market vanish (Part 2 of Proposition 5), market tightness approaches a finite and positive limit defined in (46). The job finding rate, $B\overline{f}(\theta_\infty^\ell)$, and the vacancy filling rate, $B\overline{f}(\theta_\infty^\ell)/\theta_\infty^\ell$, go to infinity. Hence, unemployment vanishes asymptotically and the Phillips curve becomes vertical at $u = 0$. The real wage approaches the average productivity of a worker so that the markdown tends to 0.

Finally, in Part 3 of Proposition 5, we describe the limit as both frictions in the labor and goods market vanish. A competitive outcome is obtained in both markets. In the goods market, the value of consumers' outside options exhausts all rents, $Z \to \upsilon(y^*) - \varphi(y^*)$, and markups tend to zero. In the labor market, the wage approaches workers' productivity, $w \to x$, i.e., markdowns go to zero. From (50), market tightness is bounded at the limit and it decreases with workers' bargaining power.

3.8 Welfare

In the presence of search externalities in decentralized goods and labor markets, the monetary policy that minimizes the unemployment rate is not necessarily the one that maximizes social welfare. As shown in Proposition 4, an increase in π above the Friedman rule reduces unemployment by raising firms' market power. If firms' market power is at a level that generates efficient entry, then a deviation from the Friedman rule might be welfare-reducing. We explore this conjecture by first characterizing the constrained-efficient allocations, and by comparing them to the equilibrium allocation.

The planner's problem is:

$$\max \int_0^{+\infty} e^{-\rho t} \varpi_t dt \tag{51}$$

subject to

$$\begin{aligned} \dot{\omega}_{1,t} &= \lambda(\omega - \omega_{1,t}) - \left[\alpha\left(\frac{n_t}{\omega_{1,t}}\right) + \gamma\right]\omega_{1,t} &\quad (52) \\ \dot{n}_t &= f(\theta_t)(1 - n_t) - \delta n_t, &\quad (53) \end{aligned}$$

where n_0 and $\omega_{1,0}$ are given and the instantaneous social welfare, ϖ_t, is given by

$$\begin{aligned} \varpi_t &= \omega_{1,t}\alpha\left(\frac{n_t}{\omega_{1,t}}\right)\left\{\chi^m\left[\upsilon(y_{m,t}) - \varphi(y_{m,t})\right] + \chi^d\left[y_{d,t} - \varphi(y_{d,t})\right]\right\} &\quad (54) \\ &\quad + n_t(x - b) + b - (1 - n_t)\theta_t k. \end{aligned}$$

The state variables are $\omega_{1,t}$ and n_t. The control variables are $y_{m,t}$, $y_{d,t}$, and θ_t. According to (54), welfare is the sum of the trade surpluses in pairwise meetings plus the production of the numeraire good by employed workers, x, and unemployed workers, b, net of the vacancy posting costs. (Here, b is not treated as a transfer).

We denote ξ_t the current-value co-state variable associated with n_t, and ζ_t the current-value co-state associated with $\omega_{1,t}$. So ξ is the shadow value of a match in the labor market while ζ is the shadow value of an active buyer. The social surplus from a trade in a pairwise meeting is $\upsilon(y_{s,t}) - \varphi(y_{s,t}) - \zeta_t$ where $s \in \{m, d\}$. The first-order conditions with respect to $y_{m,t}$ and $y_{d,t}$ give

$$y_{m,t} = y_{d,t} = y^* \text{ for all } t.$$

The first-order condition with respect to θ_t gives

$$k = \xi_t f'(\theta_t). \tag{55}$$

The planner equalizes the cost of posting a vacancy with its marginal benefit as measured by the product of the shadow value of an employed worker, ξ_t, and the marginal increase in the job finding rate, $f'(\theta_t)$. As shown in the proof of Proposition 6 below, at a stationary solution to the planner's problem, ζ and θ solve

$$(\rho+\lambda+\gamma)\,\zeta = \alpha\,(q)\,[1-\epsilon_\alpha\,(q)]\,[\upsilon(y^*)-\varphi(y^*)-\zeta] \tag{56}$$

$$(\rho+\delta)\,\frac{k\theta}{f(\theta)} = \epsilon_f(\theta)\left\{\frac{\alpha\,(q)}{q}\epsilon_\alpha(q)\,[\upsilon(y^*)-\varphi(y^*)-\zeta]+x-b\right\}-[1-\epsilon_f(\theta)]\,k\theta, \tag{57}$$

where $\epsilon_\alpha \equiv \alpha'\,(q)\,q/\alpha(q)$ and $\epsilon_f \equiv f'(\theta)\theta/f(\theta)$ denote the elasticities of the matching rates in the goods and labor markets. Equation (56) is the analog to (35) that determines Z in equilibrium. Equation (57) is the analog to (34) that determines θ in equilibrium. In the proof of Proposition 6 we show that there is a unique $(\theta^s, q^s, n^s, \omega_1^s)$ solution to (26), (29), and (56)-(57). It is the solution to the planner's optimal control problem if $n_0 = n^s$ and $\omega_{1,0} = \omega_1^s$.

Proposition 6 ***(Constrained efficiency)***. *A decentralized equilibrium implements the constrained-efficient allocation if* $i_t = 0$,

$$\mu = \epsilon_\alpha\,(q^s) \tag{58}$$

$$1-\beta = \epsilon_f(\theta^s), \tag{59}$$

and the initial value n_0 *and* $\omega_{1,0}$ *equal the steady state value* n^s *and* ω_1^s.

A decentralized equilibrium maximizes social welfare when monetary policy implements the Friedman rule, $i_t = 0$, and the Hosios conditions in both the labor and goods markets hold.[15] It should be noted that imposing the Hosios condition in each market is not necessary to implement the constrained-efficient allocation. Indeed, from (38), labor market tightness at the Friedman rule is the unique solution of

$$(\rho+\delta)\,\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\alpha^s(\theta)\frac{\mu\,(\rho+\lambda+\gamma)}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}\,[\upsilon(y^*)-\varphi(y^*)]+x-b\right\}-\beta k\theta. \tag{60}$$

From Proposition 6, the constrained-efficient allocation is achieved for $\beta = 1-\epsilon_f(\theta^{sp})$ and $\mu = \epsilon_\alpha\,[q(\theta^{sp})]$, where θ^{sp} is the constrained-efficient labor market tightness. It is easy to see that there are other combinations of (β,μ) that achieve the efficient level of market tightness. For instance, if β is slightly above $1-\epsilon_f(\theta^{sp})$, then a μ slightly below $\epsilon_\alpha(q^{sp})$ will implement the same θ^{sp}. This multiplicity arises because the planner can control two bargaining shares to target a single variable, θ. If we also had entry by consumers in the goods market, then this logic would not hold.

[15] This result is a generalization of Berentsen et al. (2007) to an economy where both goods and labor markets are frictional. Relatedly, Mangin and Julien (2021) derive a "generalized Hosios condition" for a static version of BMW. Petrosky-Nadeau and Wasmer (2017) obtain a similar condition in a nonmonetary economy where credit, labor, and goods markets are frictional. In Appendix D, we show that if there is competitive search in both the goods and labor markets, as in Moen (1997), then the Hosios conditions are satisfied and the Friedman rule is optimal.

3.9 Optimal monetary policy

We now study the optimality of the Friedman rule. In order to simplify the analysis, we follow Hosios (1990) and Pissarides (2000) and assume that agents are infinitely patient, i.e. $\rho \to 0$, which allows us to focus on steady-state welfare. The planner maximizes ϖ in (54) by choosing the inflation rate (or the nominal interest rate) subject to the law of motion of workers and consumers, and the free entry of firms.

Proposition 7 ***(Optimal monetary policy)***

1. *If $\lambda = +\infty$, then the Friedman rule, $\pi = -\rho$, is optimal.*

2. *If $\lambda < +\infty$, then there exists a $\bar{\beta}(\mu) \in [0, 1)$ such that $\pi > -\rho$ is optimal if and only if $\beta > \bar{\beta}(\mu)$. The cutoff, $\bar{\beta}(\mu)$, rises in μ.*

When $\lambda = +\infty$, the Friedman rule ($\pi + \rho = 0$) is optimal regardless of whether labor market tightness is inefficiently high or inefficiently low (relative to the planner's solution). If θ is too high because firms have too much bargaining power, a deviation from the Friedman rule can raise firm entry, thereby improving welfare. In addition, there is a negative effect on quantities traded, which reduces welfare. The second effect dominates so that the Friedman rule is always optimal.[16]

When $\lambda < +\infty$, a small deviation from the Friedman rule, i.e. $i > 0$ (or, equivalently, $\pi > -\rho$), has a first-order impact on firm entry by reducing the value of consumers' outside option, Z. This deviation improves social welfare if the market tightness, θ, is inefficiently low at the steady-state equilibrium, which happens when firms have too little bargaining power. The line $\bar{\beta}(\mu)$ is illustrated in the left panel of Figure 12 as an upward sloping black line. It contains the combinations of (β, μ) that achieve the efficient level of market tightness characterized in Section 3.8. Therefore, along the black line, the Friedman rule implements the constrained-efficient allocation. If (β, μ) is to the right of the black line, then the optimal policy is $i > 0$. This result highlights that the unemployment-minimizing inflation as well as the welfare-maximizing inflation can both exceed the Friedman rule level when $\lambda < +\infty$.

4 Calibration

We now carry out a quantitative analysis by calibrating our model to the US between 1955-2005.

4.1 The generalized model

We generalize the model by introducing heterogeneity across consumers in terms of λ's. Suppose there are $J \in \mathbb{N}$ categories of goods and J types of consumers. A consumer of type $j \in \{1, ..., J\}$ is specialized in the consumption of good j. The measure of consumer of type j is ω_j with $\sum_{j=1}^{J} \omega_j = \omega$. We assume that the only element that distinguishes goods is the frequency at which they are consumed, λ_j. We interpret

[16] A similar result is derived by Rocheteau and Wright (2005) in the context of a search-and-bargaining monetary model with free entry of sellers.

the economy as one composed of large households composed of workers and shoppers where shoppers are specialized in terms of the goods they purchase. For tractability, we assume that firms are identical and can produce all categories of goods.

From (32), the value of the outside option of consumer j is determined by

$$(\rho + \lambda_j + \gamma) Z_j = \max_{a \geq 0} \left\{ -ia + \alpha(1-\mu) \left[\chi^m S^m(a, Z_j) + \chi^d S^d(Z_j) \right] \right\}.$$

So consumers of different types will carry different amounts of real balances because they have different market powers due to different outside options. The measure of active consumers of type j is $\omega_{1,j} \equiv \omega_j \lambda_j / (\lambda_j + \gamma + \alpha)$ and the measure of all active consumers is $\omega_1 \equiv \sum_{j=1}^{J} \omega_{1,j}$. Market tightness in the goods market, $q = n/\omega_1$. Using that $n = f(\theta)/[\delta + f(\theta)]$, we can express tightness in the labor market as a function of tightness in the goods market as follows:

$$\frac{f(\theta)}{\delta + f(\theta)} = \sum_{j=1}^{J} \frac{\lambda_j \omega_j q}{\lambda_j + \gamma + \alpha(q)}.$$

This gives an implicit solution $q = Q(\theta)$. From the free entry condition,

$$(\rho + \delta) \frac{k\theta}{f(\theta)} = (1-\beta) \left\{ \frac{\alpha \left[Q(\theta) \right]}{Q(\theta)} \mu \sum_{j=1}^{J} \frac{\omega_{1,j}}{\omega_1} \left[\chi^m S^m(a_j, Z_j) + \chi^d S^d(Z_j) \right] + x - b \right\} - \beta k \theta.$$

An equilibrium is a list of $\{(a_j, Z_j)\}_{j=1}^{J}$, and a positive real number, θ, solutions to the equations above.

4.2 Calibration strategy

A unit of time corresponds to one month. We assume the matching function in both labor and goods markets are Cobb-Douglas. The job finding rate is given by $f(\theta) = \Xi \theta^{\xi}$ and the matching rate for consumers in the goods market is given by $\alpha(q) = \Psi q^{\psi}$. The cost of production in bilateral meetings is $\varphi(y) = G y^{g}$.

We set $\gamma = 0$ so that when the desire to consume arrives, it never disappears. To calibrate λ, we estimate the frequency of purchases using evidence from the Consumer Expenditure Survey (CEX). The CEX reports the fraction of households who spent a positive amount in narrowly-defined expenditure categories (e.g. prepared flour mixes) in a given time period. We interpret these fractions as the probability a representative household purchases a product in a given expenditure category in a given time period, which gives an estimate of the frequency of purchases for that category. We then compute the frequency of purchases for an average product by aggregating across categories using aggregate expenditure shares. Table 3 and Appendix C report our estimated frequency of purchases and estimation procedure. On average, households may purchase a given product category 1.53 times per month. Some categories are purchased frequently, like food at home (4.25 times per month) or shelter (3.45 times per month). However many categories are purchased infrequently, like education expenses (0.01 times per month) or medical supplies (0.02 times per month). As our baseline, we set $\lambda = 1.5$ but we will also illustrate how our results depend on λ.

The parameters ρ, k, δ, Ξ, ξ, β, and b are fixed using an approach close to that in BMW. We set $\rho = 0.001$ so that the real interest rate in the model matches the difference between the rate on Aaa bonds and realized inflation, on average. We use k and δ to match the average unemployment rate and unemployment-to-employment (UE) transition rate. The parameter, Ξ, is normalized so that the vacancy rate is 1. The elasticity of the matching function, ξ, targets the regression coefficient of labor market tightness θ on the UE rate of 0.6. Firms' bargaining power in the labor market, $1-\beta$, is set to match an average wage markdown relative to the firm's marginal revenue product of labor, $1 - w/\hat{x}$, of 0.35 following evidence in Yeh et al. (2022).

Unemployment benefits b represent both unemployment income and the value of non-work. We follow Hall and Milgrom (2008), and set $b = 0.71\hat{x}$, where $\hat{x} = \alpha^s \mu \left(\chi^m S^m + \chi^d S^d\right) + x$ is average output per worker.[17] We set the level of the goods market matching function $\Psi = 1$ and assume an equal elasticity between firms and consumers, $\psi = 0.5$. We calibrate G, g, and μ as in the New Monetarist literature. We set G and g to match the relationship between money demand M/pY and i in the data, using the adjusted M1 series in Lucas and Nicolini (2015) as our measure of money. We set μ so the markup in the goods market is 30%, as discussed by Faig and Jerez (2005). We set $\chi^m = 0.8$ because in the Atlanta Fed data discussed by Foster et al. (2013), credit cards account for 23% of purchases in volume. In the Bank of Canada data discussed by Arango and Welte (2012), this number is 19%.

The targets and parameter values discussed above are summarized in Table 2.

Parameter	Description	Targets	Value
ρ	Rate of time preference	Average real interest rate	0.001
k	Vacancy cost	Average unemployment rate	0.39
δ	Job destruction rate	Unemployment-to-employment rate	0.03
Ξ	Level of labor market matching	Average vacancies (normalization)	0.08
ξ	Elasticity of labor market matching	Elasticity of UE rate	0.60
Ψ	Level of goods market matching	—	1.00
ψ	Elasticity of goods market matching	Equal contribution to matching	0.50
β	Bargaining power of worker in labor market	Wage markdown	0.03
b	Unemployment benefits	$b = 0.71\hat{x}$	0.62
λ	Arrival of desire to consume	CEX evidence, see Table 3	1.5
γ	Desire to consume disappears	—	0
χ^m	Fraction of monetary meetings	Fraction of credit card transactions	0.80
G	Level of production cost	Level of money demand	0.58
g	Elasticity of production cost	Elasticity of money demand	1.28
μ	Bargaining power of firms in goods market	Retail sector markup	0.87

Table 2: Calibrated parameters

In the top row of Figure 9, we illustrate how long-run inflation affects the unemployment rate (top-left panel), real money balances (top-middle panel), and consumers' outside option (top-right panel) under the

[17] The effects of inflation on unemployment are channeled through labor productivity. Hence, we aim to capture the extent to which movements in labor productivity affect unemployment, as is studied in the large literature following Shimer (2005). Our target for b represents a conservative estimate between the calibrations of Shimer (2005), Hall and Milgrom (2008), and Hagedorn and Manovskii (2008).

baseline calibration. As predicted by Proposition 4, the Phillips curve is non-monotone in π. The reason is that consumers' outside option, Z, decreases with inflation since money holdings become more costly (right panel). Even though lower money holdings constrain the payment that consumers can make to firms, their lower outside option improves firms' market power in goods markets, increases their markup as illustrated in the bottom-left panel of Figure 9, and increases their expected revenue from a filled vacancy.[18]

In the top-left panel of Figure 9, we illustrate counter-factual Phillips curves as responds in unemployment to inflation due to changes only in real money balances (dash-dotted red line) or changes only in consumers' outside option (dashed green line). The outside option effect is quantitatively dominant for annual inflation rates between the Friedman rule and -0.2%, the latter representing the unemployment-minimizing inflation rate. As inflation rises beyond -0.2%, the effect on liquidity constraints dominates and the Phillips curve turns upward-sloping.

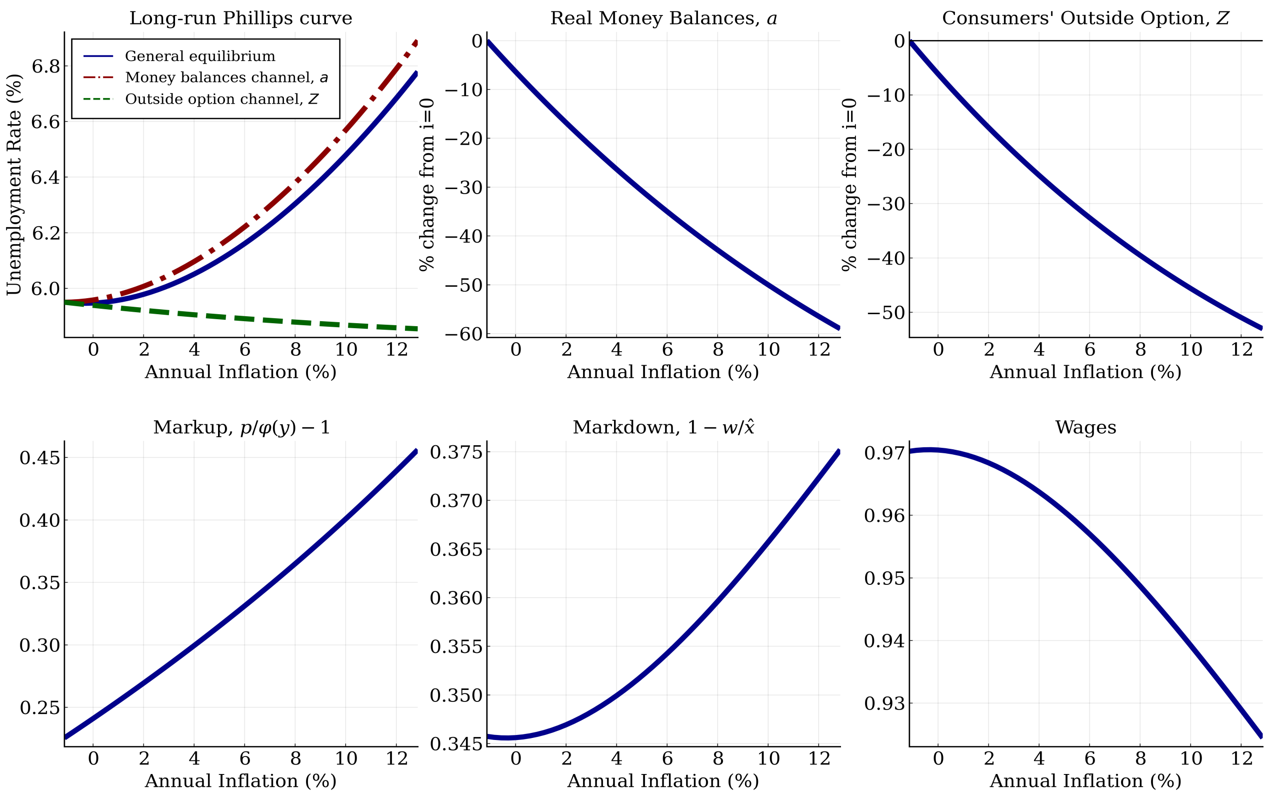

Figure 9: The effects of inflation on unemployment (top-left), real money balances (top-middle), consumers' outside options (top-right), markups (bottom-left), markdowns (bottom-middle), and wages (bottom-right)

The bottom-right and bottom-middle panels of Figure 9 illustrate the effect of inflation on wages and the wage markdown, respectively. While inflation unambiguously increases firms' markup in the goods market, it has a non-monotone effect on the markdown in the labor market. For low inflation rates, higher inflation

[18] The model produces a quantitatively-similar relationship between inflation and firms' markup to that documented during the recent inflation surge. For instance, Glover et al. (2023) document that firm-level markups increased by 3.4 percentage points during 2021 while Personal Consumption Expenditure inflation increased by 2.9 percentage points. For the same increase in inflation, our model predicts that firm markups should increase by 4.6 percentage points.

increases wages and reduces the wage markdown. For high inflation rates, inflation reduces consumers' payment capacity, firm's profits, and hence wages.

The strength of the consumer search channel depends on the rate at which the desire to consume arrives, λ. If it occurs relatively infrequently, then consumers' opportunity cost of consumption is relatively large since the value of returning to being idle is relatively low. If, however, consumers spend little time idle, the opportunity cost of consumption is low. In Figure 10, we illustrate the effects of λ on the shape of the long-run Phillips curve and the unemployment-minimizing level of inflation. For each value of λ, we recalibrate all other model parameters as outlined above.

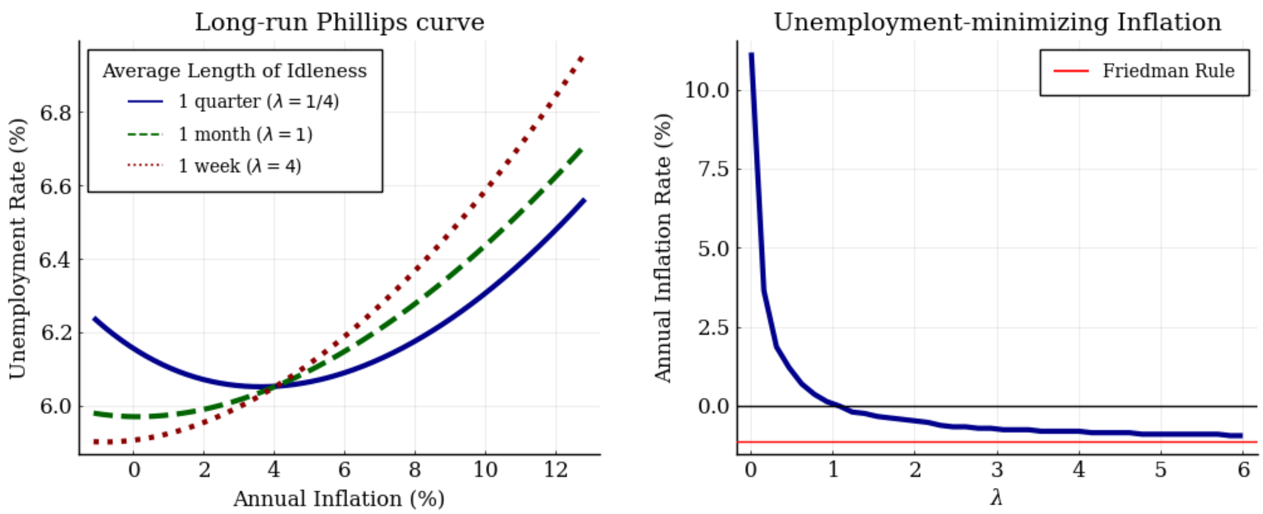

Figure 10: The Long-run Phillips curve and λ (left); Unemployment-minimizing inflation rate and λ (right)

The left panel of Figure 10 shows that for low values of λ the consumer search channel is quantitatively strong. For instance, if the desire to consume occurs on average once per quarter, illustrated in the solid-blue curve, the long-run Phillips curve is downward sloping for annual inflation rates up to 4%. The right panel illustrates how the unemployment-minimizing inflation rate changes as λ varies from close to zero to 6. As λ approaches zero, the unemployment-minimizing inflation rate is just under 12%. As the speed of the desire to consume increases, the unemployment-minimizing inflation rate decreases towards the Friedman Rule, without hitting it.

Figure 11 illustrates how the unemployment-minimizing rate of inflation depends on firm's bargaining power, μ. As μ increases, consumers' outside option falls and the consumer search channel is diminished. This leads to a more positively-sloped long-run Phillips curve and a lower unemployment-minimizing inflation rate.

Figure 12 represents the welfare-maximizing inflation rate as a function of two bargaining power parameters, β and μ, with countour lines. The combination of the bargaining powers that implement the constrained efficient allocation are represented by the solid-black line. The welfare-maximizing inflation rate is above the Friedman rule, $\pi_{FR} = -\rho$, when households' bargaining power in the labor market, β, is larger

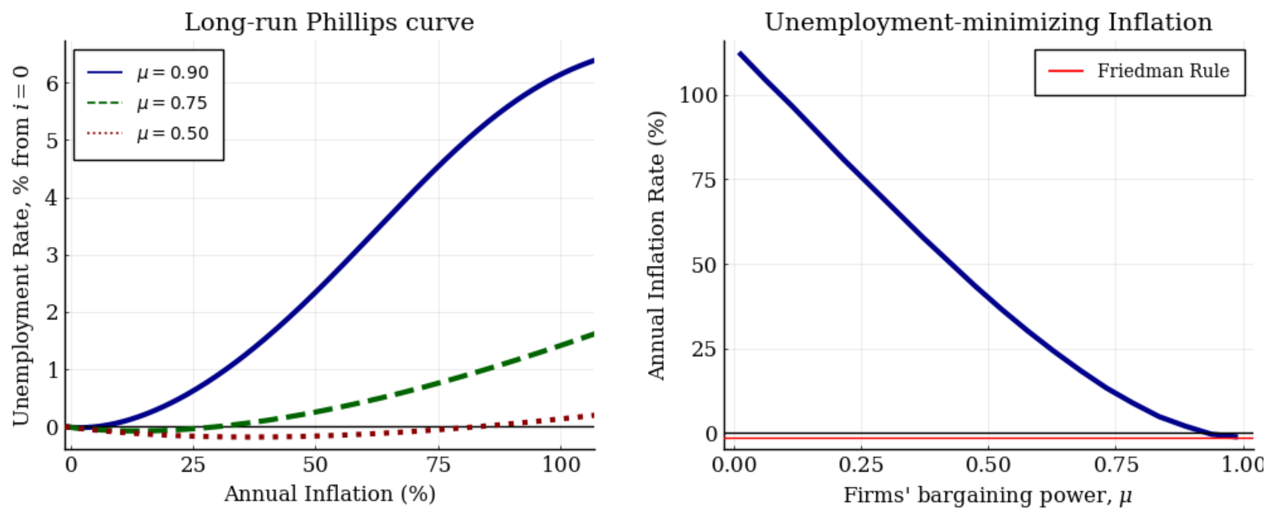

Figure 11: The Long-run Phillips curve and μ (left); Unemployment-minimizing inflation rate and μ (right).

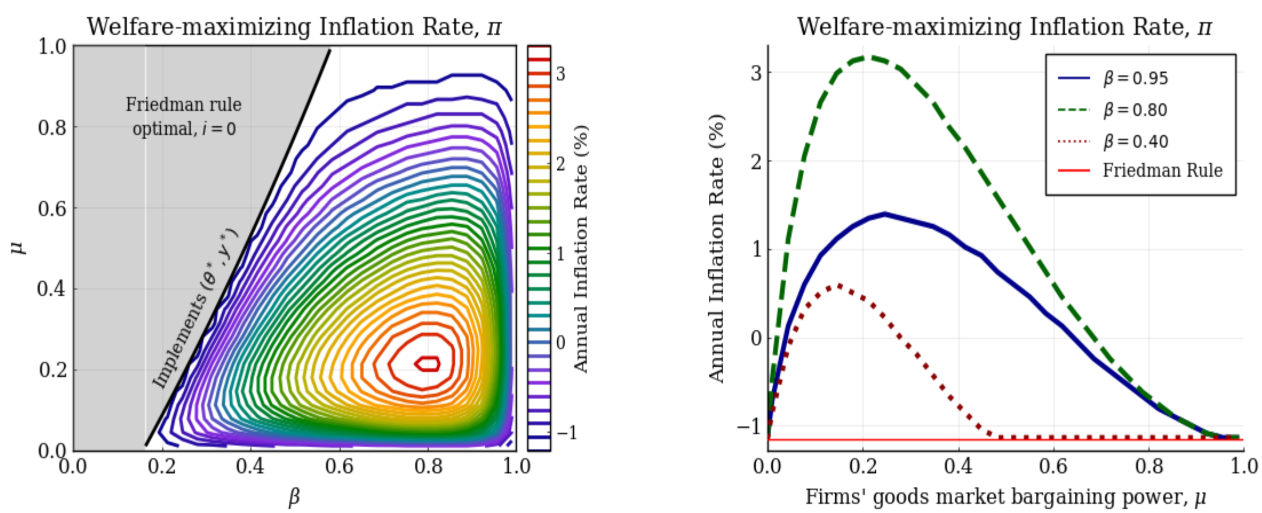

Figure 12: Welfare-maximizing inflation rate, given μ and β.

than a threshold, which is a little less than 20% for our calibrated example, and firm's bargaining power in the goods market, μ, is positive but lower than some threshold given by the solid black line. In this region, firm entry and labor market tightness are too low compared to the planner's solution in (56)-(57). Increasing inflation above the Friedman rule reduces consumers' outside option, thereby increasing firms' market power and incentivizing entry. It also raises the cost of holding money but, in the neighborhood of $y = y^*$, the effect on social welfare is second order. It can also be seen from Figure 12 that for a given β, the welfare-maximizing inflation rate is a nonmonotone function of μ. When $\mu = 0$, firms have no market power in the goods market and, from (30), θ is independent of Z. As a result, an increase in inflation does not shift market toward the firm. It follows that the welfare-maximizing inflation is $\pi = \pi_{FR} \equiv -\rho$. (At an annual frequency, $\pi_{FR} \approx -0.012$.) As μ increases above 0, Z affects θ and it becomes optimal to raise π above $-\rho$. As μ reaches the value that impements the first-best market tightness, which corresponds to the black solid line, π returns to π_{FR}.

In the right panel of Figure 12, the welfare maximizing inflation rate when $\beta = 0.95$ is lower than when $\beta = 0.80$. The reason is that when β is very high, θ is low. An increase in inflation reduces Z (so increases θ), but reduces real balances also. As the effect on real balances is first-order, inflation should not increase too much to maximize welfare.

5 Equilibrium consumer search

In the baseline model of Section 3, consumers never exert their option to search once matched with a firm on the equilibrium path. Searching for alternative producers is a threat that reduces firms' market power and that affects the surplus of the match and the terms of trade. We now extend our model with horizontally differentiated products to introduce search on the equilibrium path.

The preference of a consumer for good y is now $\varepsilon \upsilon(y)$ where ε is a random variable capturing the idiosyncratic taste of the consumer. When a consumer meets a firm, $\varepsilon \in [0, \bar{\varepsilon}]$ is drawn from a cumulative distribution $F(\varepsilon)$ and is common-knowledge in the match.[19] The consumer can then decide to engage a negotiation with the firm or to keep searching for an alternative producer.

Consider a match between a consumer with a real balances and a firm when the realization of the preference shock is ε. There are gains from trade if

$$\max_{y \geq 0} \{\varepsilon \upsilon(y) - \varphi(y) : \varphi(y) \leq a\} > Z, \tag{61}$$

which has a similar interpretation as (4). The next lemma characterizes the threshold for ε above which the condition (61) holds.

Lemma 1 ***(Optimal search)*** *The threshold for ε above which gains from trade are positive obeys:*

$$\begin{aligned} \varepsilon_R(a, Z) &= \hat{\varepsilon}(Z) \quad \textit{if } \hat{\varepsilon}(Z) \leq \tilde{\varepsilon}(a) \\ &= \frac{a + Z}{\upsilon[\varphi^{-1}(a)]} \quad \textit{otherwise,} \end{aligned} \tag{62}$$

where $\hat{\varepsilon}(Z)$ is the solution to $\varepsilon \upsilon(y^*_\varepsilon) - \varphi(y^*_\varepsilon) = Z$ and $\tilde{\varepsilon}(a) \equiv \varphi'[\varphi^{-1}(a)]/\upsilon'[\varphi^{-1}(a)]$.

The threshold ε_R can take two values. If a is large, it is equal to $\hat{\varepsilon}(Z)$ which only depends on the consumer's outside options. It is the value of ε such that the match surplus is zero when the quantity traded, y, is efficient. If a is small, the liquidity constraint binds and ε_R depends on both a and Z. It increases with Z, i.e., if the consumers' outside options improve, consumers become pickier. It decreases with a, i.e., if consumers hold more real balances, then they are willing to buy varieties that they value less because they can compensate the lowest match quality by purchasing larger quantities.

[19] One can give several interpretations for ε. For instance, firms produce different varieties of good y and consumers value these different varieties differently. Alternatively, the intensity for the desire to consume could be varying over time.

For all $\varepsilon \geq \varepsilon_R$, the determination of the terms of trade is given by the proportional solution,

$$\max_{p,y} \{p - \varphi(y)\} \quad \text{s.t. } p - \varphi(y) = \mu \left[\varepsilon v(y) - \varphi(y) - Z\right] \quad \text{and} \quad p \leq a. \tag{63}$$

We denote $y_\varepsilon (a, Z)$ the solution to (63). The surpluses in monetary and credit matches are

$$S_\varepsilon^m(a, Z) \equiv \varepsilon v \left[y_\varepsilon (a, Z)\right] - \varphi \left[y_\varepsilon (a, Z)\right] - Z \quad \text{if } \varepsilon \geq \varepsilon_R(a, Z) \tag{64}$$

$$S_\varepsilon^d(Z) \equiv \varepsilon v(y_\varepsilon^*) - \varphi (y_\varepsilon^*) - Z \quad \text{if } \varepsilon \geq \hat{\varepsilon}(Z). \tag{65}$$

If there are no gains from trade, then $S_\varepsilon^\chi = 0$ for $\chi \in \{m, d\}$. Following the same reasoning as above, the outside option of the consumer, Z, solves:

$$(\rho + \lambda + \gamma) Z = \max_{a \geq 0} \left\{-ia + \alpha(1 - \mu) \int \left[\chi^m S_\varepsilon^m(a, Z) + \chi^d S_\varepsilon^d(Z)\right] dF(\varepsilon)\right\}. \tag{66}$$

This equation, which has a similar interpretation as (14), admits a unique solution, $Z \in [0, \bar{\varepsilon} y_{\bar{\varepsilon}}^* - \varphi(y_{\bar{\varepsilon}}^*)]$. It is an increasing function of α and a decreasing function of μ. If the optimal choice of real balances is interior, it solves

$$\int_{\varepsilon_R(a,Z)}^{\bar{\varepsilon}} \frac{\alpha \chi^m (1 - \mu) \left[\varepsilon v'(y_\varepsilon) - \varphi'(y_\varepsilon)\right]}{\mu \varepsilon v'(y_\varepsilon) + (1 - \mu)\varphi'(y_\varepsilon)} dF(\varepsilon) = i, \tag{67}$$

where $y_\varepsilon = y_\varepsilon (a, Z)$. The left side, which represents the marginal benefits from holding real balances, is decreasing in a. So, if it exists, there is a unique solution to (67).

The free-entry condition (22) in the labor market is generalized to give:

$$\begin{aligned}(\rho + \delta) \frac{k\theta}{f(\theta)} + \beta k\theta &= (1 - \beta) \times \\ &\left\{\alpha^s(\theta)\mu \left[\chi^m \int_{\varepsilon_R(a,Z)}^{\bar{\varepsilon}} S_\varepsilon^m(a, Z) dF(\varepsilon) + \chi^d \int_{\hat{\varepsilon}(Z)}^{\bar{\varepsilon}} S_\varepsilon^d(Z) dF(\varepsilon)\right] + x - b\right\}. \end{aligned} \tag{68}$$

The two integrals on the right side represent the firm surpluses in all monetary and credit matches where the gains from trade are positive.

The steady-state measure of active consumers solves

$$\left\{\gamma + \alpha\chi^m \left[1 - F(\varepsilon_R)\right] + \alpha\chi^d \left[1 - F(\hat{\varepsilon})\right]\right\} \omega_1 = \lambda(\omega - \omega_1). \tag{69}$$

The left side is the flow of consumers who become inactive, either because their desire for consumption vanishes at rate γ or because they meet a firm that produces a variety of the good that they want to consume. The right side represents the flow of inactive consumers that become active at rate λ. Using that $q\omega_1 = n$, (69) can be rewritten to obtain the following relationship between q and θ:

$$\frac{\lambda\omega q}{\gamma + \alpha(q)\chi^m \left[1 - F(\varepsilon_R)\right] + \alpha(q)\chi^d \left[1 - F(\hat{\varepsilon})\right] + \lambda} = \frac{f(\theta)}{\delta + f(\theta)}. \tag{70}$$

So the tightness of the goods market increases with θ but decreases with ε_R and $\hat{\varepsilon}$.

An equilibrium is defined as a list, $\langle \varepsilon_R, Z, a, \theta, q\rangle$, solution to (62), (66), (67), (68), and (70). In the following, we consider equilibria in the neighborhood of the Friedman rule when $i = 0^+$.

Proposition 8 ***(Long-run Phillips curve and consumer search)*** *Assume $x > b$, $\rho + \lambda + \gamma$ is small, and $i = 0^+$.*

1. *Conditional on a trade taking place, quantities traded are efficient, $y_\varepsilon = y_\varepsilon^*$ for all $\varepsilon \geq \varepsilon_R(a, Z) = \hat{\varepsilon}(Z)$, where a and Z are the equilibrium values when $i = 0^+$.*

2. *An increase in λ or γ reduces the value of consumer search, Z, and makes consumers less picky, i.e., ε_R decreases. It raises labor market tightness, θ, and reduces unemployment, u.*

3. *A small increase in i from $i = 0^+$ generates an increase in θ, and a decrease in u, Z, and ε_R.*

The proof operates some substitutions to reduce an equilibrium to a pair, (θ, Z), solution to (66) and (68). The assumption that the effective discount rate, $\rho + \lambda + \gamma$, is small allows us to focus on equilibria where the curve representing (68) cuts the curve representing (66) from above in the (θ, Z) space, as shown in Figure 13. We also focus on monetary policy in the neighborhood of the Friedman rule so that, conditional on a trade, quantities are efficient, as shown by the first part of the proposition.

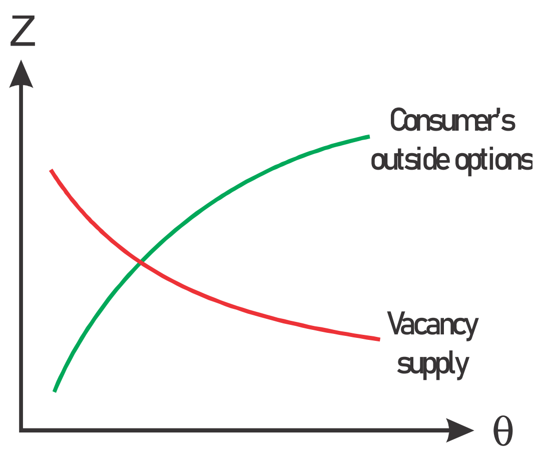

Figure 13: Equilibrium with consumer search when $i = 0^+$

The second part of Proposition 8 establishes the links between market power, consumer search, and unemployment. As λ or γ rises, the value of searching falls and hence firms' market power increases. Consumers spend their real balances on goods that they value less, i.e., ε_R decreases. Firms' expected revenue rises, which incites them to open more vacancies and reduces unemployment.

The third part of Proposition 8 shows that the result according to which the long-run Phillips curve is downward sloping at low inflation rates is robust when one introduces differentiated goods and ex post match heterogeneity. As the inflation rate increases, consumer search becomes more costly. As a result, consumers become less choosy, ε_R decreases, and the value of their outside options decreases. As firms' market power increases, they open more vacancies, θ increases, and the unemployment rate decreases.

6 Nominal interest rates and unemployment

So far, monetary policy took the form of a constant money growth rate, π, that could be mapped one-to-one into the nominal interest rate of an illiquid bond, $i = \rho + \pi$. As a result, the relationship between u and π was isomorphic to the one between u and i. In its current practice, however, the central bank sets the nominal interest rate on highly liquid, short-term bonds. Hence, in this section, we revisit the relationship between unemployment and nominal interest rates by distinguishing between illiquid and liquid bonds. Our objective is twofold: we determine how changes in the short-term nominal interest rate affect firms' market power and unemployment; and we characterize the combination of two policy instruments, the money growth rate and the short-term nominal interest rate, that minimizes unemployment.

6.1 Liquid bonds and market segmentation

Liquid government bonds are of the pure discount (or zero coupon) variety. A bond pays one unit of numeraire when it matures at Poisson rate $\mu > 0$. So, $1/\mu$ is the expected maturity of the bond. The real interest rate of the bond is denoted r_g and its price in terms of the numéraire is ϕ_g. Hence, by standard asset pricing,

$$r_g \phi_g = \mu(1 - \phi_g) \Rightarrow \phi_g = \frac{\mu}{r_g + \mu}.$$

We consider the limit as the maturity of the bond is close to 0, i.e., $\mu \to +\infty$. Hence, $\phi_g = 1$. The nominal interest rate on liquid bonds is denoted $i_g = r_g + \pi$. The total supply of bonds is denoted A_g. Since $\phi_g = 1$, A_g is also the real value of the bond supply.

According to the rate-of-return dominance puzzle, if bonds are perfectly liquid, then $r_g = -\pi$ and $i_g = 0$, i.e., bonds do not bear interest. We address this puzzle by assuming limited participation in the bonds market. We divide consumers into two types, $\kappa \in \{1, 2\}$, where types differ in terms of the assets, money or bonds, they can hold in their portfolios. A fraction ψ_1 of consumers, those of type $\kappa = 1$, can only use money to finance the consumption of good y. The remaining fraction, $\psi_2 = 1 - \psi_1$, corresponds to type-2 consumers who can use both bonds and money as means of payment.[20] One can think of type-1 consumers as unsophisticated investors who do not participate in the bonds market while type-2 buyers are sophisticated investors who can hold financial assets.[21]

6.2 Definition of equilibrium

Let a_g be a consumer's holding of bonds measured in numeraire, and a_m her real balances. The total liquid wealth of an agent is denoted $a \equiv a_m + a_g$. As before, in a fraction χ^d of meetings, buyers have access to credit. In the remaining fraction, $\chi^\ell \equiv 1 - \chi^d$, buyers can use their liquid assets (money for type-1 buyers,

[20] The assumption of limited participation in some asset markets has been used, e.g., by Alvarez et al. (2001), Alvarez et al. (2002), Williamson (2006), among others.

[21] One could endogenize participation in the bonds market, and hence the measure ψ_κ, by introducing a distribution of participation costs across buyers. See, e.g., Rocheteau et al. (2018).

and money and bonds for type-2 buyers). The value function of a buyer of type $\kappa \in \{1, 2\}$ is denoted $V^b_\kappa(a) = a + V^b_\kappa$.

The decision problem of type-1 consumers is the same as that in the baseline model. Their real balances are denoted a^1_m, and their real bond holdings are $a^1_g = 0$. The HJB equation for a type-2 consumer in a steady state is

$$\rho V^b_2 = \max_{a_m, a_g \geq 0} \left\{ -s_m a_m - s_g a_g + \tau + \alpha(1-\mu) \left[\chi^\ell S^\ell(a, Z_2) + \chi^d S^d(Z_2) \right] - \gamma(V^b_2 - W^b_2) \right\}, \tag{71}$$

where $s_g \equiv \rho - r_g$ is the spread between the real interest rate on an illiquid bond and the real interest rate on a liquid government bond, i.e., it is the opportunity cost of holding liquid bonds. Similarly, the spread between the real interest rate on an illiquid bond and the real interest rate on money (r_m) is denoted $s_m \equiv \rho - r_m$ and it is equal to $i = \rho + \pi$. By the logic leading to (14), the outside option of the consumer, Z_2, solves:

$$(\rho + \lambda + \gamma) Z_2 = \max_{a_m, a_g \geq 0} \left\{ -s_m a_m - s_g a_g + \alpha(1-\mu) \left[\chi^\ell S^\ell(a, Z_2) + \chi^d S^d(Z_2) \right] \right\}, \tag{72}$$

which defines a negative relationship between s_g and Z_2. The first-order conditions with respect to a_j, $j \in \{m, g\}$, are

$$s_j \geq \frac{\alpha \chi^\ell (1-\mu) \left[\upsilon'(y_2) - \varphi'(y_2) \right]}{\mu \upsilon'(y_2) + (1-\mu)\varphi'(y_2)} \qquad \text{“ = ” if } a_j > 0, \text{ for } j \in \{m, g\},$$

where $y_2 = y(a, Z_2)$ is defined by (6). For type-2 consumers, money and bonds are perfect substitutes as means of payment. Hence, if $s_g < s_m$, they go cashless. They hold both money and bonds only if $s_g = s_m$, i.e., bonds do not bear interest.

By market clearing, bonds must be held, and therefore $s_g \leq s_m$. It follows from (72) that we can compute Z_2 by assuming that sophisticated buyers can only hold bonds. Hence, the outside options of type-1 and type-2 consumers, Z_1 and Z_2, are solution to

$$(\rho + \lambda + \gamma) Z_\kappa = \max_{a \geq 0} \left\{ -s_j a + \alpha(1-\mu) \left[\chi^\ell S^\ell(a, Z_\kappa) + \chi^d S^d(Z_\kappa) \right] \right\}, \quad \kappa \in \{1, 2\}. \tag{73}$$

The only difference between the outside options of type-1 and type-2 consumers is the holding cost of liquid assets — type-1 faces s_m while type-2 faces s_g. Sophisticated consumers have better outside options than unsophisticated ones because they can hold liquidity at a lower cost, thereby reducing the cost of searching for an alternative seller.

In a steady-state equilibrium, the market-clearing condition of the bonds market implies

$$A_g = \psi_2 \omega_1 a^2_g,$$

where $\psi_2 \omega_1$ is the measure of active type-2 consumers and a^2_g represents their bond holdings. By fixing the supply of bond, A_g, the government can control r_g, and thus s_g. The real balances held by an active type-2 consumer are denoted a^2_m. Hence, aggregate real balances are

$$\phi_m M = \omega_1(\psi_1 a^1_m + \psi_2 a^2_m).$$

By the same logic as before, market tightness in the labor market solves

$$(\rho+\delta)\frac{k\theta}{f(\theta)}=(1-\beta)\left\{\alpha^s\mu\sum_{\kappa\in\{1,2\}}\psi_\kappa\left[\chi^\ell S^\ell(a^\kappa,Z_\kappa)+\chi^d S^d(Z_\kappa)\right]+x-b\right\}-\beta k\theta, \tag{74}$$

where $a^\kappa = a_m^\kappa + a_g^\kappa$ is a type-κ consumer's choice of liquidity. The equilibrium wage solves

$$w=\beta\left\{\alpha^s\mu\sum_{\kappa\in\{1,2\}}\psi_\kappa\left[\chi^\ell S^\ell(a^\kappa,Z_\kappa)+\chi^d S^d(Z_\kappa)\right]+x\right\}+(1-\beta)b+\beta k\theta. \tag{75}$$

An equilibrium is a tuple, $(\theta, a_m^1, a_m^2, a_g^2, Z_1, Z_2, w)$ where θ solves (74), $\left(a_m^1, a_m^2, a_g^2\right)$, Z_κ solve (73) for $\kappa \in \{1,2\}$, and w solves (75). By the proof logic of Proposition 1, there exists an active steady-state equilibrium provided that $\chi^d > 0$.

6.3 Typology of equilibria

We distinguish two types of equilibria. There is a *liquidity trap* equilibrium where $s_g = s_m = i$, i.e., the nominal interest on liquid bonds is $i_g = 0$.[22] In such equilibria, type-2 consumers hold a portfolio of money and bonds as they are indifferent between the two assets. Both types of consumers have the same lifetime utility and the same outside options. Liquidity-trap equilibria are isomorphic to the monetary equilibria studied in Section 3.

A decrease in A_g reduces the bond holdings of type-2 consumers but it has no impact on the value of their portfolio, a^2. Buyers compensate the decrease in a_g^2 by raising a_m^2 so as to keep their liquid wealth constant. Therefore, changes in A_g do not affect production and unemployment. A liquidity trap equilibrium occurs when

$$A_g \leq a(i)\psi_2\omega_1,$$

where $a(i)$ is the choice of liquid wealth of a consumer when the user cost of liquidity is i. So a liquidity trap equilibrium exists when the supply of liquid bonds is lower than a threshold that decreases with inflation.

If $A_g > a(i)\psi_2\omega_1$, then the economy is outside of the liquidity trap and $s_g < s_m$, i.e. $i_g > 0$. In that case, type-2 consumers only hold interest-bearing liquid bonds. By market clearing and bargaining,

$$a_g^2=\frac{A_g}{\psi_2\omega_1}=(1-\mu)\varphi(y_2)+\mu y_2-\mu Z_2 \quad\text{where}\quad \frac{\alpha\chi^\ell(1-\mu)\left[\upsilon'(y_2)-\varphi'(y_2)\right]}{\mu\upsilon'(y_2)+(1-\mu)\varphi'(y_2)}=s_g. \tag{76}$$

Note that y_2 is a function of s_g by the first-order condition, and Z_2 is also a function of s_g. So, the supply of bonds determines the liquidity spread of government bonds, i.e., changes in A_g affect s_g and allocations.

6.4 The relation between unemployment and the nominal interest rate

The traditional Phillips curve provides a relationship between the unemployment rate rate and the inflation rate or, equivalently by the Fisher equation, a relationship between u and the interest rate on illiquid bonds,

[22] Williamson (2012) provides another example of a New Monetarist model that delivers a liquidity trap equilibrium.

i. In the following proposition, we describe a *modified* Phillips curve that gives the relationship between u and the short-term nominal interest rate, i_g. Moreover, we characterize the choice of two policy instruments, π and i_g, to minimize the unemployment rate.

Proposition 9 ***(Unemployment and the nominal interest rate.)*** *Assume type-1 and type-2 consumers have identical preferences.*

1. *For given $\pi > -\rho$, the unemployment rate decreases as i_g falls below i.*
2. *There exists $\bar{\imath} > 0$ such that if $i > \bar{\imath}$ then the unemployment rate at $i_g = i$ is lower than the unemployment rate at $i_g = 0$.*
3. *Unemployment is minimized when $s_m = s_g > 0$, i.e., $i \equiv \rho + \pi > 0$ and $i_g = 0$.*

The first two parts of Proposition 9 taken together show that the relationship between u and i_g is nonmonotone provided that π is sufficiently large. Indeed, when $i_g = i$, the opportunity cost of holding liquid bonds is zero, and hence type-2 buyers hold enough bonds to purchase the first-best level of output, i.e., $y_2 = y^*$. If i_g falls slightly below i, y_2 falls as well, but it only has a second-order effect on the output net of the production cost, $\upsilon(y) - \varphi(y)$. However, the increase of s_g above 0 has a first-order effect on consumer's outside option, Z_2, which then raises firms' market power. It induces more firm entry and a lower unemployment rate.

The other polar case is when s_g tends to infinity. By the same logic as in the pure monetary economy, we can establish that the unemployment rate is lower when $s_g = 0$ compared to $s_g = +\infty$. However, s_g is bounded above by i, i.e., i_g is bounded below by 0. So, in order to u to be nonmonotone as i_g varies from 0 to i, it must be that i (or π) is sufficiently large. We illustrate these findings graphically in Figure 14 by plotting the *modified* Phillips curve for different inflation rates.

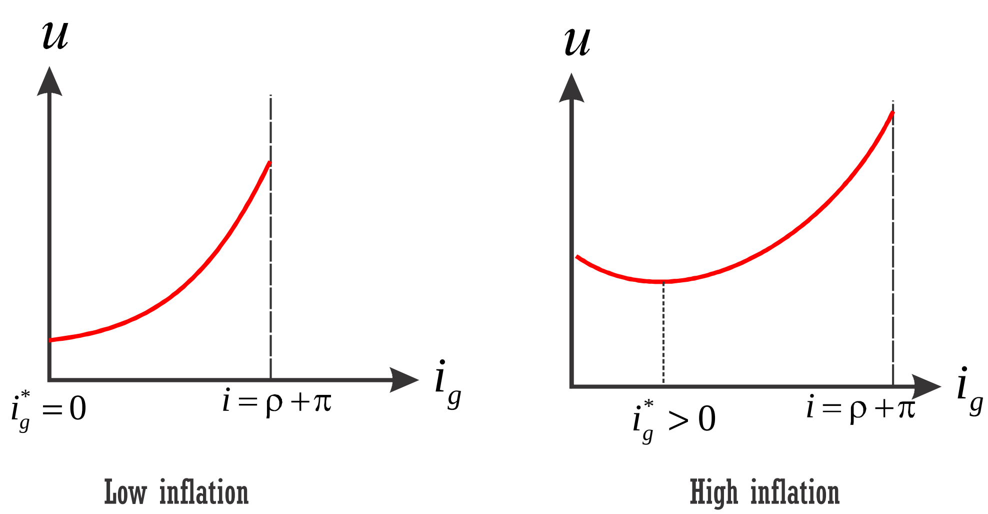

Figure 14: The modified Phillips curve: The relationship between u and i_g at low and high inflation rates

The third part of Proposition 9 determines the monetary policy, in terms of inflation and nominal interest rate, that minimizes unemployment. From (74), the policymaker chooses the spreads s_m and s_g as follows:

$$s_j \in \arg\max \left\{ \chi^{\ell} S^{\ell} \left[a^{\kappa}(s_j), Z_{\kappa}(s_j) \right] + \chi^d S^d \left[Z_{\kappa}(s_j) \right] \right\}, \quad (j, \kappa) \in \{m, g\} \times \{1, 2\}$$

The problems to determine s_m and s_g are independent (since the profits arising from the two types of buyers are additively separable) and identical. Hence, if s_m maximizes the profits from type-1 consumers, then $s_g = s_m$ maximizes the profits from type-2 consumers. An implication of this result is that the unemployment-minimizing nominal interest rate is zero, i.e., when the unemployment rate is minimum, the economy is in a liquidity trap. Note that the reverse is not always true. An economy can be in a liquidity trap but the inflation rate might not be at the level that minimizes unemployment. In addition, as in Proposition 4, the unemployment rate is minimum when interest rate spreads are positive, i.e., the inflation rate is above the Friedman rule.

We illustrate these effects in Figure 15 under the calibration in Section 4. We set $\psi_1 = 0.4$ to match the fraction of households in the Survey of Consumer Finances with no directly- or indirectly-held financial assets (i.e. they only hold currency, transaction accounts, or durable assets). The left panel illustrates the relationship between unemployment and the liquid bond rate, for a given inflation rate. For inflation rates below -0.2%, unemployment is increasing in the liquid bond rate and the unemployment minimizing i_g is achieved at $i_g = 0$. For higher inflation rates, unemployment is u-shaped in i_g. The unemployment minimizing liquid bond rate increases as inflation increases (middle panel), however the minimum-unemployment joint policy (π, i_g) is achieved at $\pi = -0.2\%$ and $i_g = 0$.

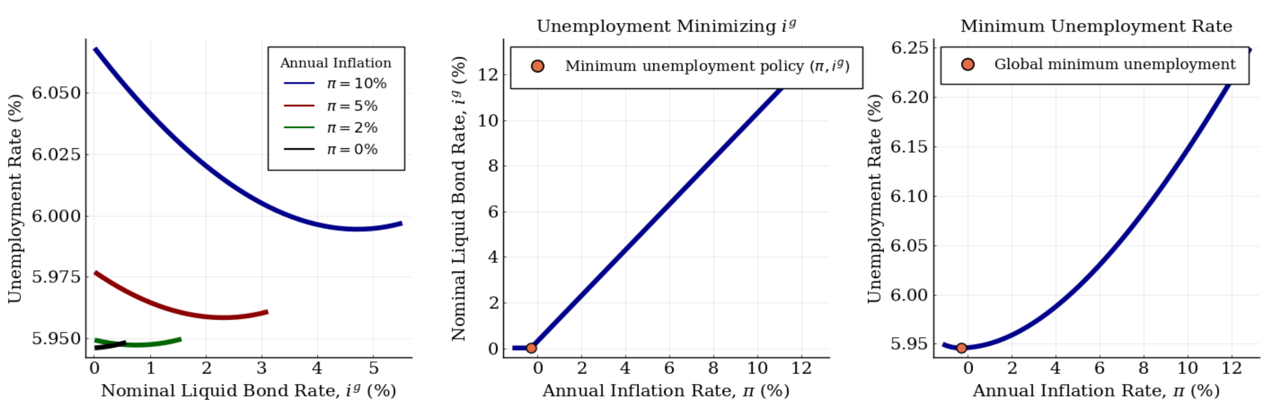

Figure 15: The effect of the nominal rate i^g on unemployment (left), the unemployment-minimizing nominal rate i^g given π (middle), and the minimum unemployment achieved given π (right).

So far, we have assumed that consumers of type 1 and 2 were identical in terms of their preferences and opportunities to consume. Suppose now that type-1 and type-2 consumers receive preference shocks at different rates, λ_1 and λ_2. This heterogeneity can explain $s_m \neq s_g$ and hence $i_g > 0$. To see this, note that for $(j, \kappa) \in \{(m, 1), (g, 2)\}$,

$$\left. \frac{\partial Z_{\kappa}}{\partial s_j} \right|_{s_j = 0^+} = \frac{-p(y^*)}{\rho + \lambda_{\kappa} + \gamma + \alpha(1 - \mu)}.$$

Intuitively, in absolute value, the impact of s_j on Z_κ is smaller when λ_κ is larger. Hence, as λ_κ increases, the spread that minimizes unemployment should decrease. Therefore, we conjecture that if $\lambda_2 > \lambda_1$, then $s_g < s_m$ and $i_g > 0$. The next proposition shows that this conjecture is true when λ_1 is sufficiently large, and we numerically illustrate that the conjecture holds in our calibrated economy.

Proposition 10 ***(Asymmetric consumers.)*** *If $\lambda_2 > \lambda_1$ and λ_1 is sufficiently large, then the unemployment minimizing policy features $s_g < s_m$ and $i_g > 0$.*

In Figure 16 we illustrate the unemployment-minimizing joint policy, either as a combination of nominal interest rates (i, i^g) or as a combination of spreads (s_m, s_g). For $\lambda_1 < \lambda_2$, unemployment is minimized for positive inflation rates $i > 0$ and positive nominal liquid bond rates $i^g > 0$. As λ_1 increases, both i and i^g fall until λ_1 rises above λ_2, in which case $i^g = 0$ minimizes unemployment. These results illustrate that when $\lambda_1 < \lambda_2$, a policy maker who wants to minimize unemployment might prefer $i, i_g > 0$.

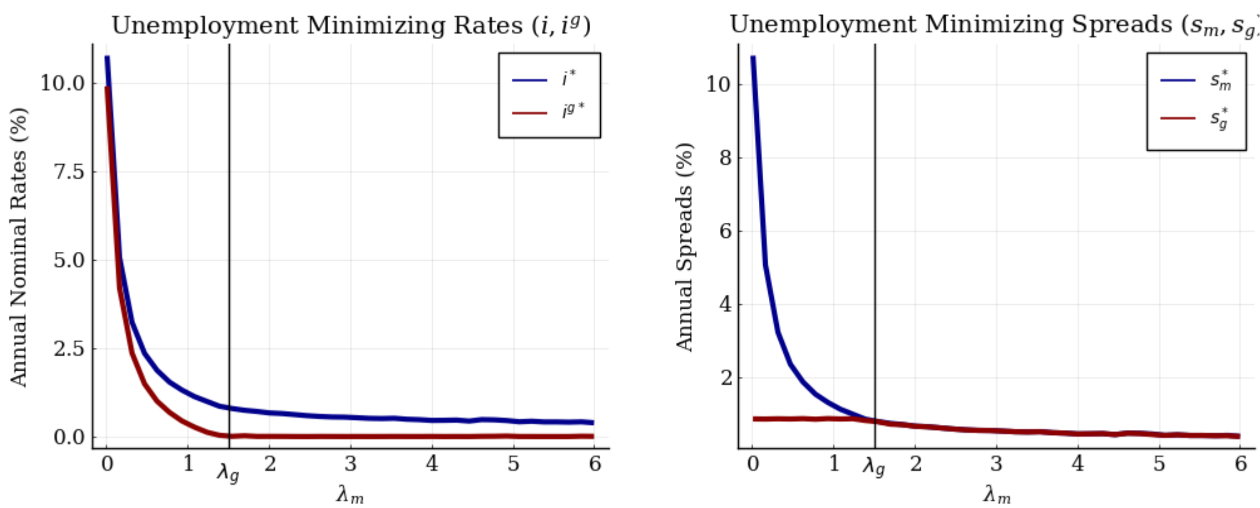

Figure 16: Unemployment-minimizing policy under asymmetric preferences (λ_1, λ_2)

7 Conclusion

The objective of this paper was to make a simple but robust observation regarding the long-run trade-off between unemployment and inflation. In the class of models pioneered by BMW, where goods and labor markets are frictional, and money plays an essential role to facilitate the exchange of goods and services, the relation between unemployment and inflation is U-shaped. As a result, the inflation rate that minimizes unemployment is above the one prescribed by the Friedman rule. This result is robust – it does not require parametric conditions to hold – once one introduces consumer search in order to endogenize consumer outside options and firms' market power. We extend the model by introducing horizontally differentiated products and liquid government bonds, and show that our main result persists.

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A Proofs of propositions and lemmas

Proof of Proposition 1.

Throughout we assume $\chi^d > 0$. Define $Z(\theta)$ the solution to (32) as a function of θ. To see that $Z(\theta)$ exists and is unique, note that the left side of (32) is linear, increasing in Z while the right side (RHS) is decreasing for all $Z \in (0, \upsilon(y^*) - \varphi(y^*))$. If $Z = 0$ then $RHS \geq \alpha^b(\theta)(1-\mu)\chi^d[\upsilon(y^*) - \varphi(y^*)] > 0$ if $\theta > 0$. If $Z = \upsilon(y^*) - \varphi(y^*)$ then $RHS = 0$. Hence, for all $\theta > 0$, there is a unique $Z \in (0, \upsilon(y^*) - \varphi(y^*))$ solving (32). Since $\alpha^b(\theta)$ is increasing in θ, it follows that RHS is increasing in θ, and hence $Z'(\theta) > 0$. Moreover, $\alpha^b(0) = 0$ implies $Z(0) = 0$.

We have seen that the set of solutions to (11) is either the corner solution 0, the interior solution given by (12), which we denote by $a_I^*(\theta)$, or both. Note that $a_I^*(\theta)$ is increasing continuously in θ. Also the best response transitions from 0 to $a_I^*(\theta)$ as θ rises, and not the other way around. Hence $S^m[a^*(\theta), Z(\theta)]$ can only jump upward as θ rises. We now define the following function:

$$\Gamma(\theta) \equiv \Big\{(1-\beta)\left\{\alpha^s(\theta)\mu\left\{\chi^m S^m[a^*(\theta), Z(\theta)] + \chi^d S^d[Z(\theta)]\right\} + x - b\right\} \\ -\beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}\Big\}. \tag{77}$$

When both 0 and $a_I^*(\theta)$ are optimal for the consumers, we assume $\alpha^*(\theta) = a_I^*(\theta)$, and hence $\Gamma(\theta)$ is right continuous. An equilibrium can be reduced to a θ solution to $\Gamma(\theta) = 0$. Since $S^m[a^*(\theta), Z(\theta)]$ only jumps upward, so does $\Gamma(\theta)$. As $\theta \to 0$, $a^*(\theta) \to 0$, so $\Gamma(\theta)$ converges to the value

$$(1-\beta)\left\{\alpha^s(0)\mu\chi^d[\upsilon(y^*) - \varphi(y^*)] + x - b\right\}.$$

Since $\alpha^s(0) = +\infty$, $\Gamma(0) = +\infty$. As $\theta \to +\infty$, $\alpha^s(\theta) \to 0$, $\theta/f(\theta) \to 1/f'(+\infty) = +\infty$ and hence $\Gamma(\theta)$ converges to $-\infty$. Since $\Gamma(\theta)$ can only have upward jumps, it must cut the x-axis and therefore a steady state equilibrium exists. ■

Proof of Proposition 2. Equation (38) determines θ. The left side is increasing in θ from 0 to $+\infty$ while the right side is decreasing in θ from $+\infty$ to $z-b$. Hence, θ is unique. The right side of (38) is increasing in $\lambda+\gamma$. Hence, θ increases with λ and γ. It follows that the unemployment rate, $u = \delta/[\delta + f(\theta)]$, decreases with λ and γ. From (34), Z increases if and only if θ decreases. So Z is a decreasing function of λ and γ. Finally, by (36), w increases in λ and in γ. ■

Proof of Proposition 4. Part 1: Labor market tightness is determined by $\Gamma(\theta; i) = 0$ where from (77)

$$\Gamma(\theta; i) \equiv (1-\beta)\left\{\alpha^s(\theta)\mu\left\{\chi^m S^m[a^*(\theta; i), Z(\theta; i)] + \chi^d S^d[Z(\theta; i)]\right\} + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)},$$

and we make the relationships between a^* and i and Z and i explicit. When $i = 0$,

$$\Gamma(\theta; 0) \equiv (1-\beta)\left\{\frac{\alpha^s(\theta)\mu(\rho+\lambda+\gamma)}{\rho+\lambda+\gamma+\alpha^b(\theta)(1-\mu)}[\upsilon(y^*) - \varphi(y^*)] + x - b\right\} - \beta k\theta - (\rho+\delta)\frac{k\theta}{f(\theta)}. \tag{78}$$

It is monotone decreasing in θ, so equilibrium at the Friedman rule is unique.

We now show that $\Gamma(\theta; i)$ is increasing in i in the neighborhood of the Friedman rule. From (8)

$$\frac{\partial S^m(a, Z)}{\partial a} \equiv [\upsilon'(y) - \varphi'(y)] \frac{\partial y}{\partial a}.$$

In the neighborhood of $i = 0^+$, $y = y^*$ and $\upsilon'(y) - \varphi'(y) = 0$. Hence, $\partial S^m(a, Z)/\partial a = 0$. The effect of a change in a^* induced by an increase in i on the match surplus is second order when i is close to 0 because the match surplus is maximum. However, from (8)-(9), when y is in the neighborhood of y^*,

$$\frac{\partial S^m(a, Z)}{\partial Z} = \frac{\partial S^d(Z)}{\partial Z} = -1.$$

From (32),

$$\frac{\partial Z}{\partial i} = \frac{-a^*}{\rho + \lambda + \gamma + \alpha^b(\theta)(1 - \mu)},$$

where, at the Friedman rule, by (12) and (37)

$$a^* = \varphi(y^*) + \frac{(\rho + \lambda + \gamma)\,\mu}{\rho + \lambda + \gamma + \alpha^b(\theta)(1 - \mu)} [y^* - \varphi(y^*)] > 0.$$

Combining these results, $\chi^m S^m [a^*(\theta; i), Z(\theta; i)] + \chi^d S^d [Z(\theta; i)]$ is increasing in i, and hence $\Gamma(\theta; i)$ is also increasing in i. It follows that θ such that $\Gamma(\theta; i) = 0$ is increasing in i. The unemployment rate, $u = \delta/[\delta + f(\theta)]$, is decreasing in θ and hence decreasing in i. By the definition of $\Gamma(\theta; i)$ and (33), we can reexpress $\Gamma(\theta; i)$ as

$$\Gamma(\theta; i) = \frac{(1 - \beta)}{\beta}(w - b) - k\theta - (\rho + \delta)\frac{k\theta}{f(\theta)}.$$

Since $\Gamma(\theta; i) = 0$ in equilibrium, w and θ must comove as i changes. Therefore, w rises in i.

Part 2: We now consider the limiting case $i = +\infty$. Since agents do not carry money, the outcome is similar to a pure credit economy but with $\chi^d < 1$. From (31) $a^* = 0$. By the steps leading to (37)

$$Z = \frac{\alpha^b(\theta)\chi^d(1 - \mu)}{\rho + \lambda + \gamma + \alpha^b(\theta)\chi^d(1 - \mu)} [\upsilon(y^*) - \varphi(y^*)]$$

and hence

$$\Gamma(\theta; +\infty) \equiv (1 - \beta)\left\{\frac{\alpha^s(\theta)\mu\chi_d\,(\rho + \lambda + \gamma)}{\rho + \lambda + \gamma + \alpha^b(\theta)\chi^d(1 - \mu)} [\upsilon(y^*) - \varphi(y^*)] + x - b\right\} - \beta k\theta - (\rho + \delta)\frac{k\theta}{f(\theta)}. \tag{79}$$

From (78) and (79), $\Gamma(\theta; +\infty) < \Gamma(\theta; 0)$ for all $\theta > 0$. So θ_0 solution to $\Gamma(\theta; 0) = 0$ is larger than $\theta_{+\infty}$ solution to $\Gamma(\theta; +\infty) = 0$. Hence, the unemployment rate when $i = 0$ is lower than the unemployment rate when $i = +\infty$. Since w and θ comove, w is lower when $i = +\infty$ than when $i = 0$. Finally, note that there is a finite upper bound for i above which a monetary equilibrium does not exist. This upper bound is $i = \chi^m(1 - \mu)\alpha^b(\theta)/\mu$ where θ is a bounded above by the market tightness of a pure credit economy with $Z = 0$. This upper bounded of θ is finite and it solves

$$(1 - \beta)\{\alpha^s(\theta)\mu [\upsilon(y^*) - \varphi(y^*)] + x - b\} - \beta k\theta - (\rho + \delta)\frac{k\theta}{f(\theta)} = 0.$$

■

Proof of Proposition 5. From (29),

$$\frac{q}{\bar{\alpha}(q)} \to \frac{Af(\theta)}{\lambda\omega\left[\delta + f(\theta)\right]} = +\infty \text{ for all } \theta > 0.$$

So, if $\theta > 0$, $q \to +\infty$ and the firm matching rate with a consumer tends to $A\bar{\alpha}(q)/q \to \lambda\omega\left[\delta + f(\theta)\right]/f(\theta)$. Since $A\bar{\alpha}(q) \to +\infty$, from (38), $Z \to \upsilon(y^*) - \varphi(y^*)$. Using that $S^m(a^*, Z) \to 0$ and $S^d(Z) \to 0$, from (30), θ solves

$$(\rho + \delta)\frac{k\theta}{f(\theta)} = (1 - \beta)(x - b) - \beta k\theta.$$

It is consistent with $\theta > 0$ iff $x > b$. Hence, if $x \leq b$ then $\theta \to 0$ as $A \to +\infty$. Suppose $q \to q_\infty > 0$. Then, $\alpha^b = A\bar{\alpha}(q) \to +\infty$ and, from (32), $Z \to \upsilon(y^*) - \varphi(y^*)$. If $q_\infty = 0$, then $A\bar{\alpha}(q)/q \to +\infty$ and, from (30), $\theta = 0$ implies $Z \to \upsilon(y^*) - \varphi(y^*)$. ■

Proof of Proposition 6. The current-value Hamiltonian is

$$\begin{aligned}
\mathcal{H} \;\equiv\;& \omega_1 \alpha\left(\frac{n}{\omega_1}\right)\left\{\chi^m\left[\upsilon(y_m) - \varphi(y_m)\right] + \chi^d\left[\upsilon(y_d) - \varphi(y_d)\right]\right\} + n(x - b) + b - (1 - n)\theta k \\
&+\xi\left\{f(\theta)(1 - n) - \delta n\right\} \\
&+\zeta\left\{\lambda(\omega - \omega_1) - \left[\alpha\left(\frac{n}{\omega_1}\right) + \gamma\right]\omega_1\right\}.
\end{aligned}$$

From Pontryagin's Maximum Principle,

$$(\theta_t, y_{m,t}, y_{d,t}) \in \arg\max \mathcal{H}(\theta_t, y_{m,t}, y_{d,t}, n_t, \omega_{1,t}) \quad \text{for all } t.$$

Hence, θ_t solves (55) and $y_{m,t} = y_{d,t} = y^*$ for all t. The law of motion for the co-state variable, ξ_t, are given by $\rho\xi_t = \partial\mathcal{H}_t/\partial n_t + \dot{\xi}_t$, i.e.,

$$\rho\xi_t = \alpha'(q_t)\left[\upsilon(y^*) - \varphi(y^*) - \zeta_t\right] + x - b + \theta_t k - \left[f(\theta_t) + \delta\right]\xi_t + \dot{\xi}_t. \tag{80}$$

Similarly, the law of motion for ζ_t are given by $\rho\zeta_t = \partial\mathcal{H}_t/\partial\omega_{1,t} + \dot{\zeta}_t$, i.e.,

$$(\rho + \lambda + \gamma)\zeta_t = \left[\alpha(q_t) - q_t\alpha'(q_t)\right]\left[\upsilon(y^*) - \varphi(y^*) - \zeta_t\right] + \dot{\zeta}_t. \tag{81}$$

Combining (55) and (81), the optimality condition for labor market tightness can be rewritten as

$$\begin{aligned}
(\rho + \delta)\frac{k\theta_t}{f(\theta_t)} \;=\;& \epsilon_f(\theta_t)\left\{\frac{\alpha(q_t)}{q_t}\epsilon_\alpha(q_t)\left[\upsilon(y^*) - \varphi(y^*) - \zeta_t\right] + x - b\right\} \\
& - \left[1 - \epsilon_f(\theta_t)\right]k\theta_t - \frac{f''(\theta_t)}{f(\theta_t)f'(\theta_t)}k\theta_t\dot{\theta}_t.
\end{aligned} \tag{82}$$

Using the definition of ϵ_α, the law of motion for ζ_t, (81) can be rewritten as

$$(\rho + \lambda + \gamma)\zeta_t = \alpha(q_t)\left[1 - \epsilon_\alpha(q_t)\right]\left[\upsilon(y^*) - \varphi(y^*) - \zeta_t\right] + \dot{\zeta}_t. \tag{83}$$

From (82) and (83), a constant solution to the planner's problem is a pair (θ, ζ) solution to (56)-(57), provided that $n_0 = n_s \equiv f(\theta)/[f(\theta)+\delta]$ and $\omega_{1,0} = \omega_1^s \equiv \lambda/[\alpha(\theta)+\gamma+\lambda]$.

To see that the stationary solution is unique, eliminate ζ from (57) by using (56), then θ solves

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = \epsilon_f(\theta)\left\{\frac{\alpha(q)}{q}\epsilon_\alpha(q)\frac{\rho+\lambda+\gamma}{\rho+\lambda+\gamma+\alpha(q)(1-\epsilon_\alpha)}\left[\upsilon(y^*)-\varphi(y^*)\right]+x-b\right\}-\left[1-\epsilon_f(\theta)\right]k\theta. \quad (84)$$

By (29), q falls in θ. Since $\alpha'', f'' < 0$, the elasticities ϵ_α and ϵ_f decrease in θ. Therefore, the right of (84) falls and the left side rises in θ. Hence the stationary solution is unique.

We now show that the constant solution is a solution to the planner's problem by invoking the Mangasarian sufficiency condition. First, the current-value Hamiltonian is jointly concave in (ι, n, ω_1) where $\iota \equiv (1-n)\theta$. To see this note that $\omega_1\alpha(n/\omega_1)$ is jointly concave in (ω_1, n). Also $f(\theta)(1-n) = f(\iota/u)u$ is jointly concave in (ι, u). Finally, the constant solution satisfies the following transversality conditions,

$$\begin{aligned}\lim_{t\to+\infty} e^{-\rho t}\xi_t n_t &= 0\\ \lim_{t\to+\infty} e^{-\rho t}\zeta_t \omega_{1,t} &= 0.\end{aligned}$$

We now compare the optimality conditions with the equilibrium conditions. Since the planner requires $y_{m,t} = y^*$, monetary policy must implement the Friedman rule, i.e., $i_t = 0$. From (14), Z at the Friedman rule solves

$$(\rho+\lambda+\gamma)Z = \alpha(1-\mu)\left[\upsilon(y^*)-\varphi(y^*)-Z\right]. \quad (85)$$

From (22), θ at the Friedman rule solves

$$(\rho+\delta)\frac{k\theta}{f(\theta)} = (1-\beta)\left\{\frac{\alpha(q)}{q}\mu\left[\upsilon(y^*)-\varphi(y^*)-Z\right]+x-b\right\}-\beta k\theta. \quad (86)$$

The comparison of the planner's optimality conditions, (56)-(57), and the equilibrium conditions (85)-(86), show that they coincide if the Hosios conditions in the goods and labor markets, (58)-(59), hold. ■

Proof of Proposition 7. *Part 1:* When $\lambda = +\infty$, by (30)-(33), , $\omega_{1,t} = \omega$, $Z = 0$, and a steady state equilibrium can be reduced to a 3-tuple, (θ, a, w), solution to:

$$\begin{aligned}(\rho+\delta)\frac{k\theta}{f(\theta)} &= (1-\beta)\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a,0)+\chi^d S^d(0)\right]+x-b\right\}-\beta k\theta \quad (87)\\ a &\in \arg\max_{\hat{a}\geq 0}\left\{-ia+\alpha^b(\theta)(1-\mu)\chi^m S^m(a,0)\right\}\\ w &= \beta\left\{\alpha^s(\theta)\mu\left[\chi^m S^m(a,0)+\chi^d S^d(0)\right]+x\right\}+(1-\beta)b+\beta k\theta.\end{aligned}$$

By (54), the instantaneous surplus becomes

$$\begin{aligned}\varpi &= \omega\alpha\left(\frac{n}{\omega}\right)\left\{\chi^m\left[\upsilon(y_m)-\varphi(y_m)\right]+\chi^d\left[\upsilon(y_d)-\varphi(y_d)\right]\right\} \quad (88)\\ &\quad +n(x-b)+b-(1-n)\theta k.\end{aligned}$$

The planner maximizes ϖ by choosing the policy rate i, subject to the law of motion of workers (26) and the free entry of firms (87).

We first eliminate y^m in ϖ with the free entry condition. By (87),

$$\alpha^s(\theta)\left[\chi^m S^m(a,0)+\chi^d S^d(0)\right]=\frac{1}{\mu(1-\beta)}\left[(\rho+\delta)\,\frac{k\theta}{f(\theta)}+\beta k\theta\right]-\frac{x-b}{\mu}. \tag{89}$$

Since $S^m(a,0)=\upsilon(y_m)-\varphi(y_m)$ and $S^d(0)=y_d-\varphi(y_d)$, we can eliminate the trade surplus in the planner's objective function in (88) by (89). Also, we can eliminate n by using $n=f(\theta)/[\delta+f(\theta)]$. Therefore, we can rewrite the planner's problem as $\max_\theta \varpi(\theta)$ where

$$\varpi(\theta)\equiv\frac{f(\theta)}{\delta+f(\theta)}\left[\frac{k\theta}{\mu(1-\beta)}\left(\frac{\rho+\delta[1-\mu(1-\beta)]}{f(\theta)}+\beta\right)-\left(\frac{1}{\mu}-1\right)(x-b)\right]+b.$$

The expression in the square bracket rises in θ. When the square bracket is positive, the right side strictly rises in θ. Therefore, the maximizer of $\varpi(\theta)$ is the largest element in the set of feasible market tightness, call it $\bar{\theta}$, provided that $\varpi(\bar{\theta})>b$. Since the free entry condition (87) defines a positive relationship between θ and y_m, the maximum tightness, $\bar{\theta}$, is achieved if $y_m=y_d=y^*$, which corresponds to the Friedman rule.

Now we argue $\varpi(\bar{\theta})>b$. Let θ_0 be the value of θ such that the right side of (89) is zero. At $\theta=\theta_0$, by (88),

$$\varpi(\theta_0)=\frac{\theta k}{\delta+f(\theta)}\left(\frac{\rho+\beta[\delta+f(\theta)]}{1-\beta}\right)+b.$$

Since $\varpi(\theta_0)>b$, $\varpi(\theta)$ rises in θ for all $\theta\geq\theta_0$. Since $\bar{\theta}>\theta_0$, $\varpi(\bar{\theta})>b$.

Part 2: When $\lambda<+\infty$, we can rewrite (54) by replacing $\omega_1=\lambda/[\alpha(\theta)+\gamma+\lambda]$ and $n=f(\theta)/[\delta+f(\theta)]$:

$$\begin{aligned}\varpi(\theta,y_m,y_d) &= \frac{\lambda\alpha(\theta)}{\alpha(\theta)+\gamma+\lambda}\left\{\chi^m\left[y_m-\varphi(y_m)\right]+\chi^d\left[y_d-\varphi(y_d)\right]\right\}\\ &\quad+\frac{f(\theta)}{f(\theta)+\delta}(x-b)+b-\frac{\delta\theta}{\delta+f(\theta)}k.\end{aligned}$$

At the Friedman rule, an increase in i reduces y_m and Z, but the change in y^m only has a second-order effect on firms' profits and market tightness. Hence the only first-order effect is due to the drop in Z. As Z falls, firms get more profits from trade and thus θ rises. Hence a local derivation from the Friedman rule exists when the derivative of $\varpi(\theta,y_m,y_d)$ with respect to θ is positive at the Friedman rule, fixing $y_m=y_d=y^*$, i.e.

$$\frac{\partial\varpi(\theta,y^*,y^*)}{\partial\theta}\propto\epsilon_f(\theta)\left[\frac{\alpha^s(\theta)S(y^*,0)(\gamma+\lambda)}{\gamma+\lambda+\alpha(\theta)(1-\epsilon_\alpha)}\epsilon_\alpha(\theta)+(x-b)\right]-\left(\frac{1}{n}-\epsilon_f(\theta)\right)k\theta \tag{90}$$

is strictly positive. The right side of (90) is equivalent to that of (84) when $\rho=0$. Hence the right side of (90) falls in θ. Since we have assumed $f'(0)=+\infty$, right side explodes as $\theta\to 0$. Thus the Friedman rule is suboptimal if and only if θ is sufficiently small.

The labor tightness, θ, at the Friedman rule solves (60). There is a unique solution of θ because the right side of (60) falls and the left side rises in θ. It is easy to check that the solution of θ rises in μ and falls in β. Given μ, if $\beta\to 1$, then $\theta\to 0$. Hence there exists a $\bar{\beta}(\mu)<1$ such that the Friedman rule is locally suboptimal if and only if $\beta>\bar{\beta}(\mu)$. Moreover, $\bar{\beta}(\mu)$ rises in μ because the market tightness θ at the FR increases in μ by (60). ■

Proof of Lemma 1. The inequality (61) holds iff $\varepsilon > \varepsilon_R$ where the reservation value for the preference shock, $\varepsilon_R(a^*, Z)$, solves

$$\max_{y \geq 0} \{\varepsilon_R \upsilon(y) - \varphi(y)\} = Z \;\text{ s.t. }\; \varphi(y) \leq a^*. \tag{91}$$

We distinguish two cases depending on whether or not the constraint, $\varphi(y) \leq a^*$, binds.

Case #1. If the constraint $\varphi(y) \leq a^*$ is not binding, then $\varepsilon_R = \hat{\varepsilon}(Z)$ is the solution to $\varepsilon\upsilon(y^*_\varepsilon) - \varphi(y^*_\varepsilon) = Z$ where $y^*_\varepsilon = \arg\max\{\varepsilon\upsilon(y) - \varphi(y)\}$. We now check the condition under which $\varphi(y^*_{\hat{\varepsilon}}) \leq a^*$ is slack. Using that φ is an increasing bijection, we have:

$$\varphi(y^*_{\hat{\varepsilon}}) \leq a^* \Longleftrightarrow y^*_{\hat{\varepsilon}} \leq \varphi^{-1}(a^*).$$

Apply the increasing function φ'/υ' on both sides to obtain:

$$\varphi(y^*_{\hat{\varepsilon}}) \leq a^* \Longleftrightarrow \frac{\varphi'(y^*_{\hat{\varepsilon}})}{\upsilon'(y^*_{\hat{\varepsilon}})} \leq \frac{\varphi' \circ \varphi^{-1}(a^*)}{\upsilon' \circ \varphi^{-1}(a^*)}.$$

Use the definition of $y^*_{\hat{\varepsilon}}$, i.e., $\varphi'(y^*_{\hat{\varepsilon}}) = \hat{\varepsilon}\upsilon'(y^*_{\hat{\varepsilon}})$, to rewrite the inequality above as:

$$\varphi(y^*_{\hat{\varepsilon}}) \leq a^* \Longleftrightarrow \hat{\varepsilon} \leq \tilde{\varepsilon}(a^*) \equiv \frac{\varphi' \circ \varphi^{-1}(a^*)}{\upsilon' \circ \varphi^{-1}(a^*)}.$$

Case #2. If the constraint, $\varphi(y) \leq a^*$, is binding then $y = \varphi^{-1}(a^*)$ so that ε_R solves $\varepsilon_R \upsilon[\varphi^{-1}(a^*)] - a^* = Z$. Solving for ε_R we obtain $\varepsilon_R = (a^* + Z)/\upsilon[\varphi^{-1}(a^*)]$. ■

Proof of Proposition 8. Part 1. From (67), $y_\varepsilon = y^*_\varepsilon$ for all $\varepsilon \geq \varepsilon_R$, i.e., agents trade the efficient quantities in all matches where there are gains from trade. This requires $a^* = \varphi(y^*_{\bar{\varepsilon}}) + \mu\left[\bar{\varepsilon}\upsilon(y^*_{\bar{\varepsilon}}) - \varphi(y^*_{\bar{\varepsilon}}) - Z\right]$.

We now prove that $\varepsilon_R(a^*, Z) = \hat{\varepsilon}(Z)$. From (66), $Z < \bar{\varepsilon}\upsilon(y^*_{\bar{\varepsilon}}) - \varphi(y^*_{\bar{\varepsilon}})$ since otherwise $S^m_\varepsilon(a^*, Z) = S^d_\varepsilon(Z) = 0$ and

$$(\rho + \lambda + \gamma) Z = \alpha(1 - \mu) \int \left[\chi^m S^m_\varepsilon(a^*, Z) + \chi^d S^d_\varepsilon(Z)\right] dF(\varepsilon) = 0,$$

which is a contradiction. Using that $Z < \bar{\varepsilon}\upsilon(y^*_{\bar{\varepsilon}}) - \varphi(y^*_{\bar{\varepsilon}})$, $\varphi^{-1}(a^*) > y^*_{\bar{\varepsilon}}$ and $\tilde{\varepsilon}(a^*) \equiv \varphi'[\varphi^{-1}(a^*)]/\upsilon'[\varphi^{-1}(a^*)] > \varphi'(y^*_{\bar{\varepsilon}})/\upsilon'(y^*_{\bar{\varepsilon}}) = \bar{\varepsilon}$. Moreover, $\hat{\varepsilon}(Z) < \bar{\varepsilon}$. Hence, by Lemma 1, $\varepsilon_R(a^*, Z) = \hat{\varepsilon}(Z)$.

We now show how to reduce an equilibrium to a pair (θ, Z) solution to two equations. From (70),

$$\frac{\lambda\omega q}{\gamma + \alpha(q)\left[1 - F(\hat{\varepsilon})\right] + \lambda} = \frac{f(\theta)}{\delta + f(\theta)}. \tag{92}$$

From (92), $q = Q(\theta, \hat{\varepsilon})$ where Q is increasing in θ and decreasing in $\hat{\varepsilon}$. From (66), Z solves

$$(\rho + \lambda + \gamma) Z = \alpha\left[Q(\theta, \hat{\varepsilon}(Z))\right](1 - \mu) \int_{\hat{\varepsilon}(Z)}^{\bar{\varepsilon}} S^d_\varepsilon(Z) dF(\varepsilon). \tag{93}$$

From (93), Z is an increasing function of θ. From (68), θ solves

$$(\rho + \delta)\frac{k\theta}{f(\theta)} + \beta k\theta = (1 - \beta)\left[\alpha^s\left[Q(\theta, \hat{\varepsilon}(Z))\right]\mu \int_{\hat{\varepsilon}(Z)}^{\bar{\varepsilon}} S^d_\varepsilon(Z) dF(\varepsilon) + x - b\right]. \tag{94}$$

When Z is close to $\bar{\varepsilon}\upsilon(y^*_{\bar{\varepsilon}}) - \varphi(y^*_{\bar{\varepsilon}})$, the effect of a change in Q on the right side is negligeable. In that case, θ is a decreasing function of Z. From (93), as $\rho + \lambda + \gamma$ approaches 0, for all $\theta > 0$, Z approaches $\bar{\varepsilon}\upsilon(y^*_{\bar{\varepsilon}}) - \varphi(y^*_{\bar{\varepsilon}})$. From (94), θ approaches the positive solution to

$$(\rho + \delta) \frac{k\theta}{f(\theta)} + \beta k\theta = (1 - \beta)(x - b).$$

An increase in λ or γ shifts the curve representing (93) downward in the space (θ, Z). See Figure 13. As a result Z decreases while θ increases. It follows that $\varepsilon_R = \hat{\varepsilon}(Z)$ decreases.

Part 2. Consider now a small increase in i from $i = 0^+$. We still have $\varepsilon_R(a^*, Z) = \hat{\varepsilon}(Z)$ so that $q = Q(\theta, \hat{\varepsilon})$ defined by (92). Any change in a^* only has a second-order effect on $S^m_\varepsilon(a, Z)$, hence (66) is approximated by

$$(\rho + \lambda + \gamma) Z = -ia^* + \alpha [Q(\theta, \hat{\varepsilon})] (1 - \mu) \int S^d_\varepsilon(Z) dF(\varepsilon).$$

As i increases, the curve representing (66) shifts downward in the (θ, Z) space. The equilibrium condition (68) is still approximated by (94). Since we start from an equilibrium where the curve representing (94) intersects the curve representing (93) by above, θ increases, Z decreases, and $\varepsilon_R = \hat{\varepsilon}$ decreases. ■

Proof of Proposition 9. Part 1. From (74), for given s_m, market tightness increases with the expected surplus in type-2 matches,

$$\chi^\ell S^\ell \left[a^2(s_g), Z_2\right] + \chi^d S^d(Z_2).$$

If $s_g = 0$, i.e., $i_g = i$, a small increase in s_g reduces y_2 below y^*, from (76), which has a second-order effect on S^ℓ. From (72), it reduces Z_2, which has a first-order and positive effect on S^ℓ and S^d. As a result, θ increases and u decreases. By the same reasoning as in the proof of Part 2 of Proposition 4, market tightness when $s_g = 0$ is larger than market tightness when $s_g = +\infty$,

$$\theta|_{s_g=0} > \theta|_{s_g=+\infty}.$$

Since $s_g \leq i$, there exists a threshold for i, denoted $\bar{\imath}$, such that if $i > \bar{\imath}$, then $\theta|_{s_g=0} > \theta|_{s_g=i}$ and $u|_{s_g=0} < u|_{s_g=i}$.

Part 2. The minimum level of unemployment is implemented when θ solution to (74) is maximum. It follows that the optimal spreads are such that

$$s^*_j \in \arg\max_{s_j \geq 0} \left\{\chi^\ell S^\ell \left[a^\kappa(s_j), Z_\kappa(s_j)\right] + \chi^d S^d[Z_\kappa(s_j)]\right\}, \ (j, \kappa) \in \{m, g\} \times \{1, 2\}.$$

By the same reasoning as before, a solution exists and it is such that $s^*_j > 0$. ■

Proof of Proposition 10. Let the expected trade surplus generated by a type-κ consumer be

$$\mathcal{S}_\kappa(s_j) \equiv \chi^\ell S^\ell \left[a^\kappa(s_j), Z_\kappa(s_j)\right] + \chi^d S^d \left[Z_\kappa(s_j)\right], \ (j, \kappa) \in \{m, g\} \times \{1, 2\}.$$

We assume s^*_j is the maximizer of $\mathcal{S}_\kappa(s_j)$. To show $s^*_g < s^*_m$, differentiate $\mathcal{S}_\kappa(s_j)$ with respect to s_j, taking the market tightness θ as given:

$$\begin{aligned}
\frac{\partial \mathcal{S}_\kappa(s_j)}{\partial s_j} &= -\frac{\partial Z_\kappa}{\partial s_j} + \chi^\ell \frac{\partial S^\ell\left[a^\kappa(s_j), Z_\kappa\right]}{\partial a^\kappa(s_j)} \frac{\partial a^\kappa(s_j)}{\partial s_j} \\
&= -\frac{\partial Z_\kappa}{\partial s_j} + \frac{s_j}{\alpha(1-\mu)} \frac{\partial a^\kappa(s_j)}{\partial s_j}.
\end{aligned}$$

Next we consider the cross derivative with respect to s_j and λ_κ by differentiating the expression above with respect to λ_κ:

$$\begin{aligned}
\frac{\partial^2 \mathcal{S}_\kappa(s_j)}{\partial s_j \partial \lambda_\kappa} &= -\frac{\partial^2 Z_\kappa}{\partial s_j \partial \lambda_\kappa} + \frac{s_j}{\alpha(1-\mu)} \frac{\partial^2 a^\kappa(s_j)}{\partial s_j \partial \lambda_\kappa} \\
&= \frac{1}{\rho + \lambda_\kappa + \gamma + \alpha(1-\mu)} \frac{\partial Z_\kappa}{\partial s_j} \left[1 - \frac{\mu Z_\kappa}{a^\kappa(s_j)} + \frac{s_j}{\alpha(1-\mu)}\mu\right].
\end{aligned}$$

By the logic of monotone comparative statics, s_j^* falls in λ_κ if the cross derivative is negative for all relevant choice of s_j. The derivative $\partial Z_\kappa / \partial s_j$ is negative. As $\lambda_\kappa \to \infty$, $Z_\kappa \to 0$ and the square bracketed term is positive. Hence the optimal choice of s_j^* falls in λ_κ. ■

B Alternative Interpretation of Production Cost

Previously we assume the production cost $\varphi(y)$ is a disutility paid by the entrepreneur or manager. An alternative interpretation is that $\varphi(y)$ is a production cost paid by the worker and it is compensated by the wage w. The interpretation of $\varphi(y)$ does not matter for allocations, but it affects the definition of wage w in (18) and thus the calculations of wage markdown. In this section we provide the formulas for wage and wage markdown under this alternative interpretation.

Given the new interpretation of w and $\varphi(y)$, the worker now pays the variable cost of productions, hence equation (18) now becomes

$$\rho E = w - \alpha^s \left[\chi^m \varphi(y^m) + \chi^d \varphi(y^*)\right] - \delta \beta J. \tag{95}$$

By the logic leading to (23), the wage w can be reexpressed as

$$w = \beta \left\{\alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] + x\right\} + \alpha^s \left[\chi^m \varphi(y^m) + \chi^d \varphi(y^*)\right] + (1-\beta) b + \beta k \theta. \tag{96}$$

When compared with (23), the key novelty is the presence of the second term on the right side, which represents the compensation to the worker for the variable cost of production.

We can compute the wage markdown as in (25). But since the variable cost of production is paid by the worker and compensated by wages, we do not need to subtract the variable cost when calculating the net expected revenue of a firm. Hence $\hat{x} = \mathbb{E}[p] + x$ or equivalently

$$\hat{x} = \alpha^s \left\{\chi^m \varphi(y^m) + \chi^d \varphi(y^*) + \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right]\right\} + x. \tag{97}$$

The markdown is

$$\begin{aligned}
MKDOWN &\equiv \frac{\hat{x} - w}{\hat{x}} \\
&= \frac{(1-\beta)\left\{\alpha^s \mu \left[\chi^m S^m(a^*, Z) + \chi^d S^d(Z)\right] + x - b\right\} - \beta k \theta}{\hat{x}}.
\end{aligned} \tag{98}$$

The numerator is the same as that in (25), which represents the net profit of the firm-work match. But now the net expected revenue of the firm in the denominator includes the variable cost of production, as captured by the difference between (24) and (97).

C Estimation Procedure for the Frequency of Purchases in the Consumer Expenditure Survey (CEX)

Consumer Expenditure Survey (CEX) Table R-1 reports the fraction of households who spent a positive amount in detailed spending categories either in a given week (in the diary survey) or in a given quarter (in the interview survey). We use this measure of the extensive margin of spending to estimate the frequency of purchases of a representative household within a month. Formally, we re-interpret the fraction of households who spent in category j as the probability a representative household had an expenditure in category j over the given time period. We first convert all probabilities to a weekly frequency, where π_j^w gives the probability of spending in expense category j over a week. If we treat the event of spending in category j in a given week as Bernoulli random variable, then the expected duration until a spending event is given by $1/\pi_j^w$ weeks or, converting to the time period of the model, $1/(4\pi_j^w)$ months. The expected number of spending events within a month is then given

This implies the average number of spending events within a month is simply $4 * \pi_j^w$.

We report 26 major spending categories, however the CEX does not report the fraction of households with positive purchases for this level of aggregation. We first estimate the average number of spending events for the lowest sub-classifications, (e.g. "ready-to-eat and cooked cereals" as a component of "food at home") and then average using spending shares.

D Competitive Search

In this section we consider a version of our model where the terms of trade in the goods and labor markets are determined by competitive search. The goal of the exercise is to show that the Hosios conditions (58) and (59) are satisfied under competitive search. We assume the monetary policy is at the Friedman rule level, $i_t = 0$.

D.1 Goods market

The goods market is similar to that in Rocheteau and Wright (2005). The firm-worker pair posts a pair of payment and market tightness to maximize consumer's surplus, subject to the profit being higher than the equilibrium level of profit Π^*

Expenditure Category	Purchases per month	Avg. Expenditures ($)	Share (%)
Housekeeping supplies	5.41	803	1.2
Food at home	4.25	5,259	7.9
Shelter	3.45	13,258	19.8
Utilities fuels and public services	3.16	4,223	6.3
Gasoline other fuels and motor oil	2.07	2,148	3.2
Food away from home	1.60	3,030	4.5
Health insurance	1.32	3,704	5.5
Personal care services	0.78	384	0.6
Drugs	0.56	498	0.7
Alcoholic beverages	0.55	554	0.8
Personal care products	0.26	385	0.6
Reading	0.22	114	0.2
Other vehicle expenses	0.15	3,534	5.3
Apparel and services	0.09	1,754	2.6
Household operations	0.07	1,638	2.4
Vehicle purchases (net outlay)	0.06	4,828	7.2
Public and other transportation	0.05	452	0.7
Household furnishings and equipment	0.04	2,701	4.0
Medical services	0.03	1,070	1.6
Tobacco products and smoking supplies	0.03	341	0.5
Personal insurance and pensions	0.03	7,873	11.8
Medical supplies	0.02	181	0.3
Entertainment	0.02	3,568	5.3
Cash contributions	0.02	2,415	3.6
Education	0.01	1,226	1.8
Miscellaneous	<0.00	986	1.5
Total expenditures	1.53	66,928	100

Table 3: Average Purchase Frequency, Mean Annual Expenditures, and Expenditure Shares from the 2021 Consumer Expenditure Survey (CEX)

$$\max_{p_m, p_d, , n} \{\alpha(n)[\chi^m (\upsilon(y_m) - p_m) + \chi^d (\upsilon(y_d) - p_d)]\}$$
$$s.t. \quad \frac{\alpha(n)}{n}\{\chi^m [p_m - \varphi(y_m)] + \chi^d [p_d - \varphi(y_d)]\} = \Pi^*.$$

Replace $\chi^m p_m + \chi^d p_d$ in the objective function by the constraint. The optimal choice of quantities are $y_m = y_d = y^*$. The first-order condition with respect to n is given by

$$\alpha'(n)[\upsilon(y^*) - \varphi(y^*) - Z] = \Pi^*.$$

Therefore, using the constraint in the optimization problem, the share of surplus to the firm equals the elasticity of the matching function, i.e.

$$\frac{\chi^m p_m + \chi^d p_d - \varphi(y^*)}{\upsilon(y^*) - \varphi(y^*) - Z} = \frac{\alpha'(n)n}{\alpha(n)} \equiv \epsilon_\alpha.$$

D.2 Labor market

Now we derive the firm's surplus in the labor market. The continuation value of an employed worker is

$$\rho E = w + \delta(U - E).$$

As mentioned above, the equilibrium surplus in the goods market is given by $S^d(Z)$ and the firm's share of surplus is ϵ_α. Therefore, the surplus of the firm-worker pair is given by

$$(\rho + \delta)J = \alpha^s \epsilon_\alpha S^d(Z) + x - \rho U. \tag{99}$$

The firms maximize workers' expected payoff subject to the free entry condition, namely

$$\max_{\theta, w} \{f(\theta)\,(E - U)\}$$
$$s.t. \quad \frac{f(\theta)}{\theta}\,\{J - E + U\} = k.$$

Using the constraint to eliminate E, the problem can be rewritten as

$$\max_{\theta}\,\{f(\theta)J - k\theta\}$$

where J is independent of θ by (99). The first-order condition implies the share of surplus for the firm is

$$\frac{J - E + U}{J} = \frac{\theta f'(\theta)}{f(\theta)} \equiv \epsilon_f.$$

Since Proposition 6 shows that $\mu = \epsilon_\alpha$ and $\beta = 1 - \epsilon_f$ lead to a constrained-efficient outcome, the allocation under competitive search is efficient.

E Tractable special case when $\rho = 0$ and $\delta \to 0$

For intuition, we would like to understand how $\partial\theta/\partial i$ depends on the size of λ. To this end, consider the special case when agents are infinitely patient, $\rho = 0$, and the job separation rate vanishes, $\delta \to 0$. As δ vanishes, $n \to 1$ and $q \to 1/\omega$. The unemployment rate is $u = 0$ but θ is still endogenous and determined by the free entry condition. This limit is not necessarily realistic but is instructive because of its tractability. At this limit, θ and Z are given by

$$\beta k\theta = (1-\beta)\left\{\alpha^s(1/\omega)\mu\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right] + x - b\right\} \tag{100}$$

$$(\lambda + \gamma)\,Z = -ia + \alpha^b(1/\omega)(1-\mu)\left[\chi^m S^m(a,Z) + \chi^d S^d(Z)\right]. \tag{101}$$

The market tightness, θ, is linear in the surplus of a firm-work pair. Hence the derivative of θ with respect to i at $i = 0$ is proportional to the change in Z, namely (43) holds. Equation (44) can be derived by differentiating (101) with respect to i. The money holding, a, and outside option, Z, are given by

$$a = (1-\mu)\varphi(y^*) + \mu\nu(y^*) - \mu Z, \quad Z = \frac{\alpha^b(1/\omega)(1-\mu)[\varphi(y^*) - \nu(y^*)]}{\lambda + \gamma + \alpha^b(1/\omega)(1-\mu)}.$$

Claim 1 *Assume $i = \rho = 0$ and consider the limit $\delta \to 0$. The derivative $\partial\theta/\partial i$ is non-negative and falls in β. As $\lambda + \gamma$ rises from 0 to $+\infty$, $\partial\theta/\partial i$ falls monotonically if $\mu \leq \varphi(y^*)/[\nu(y^*) - \varphi(y^*)]$ and otherwise it is hump-shaped. It vanishes as λ explodes.*

Proof. We only show the claim regarding increases in $\lambda + \gamma$ because other claims are straightfoward implications of (43).

Substituting the expressions for a and Z into (43),

$$\frac{\partial\theta}{\partial i} \propto \frac{1}{\lambda + \gamma + \alpha(q)(1-\mu)} \left[(1-\mu)\varphi(y^*) + \mu\nu(y^*) - \mu \frac{\alpha(q)(1-\mu)[\varphi(y^*) - \nu(y^*)]}{\lambda + \gamma + \alpha(q)(1-\mu)} \right].$$

The derivative of the right side with respect to λ is positive if and only if

$$\frac{2\alpha(q)(1-\mu)\mu[\nu(y^*) - \varphi(y^*)]}{(1-\mu)\varphi(y^*) + \mu\nu(y^*)} > \lambda + \gamma + \alpha(q)(1-\mu),$$

which fails when $\lambda + \gamma$ is sufficiently large. Hence $\partial\theta/\partial i$ is either decreasing or hump-shaped in λ. When $\lambda + \gamma = 0$, the inequality holds if and only if $\mu \leq \varphi(y^*)/[\nu(y^*) - \varphi(y^*)]$. ■