Monetary Economics: 5

Money, Credit and Banking

Randall Wright

Introduction

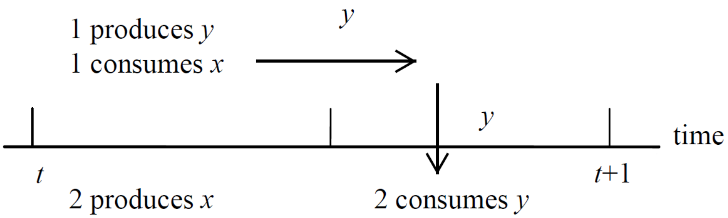
- "Financial institutions are at the mercy of our innate inclination to veer from euphoria to despondency." Niall Ferguson, *The Ascent of Money*.
- "Historically, even some of the staunchest proponents of laissez-faire have viewed banking as inherently unstable and so requiring government intervention." Rolnick & Weber (1986).
  - e.g., Friedman's *Program for Monetary Stability*.
- Literature following Diamond & Dybvig (1983) on bank runs.
- Idea: credit is a lot like money (Sanches & Williamson 2010).

A Simple Version of Kehoe-Levine

- Time is discrete and continues forever.
- Each period is divided into two subperiods.
- There equal measure of type 1 and type 2 agents.
- Two goods every period: x and y.
- Type 1 consume good x and produce good y.
- Type 2 produce good x and consume good y.
- All production takes place in first subperiod.

Timing

- Type 1 consume x in subperiod 1, produce and store y for return ρy, and deliver it in subperiod 2.



Preferences

- Type 1: $U^1 = U^1(x, y)$; Type 2: $U^2 = U^2(y, x)$.
- $U^i(\cdot)$ increasing in 1st and decreasing in 2nd argument; satisfies usual assumptions plus normal goods.
- Discount factor β.
- Type 1 can get liquidation value λy.
- Assume $U^1(x, y + y') + \lambda\rho y' \leq U^1(x, y)$.
  - implies it is not efficient for 1 to produce y for his own use
  - but with limited commitment he may do so opportunistically
- Note: y acts as collateral which is better if λ is smaller.

Endogenous Debt Limit

- Method in Alvarez-Jermann, Gu et al, etc:
  - Step 1: pick and arbitrary limit ϕ and find equil $x(\phi)$, $y(\phi)$.
  - Step 2: work out how much debt agents are willing to pay $\phi' = \Phi(\phi)$ based on punishment for deviators (renegers).
  - Step 3: look for a fixed point $\phi = \Phi(\phi)$.
- Different mechanisms can be considered, including Nash bargaining and Walrasian price-taking.
- Imperfect monitoring/detection: get caught w/prob μ.
- Assume punishment is autarky.

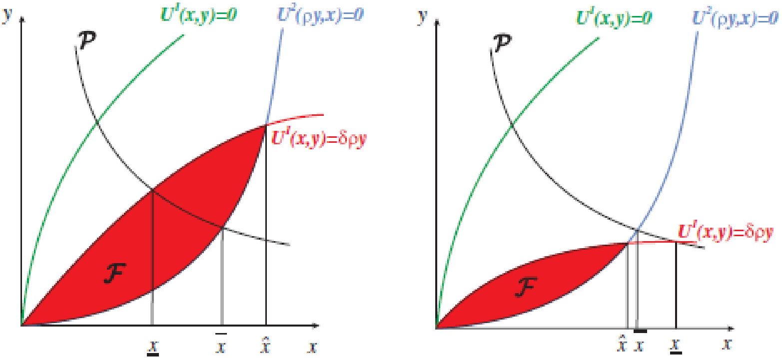


FIGURE 1

Incentive feasible allocations

Repayment Constraint

Type 1 delivers y only if

$$\beta V_{t+1}^1 \geq \lambda\rho y_t + (1-\mu)\,\beta V_{t+1}^1,$$

or $y_t \leq \phi_t$ where $\phi_t \equiv (\beta\mu/\rho\lambda)\, V_{t+1}^1$. Also, in equil

$$V_t^1 = U^1(x_t, y_t) + \beta V_{t+1}^1.$$

Use def'n of ϕ plus equil $x(\phi)$ and $y(\phi)$ to write his as

$$\phi_t = \frac{\beta\mu}{\rho\lambda} U^1\left[x_{t+1}(\phi_{t+1}), y_{t+1}(\phi_{t+1})\right] + \beta\phi_{t+1}$$

Hence, debt limit at t depends on debt limit at $t+1$.

Nash Bargaining

$$\max_{x,y} U^1(x,y)^\theta U^2(\rho y, x)^{1-\theta} \text{ st } y \leq \phi$$

Also st participation $U^i \geq 0$ but these typically do not bind.

If y^* solves bargaining problem unconstrained by RC then

$$\begin{aligned} y^* &\leq \phi \Rightarrow y = y^* \text{ and } x = h^n(y^*) \\ y^* &> \phi \Rightarrow y = \phi \text{ and } x = h^n(\phi) \end{aligned}$$

where $x = h^n(y)$ solves FOC from Nash. Then

$$\phi_{t-1} = f(\phi_t) = \begin{cases} \delta U^1[x(y^*), y^*] + \beta\phi_t & \phi \geq y^* \\ \delta U^1[x(\phi_t), \phi_t] + \beta\phi_t & \text{otherwise} \end{cases}$$

Walrasian Pricing

Similar but instead of Nash solve

$$\begin{array}{ll} \text{Type 1:} & \max U^1(x,y) \text{ st } px = y,\ y \leq \phi \\ \text{Type 2:} & \max U^2(\rho y, x) \text{ st } px = y. \end{array}$$

If y^* is Walrasian equil unconstrained by RC then

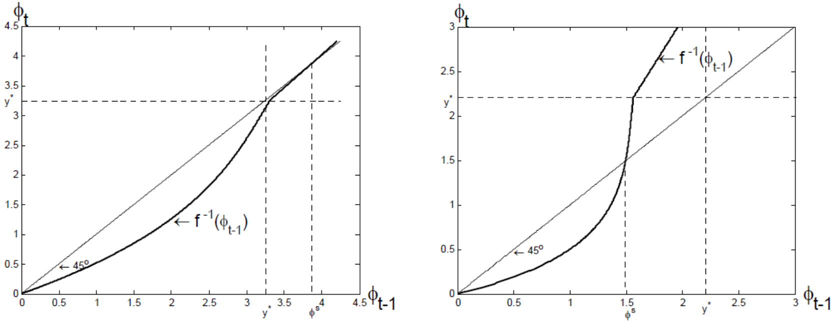
$$\begin{aligned} y^* &\leq \phi \Rightarrow y = y^* \text{ and } x = h^w(y^*) \\ y^* &> \phi \Rightarrow y = \phi \text{ and } x = h^w(\phi) \end{aligned}$$

where $x = h^w(y)$ solves FOC for type 2.

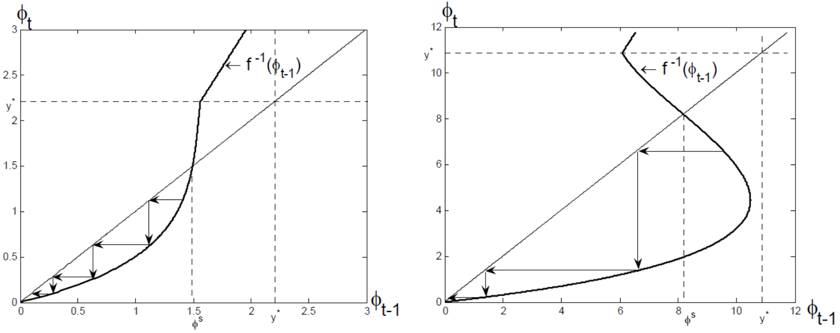
Dynamic system it the same as above except $h^w(\cdot)$ replaces $h^n(\cdot)$.

In either case notice the kink in $f(f)$ at $\phi = y^*$.

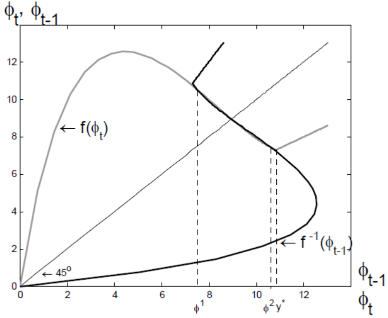
Two steady states, $\phi = 0$ and $\phi > 0$, with either $\phi > y^*$ or $\phi < y^*$



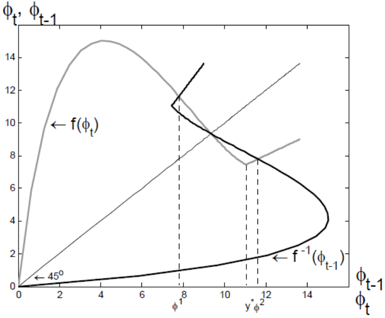
As in monetary economy, $\phi = 0$ is stable so $\exists$ equil with $\phi_t \to 0$.



As in monetary economy, $\exists$ cycles where $\phi < y^*$ at least in 1 state.

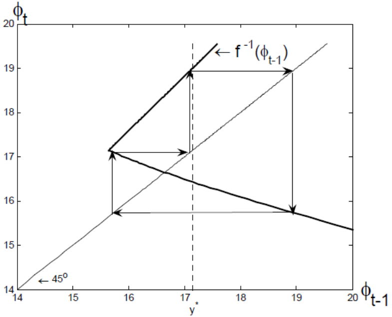


Nash cycle, $\phi^1, \phi^2 < y^*$

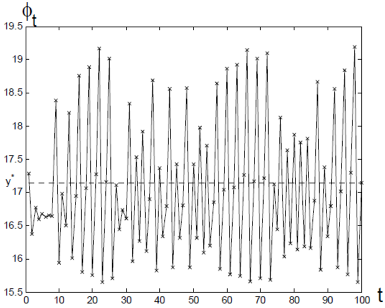


Nash cycle, $\phi^1 < y^* < \phi^2$

As in monetary economy, $\exists$ $n-$cycles and chaotic dynamics.



A three-period cycle



Chaotic dynamics

Summary and Intuition

Economies with endogenous debt limits have lots of equil with complicated deterministic dynamics.

Plus stochastic dynamics (sunspots) by the usual logic.

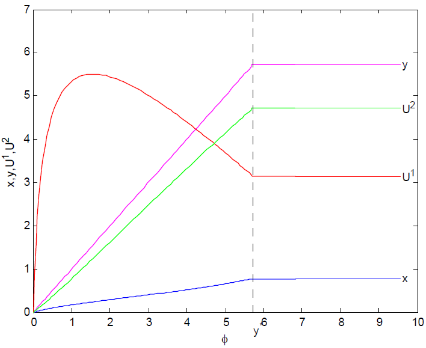
This happens when $f'(\phi^s) < -1$, so why does that happen?

- With Nash, the surplus of one agent may go down when the bargaining set expands.
- With Walras, if ϕ is not too tight making it tighter can make type 1 better off

In both cases ϕ_{t+1} high $\Rightarrow$ type 1 worse off at $t+1$ $\Rightarrow$ more inclined to default at t $\Rightarrow$ ϕ_t low $\Rightarrow$ tendency to cycle

Key Insight

- Tightening constraint affects quantity and terms of trade.
- If ϕ is close to y^* the 2nd effect is 1st order and the 1st effect is 2nd order (envelope thm).
- Like monopolist producing $q < q^*$, and similar to results in labor, trade... debt limit gives borrowers market power



Microfoundations for Banking

- Wallace (1988) Dictum: "money should not be a primitive in monetary theory – in the same way that a firm should not be a primitive in industrial organization theory or a bond a primitive in finance."
- By extension, one might think financial intermediaries should not be a primitive in banking theory.
- To say it slightly differently:
  - Debreu (GE) has two types of agents (h, f) and objects (x, y);
  - a version of the money dictum: don't just add a third object m and treat it like x or y;
  - a version of the banking dictum: don't just add a third type of agent b and treat them like h or f.

Why Microfoundations for Banking?

- Another version: banks, like money, incomplete contracts, sticky prices .... ought to be *outputs* of theory, not *inputs*.
- Townsend: "theory should explain why markets sometimes exist and sometimes do not, so that economic organization falls out in the solution to the mechanism design problem."
- Of course as always it depends on the issues under study.
  - for Debreu's purpose it may be OK to take (h, f) as primitives;
  - it would not be very good for household economics a la Becker or for the theory of the firm a la Coase.
- Yet it is hard to argue that for studying banking theory and especially banking policy/regulation it unimportant to have financial intermediation arise endogenously.

An Attempt at Microfoundation for Banks

The above environment provides one way to study banking without making prior assumptions on what banks are or what they do or who should do it.

The idea is to ask when banking is essential the same way we asked when money is essential.

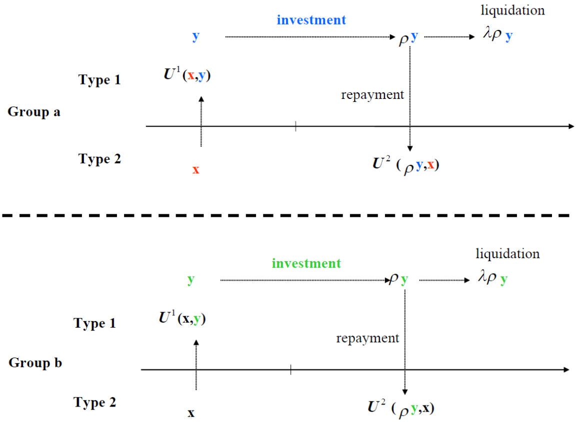
Along the way, we can ask who should be a banker?

It is desirable to make banks (self-interested) agents, not simply institutions that perform certain functions, so that we can model bankers' incentives.

Can we build a model of banking based on our model of credit with limited commitment?

Two Parallel Groups (Islands)

Imagine two groups a and b that produce different x and y $\Rightarrow$ no reason to interact for the usual reasons in intern'l trade:



Incentive Constraints

In stationary allocation, for each group repayment requires

$$\beta V^1 \geq \lambda \rho y + (1 - \mu)\, \beta V^1,$$

which simplifies to

$$U^1 (x, y) \geq \delta \rho y$$

where $\delta = \lambda (1 - \beta) / \beta \gamma \mu$ and

- λ is temptation to behave opportunistically;
- ρ is investment returns;
- β represents patience;
- μ represents visibility;
- γ represents connection to the market.

Heterogeneity

Now imagine that the repayment constraint for Type 1 agents in group *b* is less tight slack because they:

- are less tempted to misbehave;
- have better investment returns;
- are more patient;
- are more visible;
- are more connected to the market.

Then they are more *trustworthy* and hence have a comparative advantage in intertemporal trade.

Delegated Investment

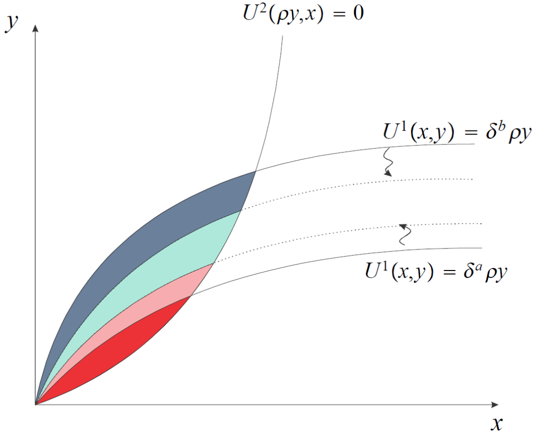
Let Type $1a$ deposit d units of output with Type $1b$, who invest and return proceeds to Type $2a$.

This loosens RC for a and tightens it for b which on net helps:

$$
\begin{aligned}
\text{group } a &: \quad U^{1a}\left(x^a, y^a\right) \geq \delta^a \rho^a \left(y^a - d\right) \\
\text{group } b &: \quad U^{1b}(x^b, y^b) \geq \delta^b \rho^b (y^b + d).
\end{aligned}
$$

- Because $1b$ are more trustworthy that $1a$, type $2a$ are more inclined to accept promises of repayment from $1b$.
- So we can support better allocations (more credit) this way.
- And hence we can say the arrangement is essential.

Gains From Trade Based on Trustworthiness



Banking

This arrangement resembles banking:

- Type $1b$ accept deposits and invest on behalf of depositors;
- and their liabilities (claims on deposits) facilitate 3rd party transactions!

Genuine banks are distinguished from other kinds of financial intermediaries by the readily transferable or 'spendable' nature of their IOUs, which allows those IOUs to serve as a means of exchange, that is, money. Commercial bank money today consists mainly of deposit balances that can be transferred either by means of paper orders known as checks or electronically using plastic 'debit' cards. George Selgin.

Who Wants to Be a Banker?

Moreover theory suggests who should be banker – those who:

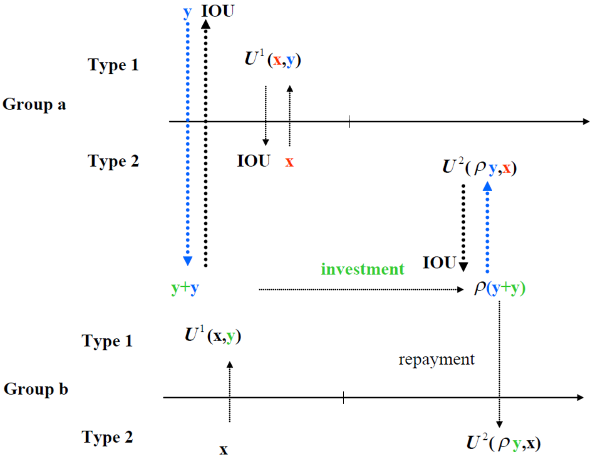
- are less tempted to misbehave;
- have better investment returns;
- are more patient;
- are more visible;
- are more connected to the market.

This is very much like saying what properties money should have (portability, storability, recognizability, divisibility).

But they don't need *all* of these – just a good combination.

- It is easy to have $\rho^b < \rho^a$ and still want Type $1b$ as bankers.

Inside Money: Using deposit receipts we can monetize this economy so all trade is spot trade.



Endogenous Monitoring (Diamond, Huang).

- Endogenize intensity of monitoring μ by making it costly.
- Natural economies of scale in monitoring, $c(\mu) = k_0 + k_1\mu$.
- Observation: typically do not want $\mu = 1$ (envelope thm).
- Even w/o heterogeneity we should pick a few agents to be bankers (although we prefer those w/ better characteristics).
- The key trade-off:
    - fewer bankers imply savings on the monitoring
    - but then they should be bigger have hence more tempted to misbehave!

Equilibrium/Efficient Number and Size of Banks (Huang).

- Over time in US trend is to fewer and bigger banks.
- Huang interprets this in terms of model parameters – monitoring costs, temptation to misbehave, etc.
- Equilibrium need not be efficient and tax/subsidy scheme or regulations may be necessary:
  - free entry – i.e., competition – is desirable as usual;
  - yet profit must be high to incentivize good behavior.
- A novel spin on the 'Occupy Wall Street' movement:
  - of course financiers get huge salaries and bonuses;
  - otherwise they would run off with your pension fund!

Other Models of Banking

Financial intermediaries perform many functions:

- Provide liquidity insurance and maturity transformation.
- Find, screen and monitor opportunities to make investments on behalf of depositors.
- Act as middlemen between asset buyers and asset sellers.
- Generate liabilities used as safe payment instruments.
- Maintain privacy about their customers or assets.

A Survey of Sorts

- As there is no all-encompassing model of these activities, consider four distinct setups:
  - Diamond-Dybvig with reputational concerns as in Gu et al
  - Delegated monitoring as in Diamond or Huang
  - Rubinstein-Wolinsky middlemen in a Duffie et al asset market
  - Bank liabilities as payment instruments as in He et al
- In each case financial intermediation engenders instability by making it more likely that there will be:
  - multiple steady states, or
  - cyclic, chaotic and stochastic dynamics

Model 1: Banking and insurance

- Diamond-Dybvig with infinite horizon as in Gu et al
- Each period in discrete time has two subperiods.
- Agents:
  - many one-period-lived households born at each t
  - some infinitely-lived candidate bankers
  - households: $u_1(c_1)$ with prob π, $u_2(c_2)$ with prob $1-\pi$.
  - candidate bankers: $u_0(c_2)$
- Households endowed with 1 unit of c, bankers with 0.
- Technology: storage or $x \longrightarrow \begin{cases} x & \text{in subperiod 1} \\ Rx & \text{in subperiod 2} \end{cases}$

Friction 1 – Private Information

Given preference types are unobservable IC contract solves:

$$\max_{c_1, c_2} \pi u_1(c_1) + (1-\pi) u_2(c_2)$$
$$\text{st } (1-\pi) c_2 = (1 - \pi c_1) R$$
$$c_2 \geq c_1$$

- $u'(1) = Ru'(R) \Rightarrow$ autarky is efficient.
- $u'(1) < Ru'(R) \Rightarrow c_1^* < 1$ and $c_2^* > R$.
- $u'(1) > Ru'(R) \Rightarrow 1 < c_1^* < c_2^* < R$.
- In none of the cases does priviate info cause a problem for the existence of efficient outcomes, but ....

How Diamond & Dybvig Got the Runs

- Are there other equil where agents all try to withdraw (a bank run) in subperiod 1?
- Given the simple demand deposit contract, the answer is *yes* in one case, $1 < c_1^* < c_2^* < R$.
- However better (maybe even more realistic) contracts rule out runs, e.g., suspension of convertibility.
- It's more complicated with aggregate uncertainty, but even then there are better options than ad hoc deposit contracts
  - Green & Lin get unique equilibrium with aggregate uncertainty.
- So ignore runs to focus on other types of fragility/volatility.

Friction 2 – Limited Commitment

- Patient agents have no incentive to deliver the goods to impatient
  - Households: finite lives imply no punishment available
  - Bankers: reputation can sustain some credibility
- Hence households might deposit some endowment with a banker, who invests on their behalf.

Deposit banking

- Banker at time t:
  - accepts deposits d_t
  - gets $b_t R$ at the end of period
  - can abscond with d_t for opportunistic payoff λd_t
  - misbehavior is detected/communicated to next gen with prob μ and punished with autarky
- The key incentive constraint:

$$u_0(b_t R) + \beta V_{t+1} \geq \lambda d_t + \beta(1-\mu) V_{t+1}$$
$$\iff \lambda d_t - u_0(b_t R) \leq \beta \mu V_{t+1} \equiv \phi_t$$

where ϕ_t is bank's franchise value.

Bank Contract Problem

$$\max_{d,r_1,r_2,b} \pi u_1 \left(dr_1 + 1 - d\right) + \left(1 - \pi\right) u_2 \left(dr_2 + \left(1 - d\right) R\right)$$

$$\text{st } \left(1 - \pi\right) dr_2 = \left(d - b - \pi dr_1\right) R$$

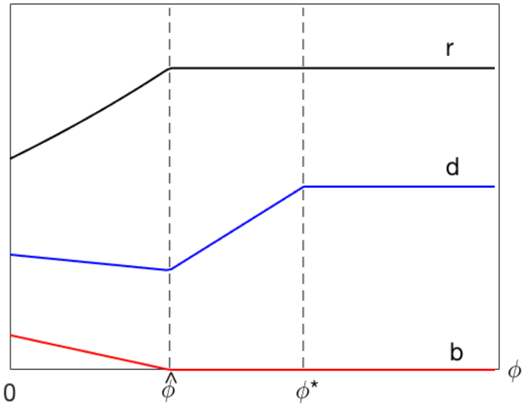
$$r_2 \geq r_1$$

$$\phi \geq \lambda d - u_0 \left(bR\right)$$

$\exists\ \hat{\phi}$ and $\phi^* > \hat{\phi}$ such that:

1. $\phi > \phi^* \implies$ IC loose $\implies d = d^*,\ b = 0$ is a solution;
2. $\hat{\phi} \leq \phi < \phi^* \implies$ IC medium $\implies d < d^*,\ b = 0$;
3. $\phi < \hat{\phi} \implies$ IC tight $\implies d < d^*,\ b > 0$.

Static Partial Equilibrium



(d, r, b) vs ϕ

Dynamic General Equilibrium

- Banker's value equation

$$V_t = u_0\left(b_t R\right) + \beta V_{t+1}$$

- Use $\phi_t \equiv \beta\mu V_{t+1}$ to get

$$\phi_{t-1} = f\left(\phi_t\right) \equiv \beta\mu u_0\left[b\left(\phi_t\right) R\right] + \beta\phi_t.$$

- **Def'n**: An equilibrium is a bounded, nonnegative solution to $\phi_{t-1} = f\left(\phi_t\right)$.

Prop: $\exists!$ stationary equil $\bar{\phi} = f(\bar{\phi})$.

- $\tilde{\phi} > 0 \implies$ stationary equil has banking and $\bar{\phi} \in (0, \hat{\phi})$.
- $\tilde{\phi} < 0 \implies$ stationary equil has no banking
- Outcome is obviously unique with no banking; specification is designed to get unique stationary equil with banking. But...

Prop: For some parameters, there are many nonstationary equil with banking.

Examples

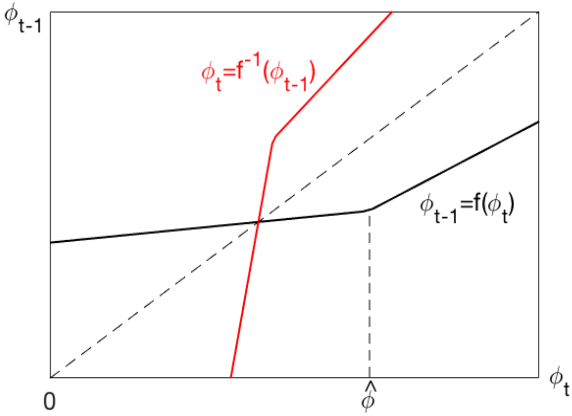
- Utility functions:
  - $u_0(c) = A_0 c$
  - $u_1(c) = A_1 \frac{(c+\varepsilon)^{1-\sigma_1} - \varepsilon^{1-\sigma_1}}{1-\sigma_1}$

    $u_2(c) = A_2 \frac{(c+\varepsilon)^{1-\sigma_2} - \varepsilon^{1-\sigma_2}}{1-\sigma_2}$
- Parameter values:

  $A_0 = 0.95,\ A_1 = 1,\ A_2 = 0.1,\ \sigma_1 = \sigma_2 = 2,\ \varepsilon = 0.01,$

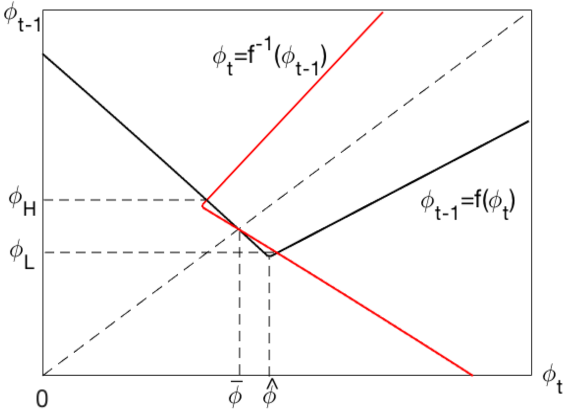
  $R = 2.1,\ \pi = 0.25,\ \lambda = 0.6,\ \beta = 0.99,\ \mu = 0.7.$
- Vary σ's for different examples.

Example with Unique Equilibrium



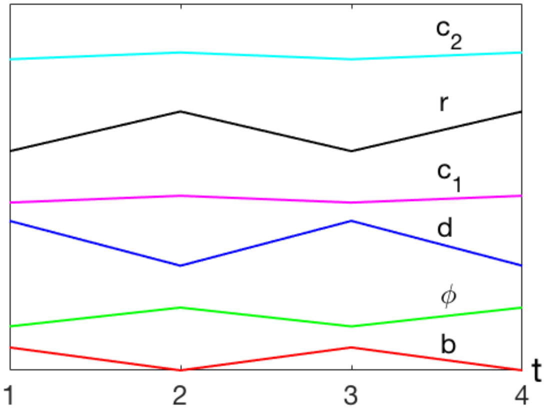
monotone f

Example with 2-cycle and Sunspot Equilibrium

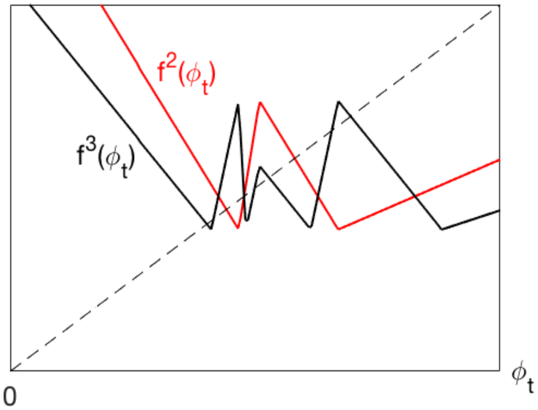


nonmonotone f

Banking Activity Over the Cycle



Example with three-cycles and chaos



cycles

Summary

Message:

- Banking is essential: if it is eliminated payoffs go down.
- But does engender instability: the equil set with (without) banking does (does not) admit cycles, chaos and sunspots.

Intuition:

- If bank franchise value ϕ_t is high his current salary b_t can be low and still satisfy IC thus making ϕ_{t-1} low.
- This can yield nonmonotone dynamics if it dominates the linear term in $\phi_{t-1} = \beta\mu u_0[b(\phi_t) R] + \beta\phi_t$.

Model 2: Delegated Investment

- Based on Diamond and recent work by Huang.
- Time is discrete and all agents live forever.
- A large number of distinct locations (islands) and we randomly relocated agents every t.
- Agents can pay a fixed cost κ to find, evaluate, screen or monitor a project that pays R at the end of the period.
- Utility: $u(x) - c(d)$, where x is consumption and d is investment

Autarky

- If an agent invests on his own, his payoff is

$$\tilde{W} = \max_{x,d} \{u(x) - c(d)\} \text{ st } x = Rd - \kappa.$$

- If an agent does not invest, his payoff is $u(0) - c(0) = 0$.
- For the sake of illustration, assume κ is big so that agents do not invest on their own.

Coalition

- Some agents (depositors, measure $1-\omega$) delegate their investment to others (bankers, measure ω) to save on fixed cost.
- But bankers lack commitment.
- The key IC constraint is

$$\beta V_{t+1} \geq \frac{\lambda\left(1-\omega_t\right)}{\omega_t} x_t + (1-\mu)\beta V_{t+1}$$

or

$$\frac{1-\omega_t}{\omega_t} x_t \leq \phi_t \equiv \frac{\beta\mu}{\lambda} V_{t+1},$$

Contract

$$W(\phi) = \max_{\omega, X, D, x, d} \left\{ \omega \left[u(X) - c(D) \right] + (1-\omega) \left[u(x) - c(d) \right] \right\}$$

$$\text{st } \omega X + (1-\omega) x = R \left[\omega D + (1-\omega) d \right] - \kappa \omega$$

$$u(x) - c(d) \geq 0$$

$$\frac{1-\omega_t}{\omega_t} x_t \leq \phi_t$$

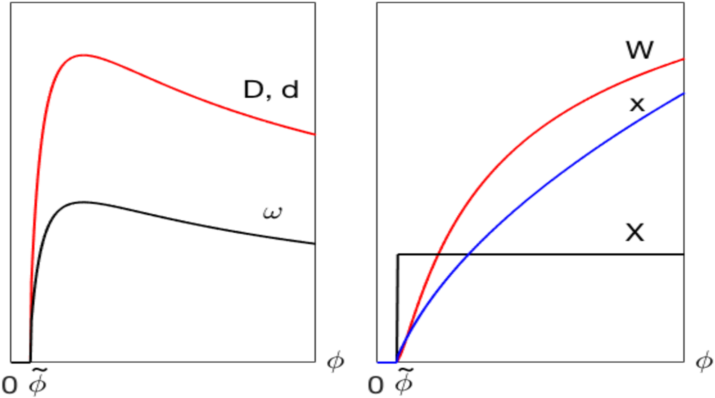
Example

- Functions:
  - $u(x) = A\frac{(x+b)^{1-\sigma} - b^{1-\sigma}}{1-\sigma}$
  - $c(d) = Bd$
- Parameter values:

  $A = b = 0.001,\ \sigma = 2,\ B = 0.1$

  $\kappa = 230, R = 1.2.$

Static Partial Equilibrium



Bank contract vs ϕ

Dynamic general equilibrium

- Agent's value equation

$$V_t = \max \{W(\phi_t), 0\} + \beta V_{t+1}.$$

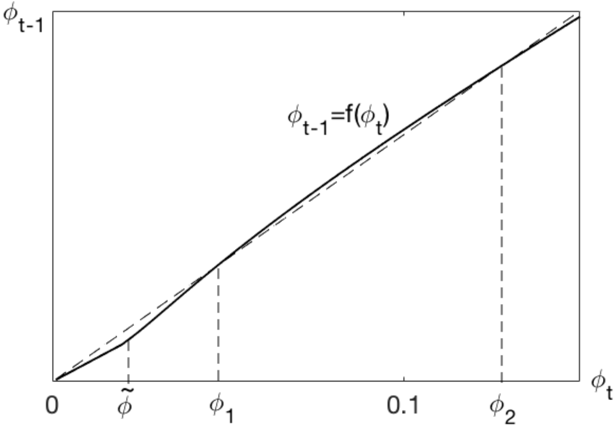
- Use $\phi_t \equiv \beta\mu V_{t+1}/\lambda$ to get

$$\phi_{t-1} = f(\phi_t) \equiv \frac{\beta\mu}{\lambda} \max \{W(\phi_t), 0\} + \beta\phi_t.$$

Prop: There is generically and odd number of stationary equil, one of which is $\bar{\phi} = 0$.

Prop: If there is a stationary equil with $\bar{\phi} > 0$ then there are sunspot equil.

Example of Sunspot Equilibrium



Monotone f with multiple steady states

Variation

- Note that this is not the same as the endogenous dynamics in Gu et al, which relies on Nash bargaining with $0 < \theta < 1$.
- That is not in the above models, but we can add it.
- Agents:
  - A one-period lived agent born at every date (depositor)
  - An infinitely lived agent (banker)
- Nash bargaining

Nash Bargaining

$$W(\phi) = \max_{X,x,D,d} [U(X) - C(D)]^{\theta} [u(x) - c(d)]^{1-\theta}$$

$$\text{st } X + x = R(D+d) - \kappa$$

$$u(x) - c(d) \geq 0$$

$$x_t \leq \phi_t$$

Solution

1. If κ is too big there is no banking.
2. Otherwise $\exists\ \tilde{\phi} < \phi^*$ such that:

   a. $\phi > \phi^* \implies$ IC loose $\implies x = x^*$ is a solution
   b. $\tilde{\phi} \leq \phi < \phi^* \implies$ IC medium $\implies x = \phi$.
   c. $\phi < \tilde{\phi} \implies$ IC tight $\implies d = x = 0$.

Dynamic General Equilibrium

- The banker's value function is

$$V_t = U(X_t) - C(D_t) + \beta V_{t+1}$$

- Use $\phi_t = \beta\mu V_{t+1}/\lambda$ to get

$$\phi_{t-1} = \begin{cases} \beta\phi_t & \text{if } \phi_t < \tilde{\phi} \\ \frac{\beta\mu}{\lambda}\left[U \circ X(\phi_t) - C \circ D(\phi_t)\right] + \beta\phi_t & \text{if } \tilde{\phi} \leq \phi_t < \phi^* \\ \frac{\beta\mu}{\lambda}\left[U(X^*) - C(D^*)\right] + \beta\phi_t. & \text{if } \phi_t \geq \phi^* \end{cases}$$

Proposition: There are odd number of steady states, one of which is $\bar{\phi} = 0$.

Proposition: If there is more than 1 steady state, there are sunspot equilibria.

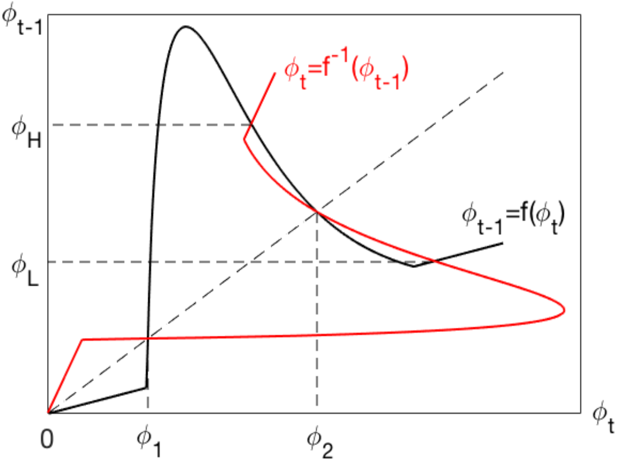
Example

- Utility functions:
  - $U(x) = u(x) = Ax$
  - $C(d) = c(d) = Bd^{\gamma}/\gamma$
- Parameter values:

  $A = 1,\ B = 0.5,\ \gamma = 5.$

  $R = 2,\ k = 1.5,\ \theta = 0.01,\ \lambda = 0.01,\ \mu = 1,\ \beta = 0.35$

Example



Delegated investment – Nash bargaining

Summary

Message:

- Banking is essential – it saves fixed cost.
- But it engender instability: the equilibrium set with (without) banking does (does not) admit cycles, chaos and sunspots.

Intuition:

- Adding fixed cost generates a stable steady state, which supports a continuum of equilibria around it.
- In Nash bargaining, one's payoff does not necessarily increase with the expansion of the bargaining set. In particular, $\partial \left[U\left(X\right) - C\left(D\right) \right] / \partial\phi|_{\phi \to \phi^*} < 0$.

Model 3: Asset Market Intermediation

- Based on RW (1987), DGP (2005) and NWW (2017)
- Time is discrete and infinite.
- Agents
  - Buyers and sellers: one-period lived, replaced by "clones"
  - Middlemen: infinitely lived
- Sellers are endowed with 1 unit of investment good.
- Buyers can transform the investment good into π, which is random with cdf $F(\pi)$, units of consumption good.
- Preferences: linear utility of consumption, and transferable utility for payments.
- Agents meet bilaterally.
- Entry by sellers or buyers or middlemen.

Agent's problem

Let $\Delta_t \equiv V_{1,t} - V_{0,t}$ and $R_t \equiv (1-\delta)\,\beta\Delta_{t+1}$

- Buyer:

$$V_{b,t}(\pi) = \frac{\alpha n_{s,t}}{N_t}\theta_{bs}\pi + \frac{\alpha n_t}{N_t}\tau(\pi, R_t)\,\theta_{bm}\left[\pi - (1-\delta)\,\beta\Delta_{t+1}\right]$$

- Seller:

$$V_{s,t} = \frac{\alpha n_b}{N_t}\theta_{sb}\mathbb{E}\pi + \frac{\alpha(n_m - n_t)}{N_t}\theta_{sm}(1-\delta)\,\beta\Delta_{t+1}$$

- Middleman:

$$\begin{aligned} V_{0,t} &= \frac{\alpha n_{s,t}}{N_t}\theta_{ms}R_t + \beta V_{0,t+1} \\ V_{1,t} &= \rho + \frac{\alpha n_b}{N_t}\theta_{mb}\int_{R_t}^{\infty}(\pi - R_t)\,dF(\pi) \\ &\quad +(1-\delta)\beta V_{1,t+1} + \delta\beta V_{0,t+1}. \end{aligned}$$

Equilibrium

- The value functions reduce to

$$R_{t-1} = (1-\delta)\,\beta\left\{\rho + R_t + \frac{\alpha n_b \theta_{mb}}{N_t}\int_{R_t}^{\infty}[1-F(\pi)]\,d\pi - \frac{\alpha n_{s,t}\theta_{ms}}{N_t}R_t\right\}$$

- Law of motion of middlemen with inventory:

$$n_{t+1} = g(n_t, N_t, R_t)$$

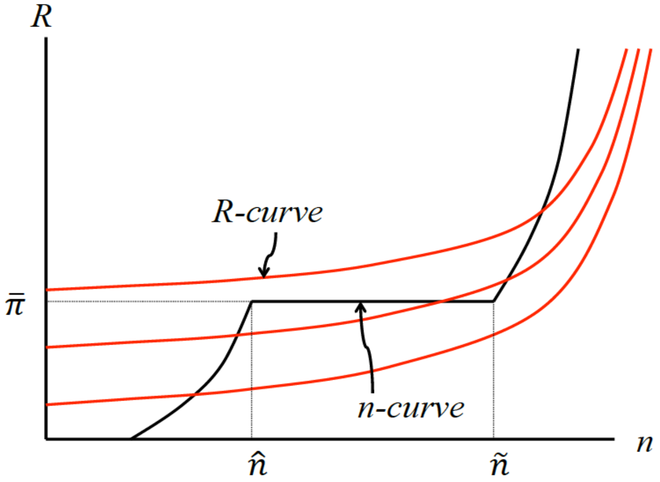
- Free entry

$$n_{s,t} = h(n_t, R_t)$$

- Equilibrium is a two-dimensional dynamical system,

$$\begin{bmatrix} n_{t+1} \\ R_{t-1} \end{bmatrix} = \begin{bmatrix} f(n_t, R_t) \\ g(n_t, R_t) \end{bmatrix}$$

Steady States – Degenerate Distribution

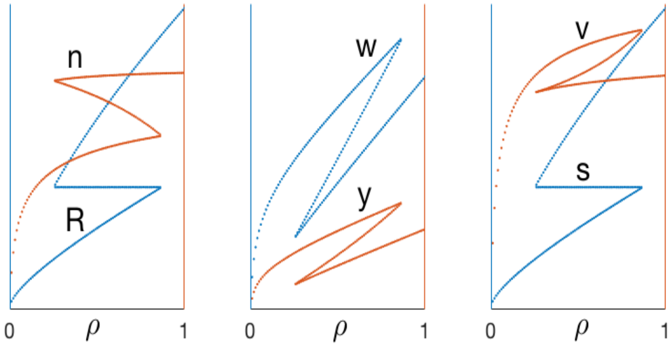


The n-curve and R-curve for different ρ

Proposition: Consider entry by type S and $\pi = \bar{\pi}$. There exist $\tilde{\rho} > 0$ and $\hat{\rho} > \tilde{\rho}$ such that:

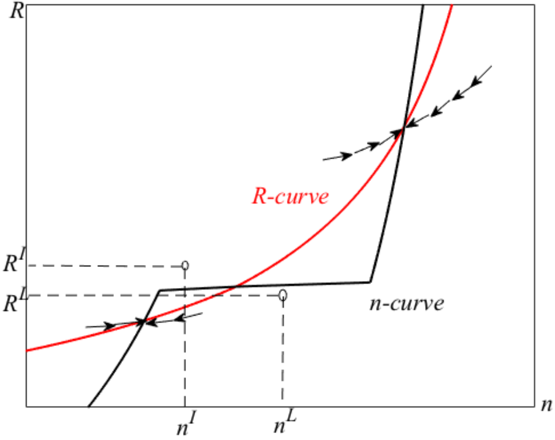
1. $\rho \in [0, \tilde{\rho})$ implies there is a unique steady state and it entails $R < \bar{\pi}$;
2. $\rho \in (\hat{\rho}, \infty)$ there is a unique steady state and it entails $R > \bar{\pi}$; and
3. $\rho \in (\tilde{\rho}, \hat{\rho})$ implies there are three steady states, one with $R < \bar{\pi}$, one with $R > \bar{\pi}$, and one with $R = \bar{\pi}$.

Steady State



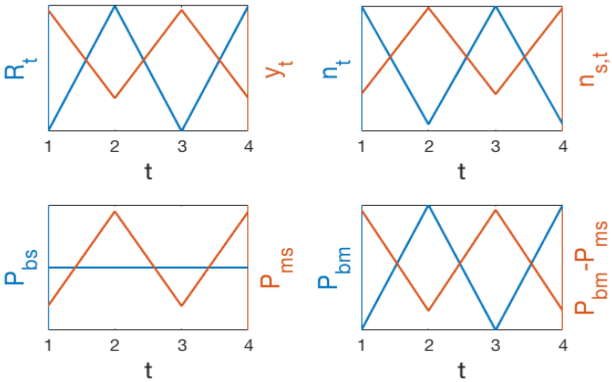
The equilibrium correspondence

Cycles



Phase plane with entry by S, including a two-cycle

Cyclic Patterns



Time series for a two-period cycle with entry by S

Summary

Proposition: For some parameter values, financial intermediation is essential. The equilibrium set admits multiple steady states, cycles and sunspots.

Interpretation:

- There is strategic complementarity between M and S.
- Asset market intermediation is a useful institution but is prone to multiplicity and excess volatility or instability.

Model 4: Safety and Liquidity

- Based on the version of LW (2005) in HHW (2005) (also see BCW 2007, SW 2010)
- Continuum of infinitely-lived buyers b and sellers s.
- Each period has two markets convene:
  - DM: s agents sell q and b agents buy q
  - CM: all trade (x, ℓ), adjust portfolios, settle debts
- Utility: $U(x) - \ell + u(q)$ & $U(x) - \ell - c(q)$
- An asset in fixed supply (Lucas tree) can be held in two ways:
  - a_1 : safe but illiquid,
  - a_2 : less safe but liquid

CM problem

- Let $A = (\phi + \rho)(a_1 + a_2)$.

$$\begin{aligned} W_t(A_t) &= \max_{x_t, \ell_t, \hat{\mathbf{a}}_t} \{U(x_t) - \ell_t + \beta V_t(\hat{\mathbf{a}}_t)\} \\ \text{st } x_t &= A_t + \ell_t - \phi_t(\hat{a}_{1,t} + \hat{a}_{2,t}) \end{aligned}$$

- Standard results: $W_t(A_t)$ linear.
- FOC for demand for $\hat{a}_{j,t+1}$:

$$\hat{a}_{j,t+1}\left[-\phi_t + \beta \frac{\partial V_{t+1}(\hat{\mathbf{a}}_{t+1})}{\partial \hat{a}_{j,t+1}}\right] = 0$$

DM problem

- A general trading mechanism $p = v(q)$, where $p \leq (\phi + \rho) a_2$

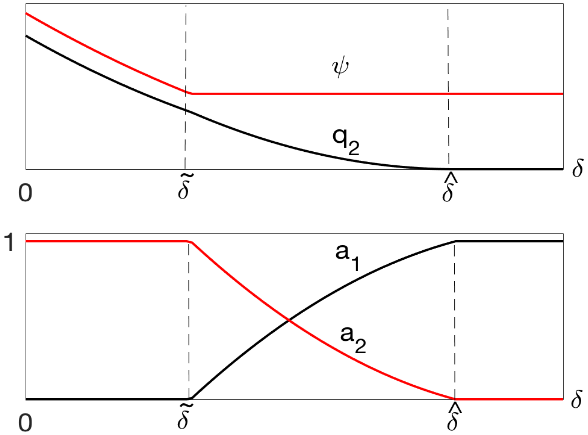
$$
\begin{aligned}
V_t(\hat{\mathbf{a}}_t) &= (1-\delta)\left\{W_t\left(A_t\right) + \alpha\left[u\left(q_t\right) - v\left(q_t\right)\right]\right\} \\
&\quad + \delta W_{t+1}\left[\left(\phi_t + \rho\right)\hat{a}_{1,t}\right]
\end{aligned}
$$

- Euler equations:

$$
\begin{aligned}
0 &= \hat{a}_{1,t}\left[\beta\left(\phi_{t+1} + \rho\right) - \phi_t\right] \\
0 &= \hat{a}_{2,t}\left\{\beta\left(\phi_{t+1} + \rho\right)(1-\delta)\left[1 + \alpha\lambda\left(q_{t+1}\right)\right] - \phi_t\right\}
\end{aligned}
$$

where $\lambda(q) = u'(q)/v'(q) - 1$.

Steady State



Steady State Regimes

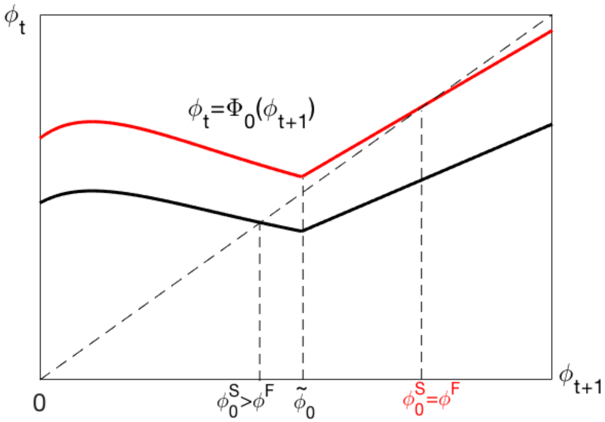
Equilibrium Without Banks

Suppose $\delta < \hat{\delta}$. The dynamic system is

$$\phi_{t-1} = \begin{cases} \beta\left(\phi_t + \rho\right)\left(1 - \delta\right)\left[1 + \alpha\lambda \circ v^{-1}\left(\phi_t + \rho\right)\right] & \text{if } \phi_t < \tilde{\phi}_0 \\ \beta\left(\phi_t + \rho\right) & \text{if } \phi_t \geq \tilde{\phi}_0 \end{cases}$$

where $\alpha\lambda \circ v^{-1}\left(\tilde{\phi}_0 + \rho\right) = \delta / \left(1 - \delta\right)$

Equilibrium Without Banking



Dynamic equilibrium

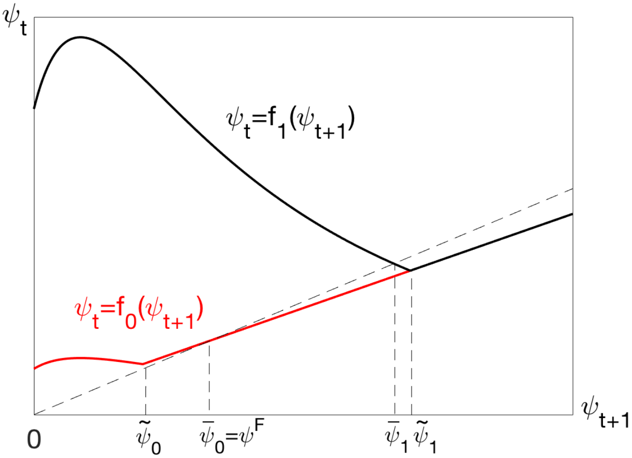
Banking

- Asset can be deposited at a bank with return ρ and $\delta = 0$.
- Equilibrium:

$$\phi_{t-1} = \begin{cases} \beta\left(\phi_t + \rho\right)\left[1 + \alpha\lambda \circ v^{-1}\left(\phi_t + \rho\right)\right] & \text{if } \phi_t < \tilde{\phi}_1 \\ \beta\left(\phi_t + \rho\right) & \text{if } \phi_t \geq \tilde{\phi}_1 \end{cases}$$

where $\lambda \circ v^{-1}\left(\tilde{\phi}_1 + \rho\right) = 0$

Equilibrium with Banking



Equilibrium with banking

Summary

Proposition: Banking is essential. But it can introduce new nonstationary equilibria including cycles, sunspots and chaos. If banking is stable, the economy without banking is stable.

Interpretation:

- Liquidity premium is amplified if the asset is safer.
- Banking for safekeeping and liquidity provision is a useful institution but is prone to multiplicity and excess volatility or instability.

Conclusion

- Banks, as several banking crisis throughout history have demonstrated, are fragile institutions. This is to a large extent unavoidable and is the direct result of the core functions they perform in the economy. *Finance Market Watch Program @ Re-Define.*
- We study various models of financial intermediation that are explicit about their core functions – i.e., they are models of these institutions, not just models with these institutions.
- The results show these are socially useful institutions but are indeed prone to excess volatility or multiplicity.
- In this way banks, or financial intermediaries more general, are much like money: institutions that are socially useful but also potentially fragile.