Monetary Economics: 4

Applications and Extensions of the Benchmark Model

by

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Dichotomy

In the baseline model $(x, \ell) \perp (q, z)$ but that changes if U is nonseparable in (x, q):

$$W(m, q) = \max_{x,\ell,\hat{m}} \{U(x, q) - \ell + \beta V(\hat{m})\} \text{ st } x = \ell + \phi(m - \hat{m}) - T$$

where (m, q) brought in from DM are state variables with

$$W_m(m, q) = \phi \text{ and } W_q(m, q) = U_q(x, q)$$

Clearly we still get $\hat{m} \perp m$ and

$$V(m) = W(m, 0) + \alpha [W(0, q) - W(\hat{m}, 0)]$$

Breaking this Dichotomy

Consider bargaining with $\theta = 1$, so $\phi m = c(q)$, leading to

$$i = \alpha \left[\frac{U_q(x, q)}{c'(q)} - 1 \right].$$

FOC's for x contingent on q are

$$U_x(x_1, q) = 1 \text{ and } U_x(x_0, 0) = 1.$$

These eqns plus CM market clearing

$$\ell = \alpha x_1 + (1 - \alpha) x_0$$

determine $\langle q, x_1, x_0, \ell \rangle$ in SME.

A Genuine Phillips Curve

Notice $(q, x_1) \perp (x_0, \ell)$, and it is easy to check

$$\partial q / \partial i < 0 \text{ and } \partial x_1 / \partial i \simeq -U_{xq}$$

- $U_{xq} > 0$ (x and q compliments) $\Rightarrow \partial x_1 / \partial i < 0$
- $U_{xq} < 0$ (x and q substitutes) $\Rightarrow \partial x_1 / \partial i > 0$

In latter case, if x_1 is labor intensive then $\partial \ell / \partial i > 0$.

- So it *is feasible* to increase LR employment with inflation
- But *not desirable:* optimal policy is as usual $i = 0$.

Indivisible Labor Version

As in Rogerson and Hansen in RBC theory, consider $\ell \in \{0, 1\}$.

CM utility is any $U(x, 1-\ell)$ and agents trade lotteries.

Get paid for $\zeta = prob(\ell = 1)$ and cons x_1 if $h = 1$, x_0 if $h = 0$.

$$\begin{aligned} W(m) &= \max_{x_1, x_0, \ell, \hat{m}} \left\{ \zeta U(x_1, 1) + (1-\zeta) U(x_0, 0) + \beta V_+(\hat{m}) \right\} \\ \text{st } \zeta w &= \zeta x_1 + (1-\zeta) x_0 + \phi(\hat{m} - m) + T. \end{aligned}$$

Define the Lagrangian

$$\begin{aligned} \mathcal{L} &= \zeta U(x_1, 1) + (1-\zeta) U(x_0, 0) + \beta V_+(\hat{m}) \\ &\quad + \eta \left[w\zeta - \zeta x_1 - (1-\zeta) x_0 - \phi(\hat{m} - m) - T \right]. \end{aligned}$$

Indivisible Labor FOC's

$$\begin{aligned}
x_1 &: \quad U_1(x_1, 1) = \eta && (1)\\
x_0 &: \quad U_1(x_0, 0) = \eta && (2)\\
\zeta &: \quad U(x_0, 0) - U(x_1, 1) = \eta\,(w - x_1 + x_0) && (3)\\
\hat{m} &: \quad \beta V'_+(\hat{m}) = \eta\phi && (4)\\
\eta &: \quad w\zeta = \zeta x_1 + (1-\zeta)x_0 + \phi\,(\hat{m} - m) + T && (5)
\end{aligned}$$

Step a: (1)-(3) $\Rightarrow (x_1, x_0, \eta) \perp (\zeta, \hat{m}, m)$

- note $x_1 = x_0$ iff $U_{12} = 0$

Step b: Given η, (4) $\Rightarrow \hat{m} \perp (\zeta, m)$

- note we get $\hat{m} \perp m$ without quasi-linear utility

Step c: (5) $\Rightarrow$ individual ζ as function of individual m

An Even More Genuine Phillips Curve

- This model has unemployment $1 - \zeta$, not just leisure $1 - \ell$.
- As usual, let $x = \zeta$ so $w = 1$, or $x = f(\zeta)$ so $w = f'(\zeta)$.
- If q enters separably then $\partial q / \partial i < 0$ but $\partial \zeta / \partial i = 0$.
- Rocheteau et al let q interact with x (or ℓ) to get Phillips curve as above, but with unemployment.
- They need $\theta = 1$ to prove results; Dong redoes it using competitive search equil with no such restriction.
- But again $i = 0$ is optimal.

Perfect Credit

- Consider baseline alternating-market model with DM credit

$$W(d) = \max_{x,\ell} \{U(x) - \ell + \beta V(0)\} \text{ st } x = \ell - d - T$$

- The same assumptions that make $\hat{m} \perp m$ imply one-period debt is sufficient.

$$\begin{aligned} V(0) &= C_0 + W(0) + \alpha [u(q) + W(d) - W(0)] \\ &= C_0 + W(0) + \alpha [u(q) - d] \end{aligned}$$

- With standard mechanism $v(q)$, unlimited credit $\Rightarrow q = q^*$ and $d = d^* = v(q^*)$.
- This is the unique equil.

Constrained Credit

$$W(a,d) = \max_{x,\ell} \left\{ \tilde{U}(x) - \ell + \beta V(0) \right\}$$
$$\text{st } x = \ell + (\rho + \phi)\, a - \phi \hat{a} - d - T$$

- Debt limit: $d \leq \bar{d} \Rightarrow q = q^*$ and $d = d^*$ if $d^* \leq \bar{d}$; else $d = \bar{d}$ and $q = v^{-1}(\bar{d})$.
- In either case FOC is: $\phi = \beta(\rho + \phi_+) \Rightarrow \phi = \phi^* = \rho / r$.
- There is a unique SME; there are no other equil – all other sol'ns to $\phi = \beta(\rho + \phi_+)$ go to ∞ or go negative.
- If a conveys no liquidity it must be priced fundamentally.

Secured Credit

- As in Kiyotaki-Moore literature impose $\bar{d} = \chi(\rho + \phi) a$.
  - usual story uses limited commitment and collateral: we can punish renegers by seizing a fraction χ of their assets
  - perhaps a better story uses private info (Li, Rocheteau & Weill)
- Now FOC is:

$$\phi = \beta(\rho + \phi_{+}) [1 + \alpha\chi\lambda(q)] \Rightarrow \phi > \phi^{*} \text{ if } \alpha\chi\lambda(q) > 0$$

- If a relaxes constraint when used as collateral, it has liquidity value, just like when a is used as money.

Colatteral vs Medium of Exchange

- Collateral story: debtor honors debt iff $d \leq \chi(\rho + \phi) a$.
- If $\chi = 1$ buyer can credibly promise payment up to value of a, but then he may as well turn over a at point of purchase
  - immediate settlement same as deferred settlement
- If $\chi < 1$ buyer stricly prefers immediate settlement
  - but private info can imply $\chi < 1$ whether a is used as collateral or money.
- Conclusion: Kiyotaki-Moore credit $\Leftrightarrow$ Kiyotaki-Wright money

Andofatto blog:

On the surface, these two methods of payment look rather different. The first [KW] entails immediate settlement, while the second [KM] entails delayed settlement. To the extent that the asset in question circulates widely as a device used for immediate settlement it is called money (in this case, backed money). To the extent it is used in support of debt, it is called collateral. But while the monetary and credit transactions just described look different on the surface, they are equivalent in the sense that capital [the asset a] is used to facilitate transactions that might not otherwise have taken place.

Lagos (JMCB):

The buyer could settle the transaction on the spot by using the asset directly as a means of payment. In some modern transactions, oftentimes the buyer would use a financial asset to enter a repurchase agreement with the seller, or as collateral to borrow the funds needed to pay the seller. Once stripped from the subsidiary contractual complexities, the essence of these transactions is that the asset helps the untrustworthy buyer to obtain what he wants from the seller. In this sense, many financial assets are routinely employed in the exchange process and play a role akin to a medium of exchange, that is, they provide liquidity —the term that monetary theorists use to refer to the usefulness of an asset in facilitating transactions.

A Case where KM Credit is Not the Same as KW Money?

Suppose asset a enters CM utility directly, e.g., housing:

$$W(a) = \max_{x,h,\hat{a}} \{U(x,a) - h + \beta V(\hat{a})\} \text{ st } x = h + \phi(a - \hat{a})$$

$\Rightarrow U_1(x,\hat{a}) = 1$, $\phi = \beta V'(\hat{a})$ and $W'(a) = U_2(x,\hat{a}) + \phi$.

Assuming a is used as money, we have

$$V(a) = W(a) + \alpha [u(q) + W(a - p) - W(a)]$$

where $p \leq a$ and $v(q) = W(\bar{a} + p) - W(\bar{a})$.

If LC binds $q'(a) = W'(\bar{a} + a) / v'(q)$ and we're stuck with

$$V'(a) = W'(a) + \alpha \left[\frac{u'(q)}{v'(q)} W'(\bar{a} + a) - W'(a) \right]$$

Housing as Collateral (He et al RED)

$$W(a,d) = \max_{x,h,\hat{a}} \{U(x,a) - h + \beta V(\hat{a})\} \text{ st } x = h + \phi(a - \hat{a}) - d$$

$\Rightarrow \phi = \beta V'(\hat{a}),\ W_1(a) = U_2(x,a) + \phi,\ W_2(a,d) = -1.$

Now assume a is used to get a *home equity loan*

$$V(a) = W(a,0) + \alpha[u(q) - d]$$

where $d \leq \phi a$, $v(q) = d$. If LC binds $q'(a) = \phi / v'(q)$

$$V'(a) = U_2(x,a) + \phi + \alpha\phi\lambda(q)$$

$$\Rightarrow \phi = \beta U_2(x,A) + \beta\phi_+ + \beta\alpha\phi_+\lambda \circ v^{-1}(\phi_+ A)$$

House price = marginal utility + capital gain + collateral value

Housing as Money

Wait – the above models assume $U(x,a)$, but it actually makes more sense to use $U(x,\hat{a})$ and

$$W(a) = \max_{x,h,\hat{a}} \left\{ U(x,\hat{a}) - h + \beta V(\hat{a}) \right\} \text{ st } x = h + \phi(a - \hat{a})$$

$\Rightarrow U_1(x,\hat{a}) = 1$, $\phi = U_2(x,\hat{a}) + \beta V'(\hat{a})$ and $W'(a) = \phi$.

Yields same EE whether a is used as money or collateral:

$$\phi = U_2(x,A) + \beta\phi_+ + \beta\alpha\phi_+\lambda \circ v^{-1}\left(\phi_+ A\right)$$

If you use a as collateral, you may as well settle in DM using a as money, since you can always reverse the trade in the CM.

So once again it is equivalent to a as money or collateral.

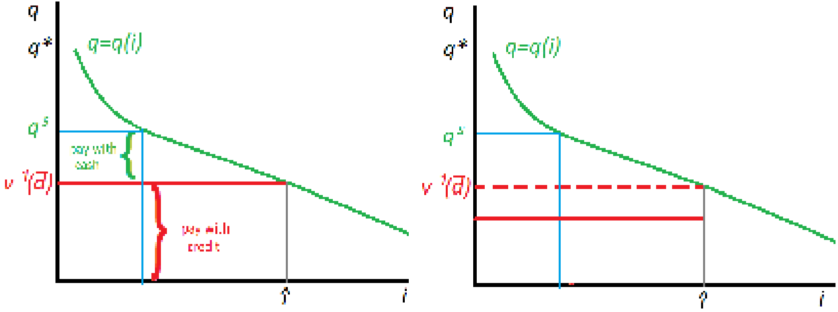
Money and Credit

$$W(m,d) = \max_{x,\ell,\hat{m}} \left\{ \tilde{U}(x) - \ell + \beta V(\hat{m},0) \right\}$$
$$\text{st } x = \ell + \phi(m - \hat{m}) - \phi\hat{m} - d - T$$

$$V(m,0) = C_0 + W(m,0) + \alpha\left[u(q) - p\right],\ p \leq \bar{d} + \phi m.$$

- If $\bar{d} \geq v(q^*)$ then $q = q^*$, $p = p^*$ and there is no ME
- If $\bar{d} < v(q^*)$ then $q < q^*$, $p = \bar{d} + \phi m$ and
  - $\iota < \hat{\iota} \Rightarrow \exists$ ME with $\phi M > 0$
  - $\iota > \hat{\iota} \Rightarrow \nexists$ ME with $\phi M > 0$
- Conclusion: if credit is tight, money can be valued and essential, for low ι

Money and Credit Redux



Prop: In SME the debt limit does not matter.

Money and Credit Redux with KM Credit

$$W(m,a,d) = \text{...st } x = \ell + \phi_m m + (\rho + \phi_a)a - \phi_m \hat{m} - \phi_a \hat{a} - d - T$$

$$V(m,a,0) = \text{..., but now } p \leq \bar{d} + \phi_m m + (\rho + \phi_a)\chi a.$$

Euler eqns

$$\phi_m = \beta \phi_m [1 + \alpha \lambda(q)] \text{ and } \phi_a = \beta(\rho + \phi_a)[1 + \alpha \chi \lambda(q)]$$

In SME

$$\begin{aligned} \iota &= \alpha \lambda(q) \\ \phi_a &= \beta(\rho + \phi_a)(1 + \chi \iota) \end{aligned}$$

Conclusion: ι affects q and ϕ_a; ρ and χ affects ϕ_a but not q.

Money and Capital

- Aruoba-Wright (JMCB): add k but k is not traded in DM.
- Aruoba et al (JME): k is not traded in DM *but* k is a factor in producing $q = f(k, e)$.
  - seller disutility is $e = c(q, k)$ with $c_1 > 0$, $c_2 < 0$...
- Planner:

$$
\begin{aligned}
J(K) &= \max_{q,X,H,K_+} \{U(X) - AH + \alpha [u(q) - c(q, K_+)] + \beta J_+(K_+) \\
\text{st } X &= F(K, H) + (1-\delta)K - K_+ - G
\end{aligned}
$$

- CM meets then DM same period, K_+ from CM is used in DM.

Solving Planner

$$\mathcal{L}(K) = U(X) - AH + \alpha\left[u(q) - c(q, K_+)\right] + \beta\mathcal{L}_+(K_+)$$
$$+\eta\left[F(K,H) + (1-\delta)K - K_+ - G - X\right]$$

$$\begin{aligned}
q &: 0 = \alpha\left[u'(q) - c_1(q, K_+)\right] \\
X &: 0 = U'(X) - \eta \\
H &: 0 = -A + \eta F_2(K,H) \\
K_+ &: 0 = -\alpha c_2(q, K_+) + \beta\mathcal{L}'_+(K_+) - \eta \\
\eta &: 0 = F(K,H) + (1-\delta)K - K_+ - G - X
\end{aligned}$$

$\mathcal{L}'(K) = \eta\left[F_1(K,H) + 1 - \delta\right]$ and K_+ eqn $\Rightarrow$

$$\eta = \beta\eta_+\left[F_1(K_+, H_+) + 1 - \delta\right] - \alpha c_2(q, K_+)$$

First Best

Eliminate multiplier η to write

$$
\begin{aligned}
u'(q) &= c_1(q, K_+) & (6)\\
A &= U'(X) F_2(K, H) & (7)\\
U'(X) &= \beta U'(X_+)\left[F_1(K_+, H_+) + 1 - \delta\right] - \alpha c_2(q, K_+) & (8)\\
X &= F(K, H) + (1-\delta)K - K_+ - G & (9)
\end{aligned}
$$

$c_2(q, K) = 0$ implies dichotomy:

- (6) $\Rightarrow$ q as in std monetary model;
- (7)-(9) $\Rightarrow$ (X, H, K_+) as std growth model.

$c_2(q, K) \neq 0$ breaks dichotomy.

Equlibrium: CM

$$W(m,k) = \max_{x,h,\hat{m},\hat{k}} \left\{ U(x) - Ah + V(\hat{m},\hat{k}) \right\}$$

st $x = \omega(1-\tau_h)h + [1-\delta+\kappa(1-\tau_k)]k + \phi(m-\hat{m}) - \hat{k} - T$

$$\begin{aligned} x &: \quad A = U'(X)(1-\tau_h)\omega \\ \hat{m} &: \quad A\phi/\omega(1-\tau_h) = V_1(\hat{m},\hat{k}) \\ \hat{k} &: \quad A/\omega(1-\tau_h) = V_2(\hat{m},\hat{k}) \end{aligned}$$

$$\begin{aligned} W_1(m,k) &= A\phi/\omega(1-\tau_h) \\ W_2(m,k) &= A[1-\delta+\kappa(1-\tau_k)]/\omega(1-\tau_h) \end{aligned}$$

Equlibrium: DM

$$\begin{aligned} V(m,k) &= \beta W(m,k) + \alpha \left\{ u[q(m,\bar{k})] - \frac{\beta A \phi m}{\omega(1-\tau_h)} \right\} \\ &\quad + \alpha \left\{ \frac{\beta A \phi \bar{m}}{\omega(1-\tau_h)} - c[q(\bar{m},k),k] \right\} \end{aligned}$$

where $q\left(m^b, k^s\right)$ solves $v(q, k^s) = \beta A \phi m^b / \omega(1-\tau_h)$ and hence

$$\begin{aligned} \partial q / \partial m^b &= \beta \phi A / \omega(1-\tau_h) / v_1(q, k^s) > 0 \\ \partial q / \partial k^s &= -v_2(q, k^s) / v_1(q, k^s) > 0 \end{aligned}$$

Note the double holdup problem:

- if you bring more m you get more q but you pay more;
- if you bring more k $c(q,k)$ falls for given q but q goes up.

Euler Eqns

As usual, insert V_1 and V_2 into FC's for $\hat{m}$ and $\hat{k}$ and use the result that (m, k) is degenerate to get:

$$\begin{aligned} U'(x)\phi &= \beta U'(x_+)\phi_+ \left[1 + \alpha\lambda(q, k_+)\right] \\ U'(x) &= \beta U'(x_+) \left[1 - \delta + \kappa\left(1 - \tau_k\right)\right] - \alpha c_2\left(q, k_+\right) \\ &\qquad + \alpha \frac{c_1\left(q, k_+\right) v_2(q, k_+)}{v_1(q, k_+)} \end{aligned}$$

where as usual

$$\lambda\left(q, k\right) = \frac{u'(q)}{v_1(q, k)} - 1$$

Efficiency vs Equilibrium

$$\begin{aligned} 1 &= u'(q)/c_1(q, K_+) \\ A &= U'(X)F_2(K, H) \\ U'(X) &= \beta U'(X_+)\left[F_1(K_+, H_+) + 1 - \delta\right] \\ &\qquad - \alpha c_2(q, K_+) \\ X &= F(K, H) + (1-\delta)K - K_+ - G \end{aligned}$$

$$\begin{aligned} \iota/\alpha + 1 &= u'(q)/v_1(q, K_+) \\ A &= U'(X)F_2(K, H)\left(1 - \tau_h\right) \\ U'(X) &= \beta U'(X_+)\left[F_1(K_+, H_+)\left(1 - \tau_k\right) + 1 - \delta\right] \\ &\qquad - \alpha c_2(q, K_+) + \alpha \frac{c_1(q, k_+)\, v_2(q, k_+)}{v_1(q, k_+)} \\ X &= F(K, H) + (1-\delta)K - K_+ - G \end{aligned}$$

Calibration

$$U(x) = B\frac{x^{1-\varepsilon}-1}{1-\varepsilon} \text{ and } F(K,H) = K^{\alpha}H^{1-\alpha}$$

$$u(q) = C\frac{q^{1-\eta}-1}{1-\eta} \text{ and } c(q,k) = q^{\psi}k^{1-\psi}$$

(a) 'Obvious' Parameters

β	ε	η	t_h	t_k	t_x	π	δ	α
.966	1	1	.242	.548	.069	.036	.070	.288

(b) Remaining Parameters

Parameters	G	ψ	A	B	σ	θ
Targets	G/Y	K/Y	H	v	ξ	μ
Target Values	.25	2.32	.33	5.29	−.23	.10

Key Results

- Bargaining model $\Rightarrow$ small impact of π on K
- Price taking model $\Rightarrow$ very big impact of π on K
- Both models $\Rightarrow$ inflation is very costly
  - bargaining: π affects q and q is very low
  - price taking: π affects K and hence X
- Taxes are costly, but even if we replace π with τ_h eliminating π may still be desirable.
- Holdup problems are important, even with DM/CM ≈ 0.05.

Stochastic Money Growth: CM

Suppose $M_{+} = (1 + \mu_{+})M$ where μ_{+} is iid draw from $G(\mu)$.

At start of DM agents learn the μ to be implemented later in CM.

Look for equil where as usual $1/\phi_{+} = \left(1 + \mu_{+}\right)/\phi$.

CM problem in terms of $\tilde{z}$ (real balances taken out of CM, not the same as real balances in DM due to the shock):

$$W(z) = \max\left\{U(x) - h + \beta \mathbb{E} V_{+}\left(\frac{\tilde{z}}{1+\mu_{+}}\right)\right\} \text{ st } x = h + z - \tilde{z} - T$$

$$\Rightarrow 1 = \beta \mathbb{E}\left[\frac{V_{+}'\left(\frac{\tilde{z}}{1+\mu_{+}}\right)}{1+\mu_{+}}\right].$$

Stochastic Money Growth: DM

Consider any $v(q)$ with $u(q) - v(q)$ increasing on $[0, q^*]$.

The realization of μ_+ is a shock to DM liquidity with critical value $\hat{\mu} = \tilde{z}/z^* - 1$.

$$\begin{array}{lcc} \mu_+ \leq \hat{\mu} \Rightarrow & q = q^* & p = v(q^*) \\ \mu_+ > \hat{\mu} \Rightarrow & v(q) = \frac{\hat{z}}{1+\mu_+} & p = \frac{\hat{z}}{1+\mu_+} \end{array}$$

If $\mu_+ > \hat{\mu}$,

$$V'\left(\frac{\tilde{z}}{1+\mu_+}\right) = W'\left(\frac{\tilde{z}}{1+\mu_+}\right) + \alpha\left[\frac{u'(q)}{v'(q)\left(1+\mu_+\right)} - \frac{1}{1+\mu_+}\right]$$

If $\mu_+ < \hat{\mu}$ the term multplying α is 0.

Stochastic Money Growth: EE

Insert $V'(\cdot)$ into FC

$$1 = \beta \mathbb{E}\left[\frac{1+\alpha\lambda(q)}{1+\mu_+}\right] = \beta \mathbb{E}\left[\frac{1}{1+\mu_+} + \frac{\alpha\lambda \circ v^{-1}\left(\frac{\tilde{z}}{1+\mu_+}\right)}{1+\mu_+}\right]$$

Stochastic version of Fisher eqn:

$$(1+\iota)\,\mathbb{E}\left(\frac{1}{1+\mu_+}\right) = 1 + r$$

Therefore EE can be written:

$$(1+\iota)\,\mathbb{E}\left(\frac{1}{1+\mu_+}\right) = \mathbb{E}\left[\frac{1}{1+\mu_+} + \frac{\alpha\lambda \circ v^{-1}\left(\frac{\tilde{z}}{1+\mu_+}\right)}{1+\mu_+}\right]$$

Stochastic Money Growth: SME

Write expectation as integral and simplify to get an eqn in $\tilde{z}$:

$$\iota = \frac{\alpha \int_{\mu > \hat{\mu}} \frac{\lambda \circ v^{-1}\left(\frac{\tilde{z}}{1+\mu_+}\right)}{1+\mu_+} dG(\mu)}{\int \frac{1}{1+\mu_+} dG(\mu)}$$

- If μ is deterministic this reduces to usual $\iota = \alpha \lambda(q)$.
- Note $\iota = 0$ is optimal (see Lagos 2009 for more).
- Note something else: Suppose at the start of DM the Fed knows the μ_+ to be implemented in CM later that period.

They should keep it a secret.

Competitive Search Equilibrium: Buyers Post

After substitutions, buyers post in CM for next DM to solve:

$$v_b = \max_{q,z,n} \left\{ \frac{\alpha(n)}{n} [u(q) - z] - \iota z \right\} \text{ st } \alpha(n) [z - c(q)] = v_s$$

Form Lagrangian with multiplier η_b and take FC's:

$$\begin{aligned}
q &: \quad \frac{\alpha(n)}{n} u'(q) - \eta_b \alpha(n) c'(q) = 0 \\
z &: \quad -\frac{\alpha(n)}{n} - \iota + \eta_b \alpha(n) = 0 \\
n &: \quad \frac{n\alpha'(n) - \alpha(n)}{n^2} [u(q) - z] + \eta_b \alpha'(n) [z - c(q)] = 0 \\
\eta_b &: \quad \alpha(n) [z - c(q)] - v_s = 0
\end{aligned}$$

Buyers Post, No Entry

Solve z eqn for η and insert into q eqn to get:

$$\iota = \frac{\alpha(n)}{n}\left[\frac{u'(q)}{c'(q)} - 1\right] = \frac{\alpha(n)}{n}\lambda(q). \tag{10}$$

The solve n eqn for

$$z = \frac{\varepsilon(n)\, u'(q)\, c(q) + [1-\varepsilon(n)]\, c'(q)\, u(q)}{\varepsilon(n)\, u'(q) + [1-\varepsilon(n)]\, c'(q)} = g(q,n). \tag{11}$$

where $\varepsilon(n) = n\alpha'(n)/\alpha(n)$.

With $n = n_b/n_s$ fixed, in CSE q solves (10); z solves (11); and

$$v_s = \alpha(n)[z - c(q)] \text{ and } v_b = \frac{\alpha(n)}{n}[u(q) - z] - \iota z.$$

Buyers Post, Seller Entry

Now $v_s = k_s$, and (n, q, z) solve $\alpha(n)[z - c(q)] = k_s$, (10)-(11).

Eliminating z, CSE with seller entry solves the MD and SE curves:

$$\begin{aligned} \text{MD} &: \quad \iota = \frac{\alpha(n)}{n}\lambda(q) \\ \text{SE} &: \quad k_s = \alpha(n)[g(q,n) - c(q)] \end{aligned}$$

$$\Rightarrow \frac{\partial q}{\partial n}_{|\text{MD}} = \frac{(1-\varepsilon)\lambda}{n\lambda'} < 0 \text{ and } \frac{\partial q}{\partial n}_{|\text{SE}} = \frac{ng_2 + \varepsilon(g-c)}{n(c' - g_1)} < 0$$

Proof: MD is easy. For SE, first, algebra implies:

$$c' - g_1 \simeq \varepsilon(u-c)\left(c'u'' - u'c''\right) - \left[\varepsilon u' + (1-\varepsilon)c'\right]\left(u' - c'\right)c'$$

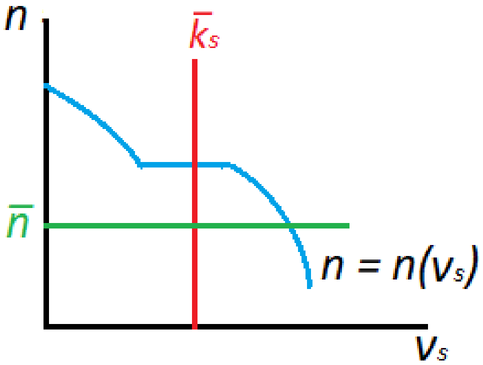
Then note $g_2 \simeq -\varepsilon' \geq 0$ for ‘standard’ mtg techologies. ■

Existence and Uniqueness

- When $\iota \to 0$ MD and SE cross uniquely at $n > 0$ and $q = q^*$.
  - although MD and SE are both downward sloping in (n, q), as $\iota \to 0$, MD becomes flat at $q = q^*$.
  - so SE gives n as unique soln to $k_s = \alpha(n)[g(q^*, n) - c(q^*)]$.
- Of course to guarantee $n > 0$ we need k_s not too big.
- When ι increases this approach fails since the FC's can have multple solns.
- But generically max problem has a unique soln and even if it is a correpondence n is decreasing in v_s.

Prop: CSE exists and it generically unique.

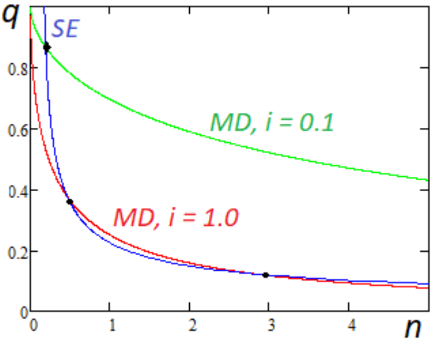
Proof: With no entry $n = \bar{n}$ pins down v_s except for measure 0 situation where $\bar{n}$ coincides with a flat part of curve. With entry $v_s = \bar{v}_s$ pins down n.



Example with Seller Entry

Standard $u(q) = q^{0.5}$, $c(q) = q$, $\alpha(n) = n^{0.4}$ and $k_s = 0.3$.

For high ι Shown are two solns to FC's at high ι but only one solves max.



Buyers Post, Buyer Entry

MD is the same but now

$$\text{BE} : \frac{\alpha(n)}{n}\left[u(q) - g(q,n)\right] = k_b + \iota g(q,n)$$

where we note $\iota g(q,n)$ on the RHS. Hence

$$\frac{\partial q}{\partial n}_{|\text{BE}} = \frac{ng_2 + (1-\varepsilon)(u-g) + \iota g_2 n^2/\alpha}{n(u' - g_1) - \iota g_1 n^2/\alpha}$$

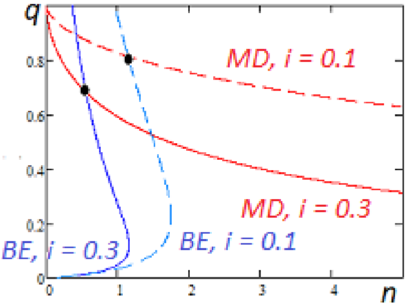
We cannot sign this, not only because of the ι terms, but because $u' - g_1$ is not well behaved like $g_1 - c'$ (related to buyer's surplus being nonmonotone as with Nash bargaining).

Still for $\iota \to 0$ we get uniqueness because then MD is flat at q^* and BE is downward sloping near q^*.

Example with Buyer Entry

With similar parameters, now a change in ι affects both money demand and the entry condition.

Shown are the unique steady state for two different ι (and *not* multple solns for the same ι)



Buyer Entry vs Seller Entry

The models look different depending on who enters.

But a reasonable conjecture is that the equil set with entry by s is actually the same as the equil set with entry by b.

Logic: Specify a model with entry by s and work out the full equil including v_b, say $v_b = v_b^*$.

Then specify a model with entry by b at cost $k_b = v_b^*$.

It seems clear that we should recover the same q, n and z.

There is also a third appraoach – let agents decide in CM to be s or b. Similar logic suggests similar results.

Competitive Search Equilibrium: Sellers Post

$$v_s = \max_{q,z,n} \{\alpha(n)[z - c(q)]\} \text{ st } \frac{\alpha(n)}{n}[u(q) - z] - \iota z = v_b$$

Form Lagrangian with mulplier η_s and take FC's

$$\begin{aligned}
q &: -\alpha(n) c'(q) - \eta_s \frac{\alpha(n)}{n} u'(q) = 0 \\
z &: \alpha(n) + \eta_s \frac{\alpha(n)}{n} - \eta_s \iota = 0 \\
n &: \alpha'(n)[z - c(q)] + \eta_s \frac{n\alpha'(n) - \alpha(n)}{n^2}[u(q) - z] = 0 \\
\eta_s &: \frac{\alpha(n)}{n}[u(q) - z] - \iota z - v_b = 0
\end{aligned}$$

Sellers Post

From the q and z eqns we get the same MD curve $\iota = \frac{\alpha(n)}{n}\lambda(q)$.

Without entry, $n = n_b / n_s$ is fixed, so this gives q, the n eqn yields $z = g(q, n)$, and v_b, v_s follow as before.

With buyer or seller entry, we add the BE curve or SE curve.

In all cases, these are the same as when buyers post.

Also, in all cases (10) gives q as if we had Nash bargaining with $\theta = 1$ while (11) gives q as if we had Nash bargaining with $\theta = \varepsilon(n)$.

Thus we avoid holdup problems with sunk costs of liquidity and of entry for buyers and sellers, resp.

CSE is Efficient iff $\iota = 0$

Normalize $n_b = 1$ and consider planner maximizing expected surplus minus cost of having $n_s = 1/n$ sellers:

$$v_p = \max_{q,n} \left\{ \frac{\alpha(n)}{n} [u(q) - c(q)] - \frac{k_s}{n} \right\}$$

Efficient q solves $u'(q) = c'(q)$ – same as equil iff $\iota = 0$.

Efficient n solves, after simplification,

$$k_s = \alpha(n) [1 - \varepsilon(n)] [u(q) - c(q)].$$

This is the same as equil entry condition for s once we simplify

$$k_s = \alpha(n) [g(q,n) - c(q)].$$