OTC Market Theory

Lecture 1 by Pierre-Olivier Weill

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Lecture 2 by Ben Lester (next week)

Federal Reserve Bank of Philadelphia

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OTC: decentralized security markets

- In contrast to all-to-all continuous auction:

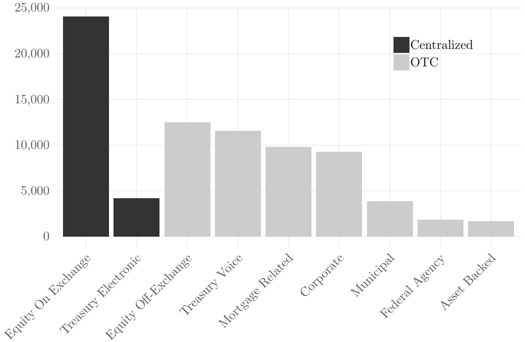
  trade is fragmented in small groups

  price setting involves a form of bargaining

  information about past transactions is often incomplete

- Most fixed income, part of equity, some derivatives

OTC markets are large



- 2018, Billion of 2019 USD, main data source is SIFMA
- Since then, some convergence:

  more off exchange for equity, more electronic for OTC

OTC markets raise policy questions

- Price transparency

  early 2000s

- Market resiliency and systemic risk

  after the Great Financial Crisis

- Unintended consequence of banking regulations

  onset of COVID-19 crisis

a rich theoretical toolbox

- Search theory

  for dynamics, many price setting mechanisms, and GE

- Network theory

  for strategic interactions

- Ben Lester and I will focus on search

a brief ancestry of the search approach

- Demsetz (68) discussed the "demand for immediacy"
- Several papers in market micro structure followed

  Garman (75), Garbade-Silber (76), Amihud-Mendelson (80)

- Search theory took off in the 1980s

  but not much on security markets!

  Bhattacharya-Hagerty (87), Spulber (96), Hall-Rust (03)

what I will do today

- A benchmark model of over-the-counter market

  Duffie-Gârleanu-Pedersen (05) and Hugonnier-Lester-Weill (21)

- Asset prices and liquidity in one particular market structure

  semi-centralized OTC market

investors' preferences

investors' preferences

- $[0, 1]$ of infinitely-lived risk-neutral investors, discount rate $r > 0$

- Can hold $q \in \{0, 1\}$ of some asset in supply $s \in (0, 1)$

- Enjoy flow utility δ for the asset

  changes with Poisson intensity γ

  new δ' drawn according to CDF $F(\delta)$ on $[0, 1]$

  type changes iid across investors

  for simplicity: initial cross-sectional distribution $= F(\delta)$

- What does δ means?

  belief, hedging, consumption opportunities

  Duffie-Gârleanu-Pedersen (02,07), Vayanos-Weill (08)

  Hugonnier (13), Praz (15), Geromichalos-Herrenbrueck (16)

investors' objective

- Given risk-neutrality

  we can substitute budget constraint into objective

- We obtain the intertemporal utility

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\left\{\delta_t q_t\,dt - P_t\,dq_t\right\}\,\middle|\,\delta_0 = \delta\right].$$

- where

  $q_t \in \{0, 1\}$ is the investor's asset holding at time t

  P_t is the price at which the investor trades at t

related specifications in the literature

- Duffie-Gârleanu-Pedersen (05)

  a special case when $F(\delta)$ has two atoms, $\delta_L < \delta_H$

- Gârleanu (09) and Lagos-Rocheteau (09)

  q is unrestricted with general utility flow $u(\delta, q)$

  our setup obtains when $u(\delta, q) = \delta \min\{q, 1\}$

centralized market

solving for equilibrium

- Suppose investors can trade continuously at P

$$\mathbb{E}\left[\int_0^\infty e^{-rt}\left\{\delta_t q_t \, dt - P \, dq_t\right\} \,\middle|\, \delta_0 = \delta\right]$$

solving for equilibrium

- Suppose investors can trade continuously at P

$$P_0 q_0 + \mathbb{E}\left[\int_0^\infty e^{-rt} q_t \left\{\delta_t - r\,P\right\} dt \,\middle|\, \delta_0 = \delta\right]$$

... after integration by part,

solving for equilibrium

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... after integration by part, hence

$$q_t = \begin{cases} 1 & \text{if } r\,P < \delta_t \\ \in \{0,1\} & \text{if } r\,P = \delta_t \\ 0 & \text{if } r\,P > \delta_t \end{cases}$$

$\Rightarrow$ market clearing condition is $1 - F(r\,P-) \le s \le 1 - F(r\;P)$

solving for equilibrium

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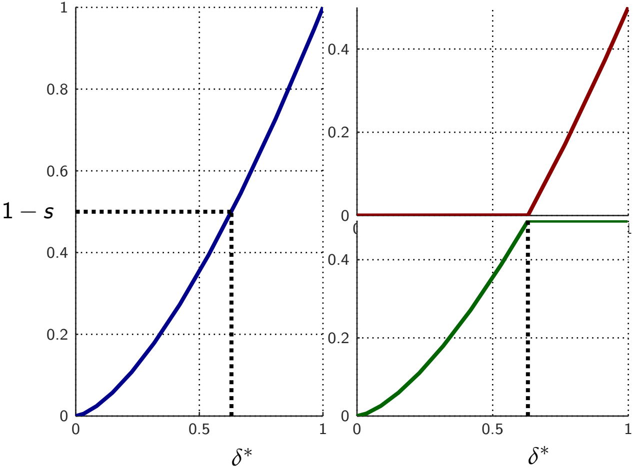
⇒ market clearing condition is $1 - F(r\,P-) \leq s \leq 1 - F(r\ P)$

⇒ equilibrium price is

$$P = \frac{\delta^\star}{r} \text{ where } 1 - F(\delta^\star -) \leq s \leq 1 - F(\delta^\star)$$

the equilibrium in a picture

$$P = \frac{\delta^\star}{r}$$



price vs. buy and hold utility

$$U(\delta) \equiv \mathbb{E}\left[\int_0^\infty e^{-rt}\delta_t \, dt \,\middle|\, \delta_0 = \delta\right] = \frac{r}{r+\gamma}\frac{\delta}{r} + \frac{\gamma}{r+\gamma}\frac{\mathbb{E}\left[\delta'\right]}{r}$$

- Investor starts at δ and then reverts to $\mathbb{E}\left[\delta'\right] = \int \delta' dF(\delta')$

  $\frac{r}{r+\gamma}$: disc fraction of time with δ

  $\frac{\gamma}{r+\gamma}$: disc fraction of time after reversion to $\mathbb{E}\left[\delta'\right]$

- When $s \simeq 0$, $P \simeq 1/r$ is greater than $U(\delta)$ <u>for all</u> δ

  larger than the buy-and-hold valuation of all investors!

  why? b/c of the option to re-trade

semi centralized market

the market structure

- Risk-neutral dealers

  flow utility $\delta = 0$ for the asset

  have access to a centralized inter-dealer market

- Investors <u>must</u> trade through dealers

  contact dealer with Poisson intensity λ

  Nash bargain over the terms of trade

  bargaining power $\theta \in [0, 1]$ for dealer

- An accurate description of many OTC markets

  e.g. corporate bonds: all-to-all trading small

  Hendershott, Livdan, Schürhoff (21)

HJ Bellman equation (1)

$$rV_1(\delta) = \delta + \gamma \int \left[V_1(\delta') - V_1(\delta) \right] dF(\delta') + \lambda \max \left\{ V_0(\delta) - V_1(\delta) + B(\delta), 0 \right\}$$

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Notice the option to re-trade!

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How is the buying “ask” price, $A(\delta)$ determined?

- Investor’s net utility is a fraction $1 - \theta$ of surplus

$$\Rightarrow A(\delta) = (1 - \theta) P + \theta \underbrace{\left[V_1(\delta) - V_0(\delta) \right]}_{\text{reservation value}}$$

HJ Bellman equation (2)

- Define reservation value $\Delta V(\delta) \equiv V_1(\delta) - V_0(\delta)$

$$r\Delta V(\delta) = \delta + \gamma \int \left[\Delta V(\delta') - \Delta V(\delta)\right] dF(\delta') + \lambda(1-\theta)\left[P - \Delta V(\delta)\right]$$

- As-if trade directly in interdealer market

  but with <u>bargaining-adjusted</u> intensity $\lambda(1-\theta)$

- Option value to re-trade

  increases $\Delta V(\delta)$ for seller, $P > \Delta V(\delta)$

  decreases $\Delta V(\delta)$ for buyers, $P < \Delta V(\delta)$

- Take derivatives: $\frac{d}{d\delta}\Delta V(\delta) = \frac{1}{r+\gamma+\lambda(1-\theta)} > 0$

market clearing

The easy way: equate gross asset supply and demand

- flow of assets brought to the market per unit of time:

  λs because contact independent from everything

- flow of investors who leave the market with one unit:

  $\lambda\,[1 - F(\delta^\star)]$, where $\Delta V(\delta^\star) = P$

- Market clearing equation is the same as in centralized market!

  same marginal investor $\delta^\star$ but different price

market price (1)

From Bellman equation we can re-write reservation value "in sequence"

$$\Delta V(\delta) = \mathbb{E}\left[\int_0^\tau e^{-rt}\delta_t\, dt \,\middle|\, \delta_0 = \delta\right] + \mathbb{E}\left[e^{-r\tau}\right] P$$

where $\tau \sim \exp$ with $\lambda(1-\theta)$,

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$$P = \frac{1}{r} \times \underbrace{\frac{\mathbb{E}\left[\int_0^\tau e^{-rt}\delta_t\,dt \,\middle|\, \delta_0 = \delta^\star\right]}{\mathbb{E}\left[\int_0^\tau e^{-rt}\,dt\right]}}_{\text{avg disc type in } [0,\tau] \text{ starting from } \delta_t = \delta^\star}$$

market price (2)

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- Buy-and-hold utility as $\lambda(1-\theta) \to 0$, centralized as $\lambda(1-\theta) \to \infty$

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  option to re-trade larger for buyer: tends to <u>decrease</u> price

- Net effect: which option is more valuable for marginal investor?

  prices increase with $\lambda(1-\theta)$ if $\delta^\star > \mathbb{E}\left[\delta'\right]$

liquidity measures

- Volume
- Liquidity yield spread
- Bid-ask spread

assume for simplicity a continuous CDF $F(\delta)$

volume

- Centralized market:

  each instant, a flow γ of asset holders switch to $\delta < \delta^\star$:

  $$\text{volume} \; = \gamma s F(\delta^\star)$$

- Semi-centralized market:

  search frictions cause volume to be lower:

  $$\text{volume} \; = \frac{\lambda}{\lambda + \gamma} \gamma s F(\delta^\star)$$

liquidity yield spread

- Assume centralized-market price $\dfrac{\delta^\star}{r}$ is PV of cash flows

- The liquidity yield spread ℓ is such that $P = \dfrac{\delta^\star}{r+\ell}$

$$\frac{\ell}{r+\ell} = \frac{\gamma}{r+\gamma+\lambda(1-\theta)} \left(1 - \int \frac{\delta'}{\delta^\star}\, dF(\delta')\right)$$

- Non-zero even if $\theta = 0$, can be positive or negative

- If positive:

  decrease with λ

  increases with θ

  increases with γ

  increases with expected distress cost of marginal investor

bid-ask spread

- Average ask to inter-dealer spread

$$\bar{A} - P = \frac{\theta}{r + \gamma + \lambda(1-\theta)} \int \left(\delta' - \delta^\star\right) \, dF(\delta' \mid \delta' > \delta^\star),$$

- Average bid to inter-dealer spread

$$P - \bar{B} = \frac{\theta}{r + \gamma + \lambda(1-\theta)} \int \left(\delta^\star - \delta'\right) \, dF(\delta' \mid \delta' < \delta^\star),$$

- Zero when $\theta = 0$

- Depends on "tail" expectations of utility flows

- Asymmetric, decreases in λ and also in γ

some extensions

alternative price setting mechanism: RFQ

- Investors often sollicit quotes from several dealers:

  Request for Quotes (RFQ) on electronic platforms

  e.g. Hendershott (15): sollicit 20-30 dealers, 25% response

- Small Burdett-Judd (83) auctions!

- Same as before with $\theta =$ proba of receiving one quote

- New predictions about quote dispersion conditional on δ

- Some references: Glebkin-Yueshen-Shen (22), Weill (20)

unrestricted asset holdings

- Demand determined by $P = V_q(\delta, q)$

  $V_q(\delta, q)$ calculated by replacing δ by $u_q(\delta, q)$ in $\Delta V(\delta)$

- A key difference: all investors are now marginal

  now search frictions change the demand of all δ

  when λ increases: high δ demand more, low δ less

- Provide a theory of trade size

  how it depends on frictions, investors' sophistication and needs

- With dealers entry: can create multiple equilibria

- Some references: Gârleanu (09), Lagos-Rocheteau (07,09)

other forms of heterogeneity

- Examples

  search intensity, λ

  bargaining power, θ

  trading needs, γ

- All are relatively easy to handle in semi-centralized markets

  asset demand can be derived as before

and much more!

- Non stationary dynamics and crises

  Weill (07), Lagos-Rocheteau-Weill (11), Feldhütter (12)

  DiMaggio (13), Biais-Hombert-Weill (14), Chiu-Koeppl (11)

- Debt pricing

  He-Milbradt (07), Chaumont (18), Chang (22)

- Search models of centralized exchange mechanisms

  Biais-Weill (08), Pagnotta-Philippon (18) Dugast (18)

- Asymmetric information

  Guerrieri-Shimer (14), Lester-Shourideh-Venkateswaran-Zetlin-Jones (18)

- General Equilibrium

  Lagos-Zhang (20), Kargar-Passadore-Silva(22)

next week, with Ben Lester

- Fully decentralized markets

- Everyone search, including dealers

- Key applications:

  endogenous intermediation

  inter-dealer markets

  inter-bank markets

  all-to-all liquidity