Monetary Economics II:

Generation 2: Divisible Goods and Prices

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Divisible Goods

- So far the terms of trade are exogenous: 1 asset buys 1 unit of the good if they trade (endogenous outcome is the *if*).
- Modeling prices: relax $m \in \{0,1\}$ or use divisible goods?
- For now we consider divisible goods, since it is easier.
- Production of quantity q costs $c(q)$, and consumption yields utility $u(q)$, where:
  - $u(0) = c(0) = 0$, $u' > 0$, $c' > 0$, $u'' < 0$ and $c'' \geq 0$;
  - efficient q^* satisfies $u'(q^*) = c'(q^*)$;
  - $\exists q > 0$ such that $u(\hat{q}) = c(\hat{q})$;
  - for some results $u'(q) / c'(q) \to \infty$ as $q \to 0$.

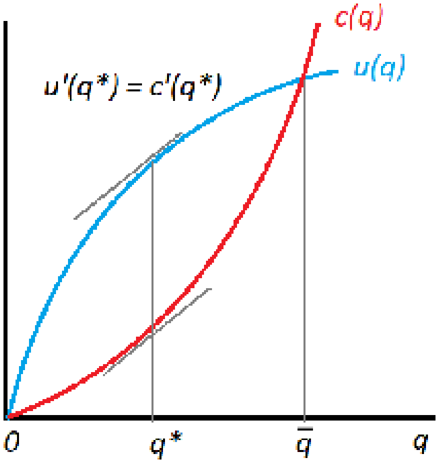


Figure: Preferences

Generalizing the Model

Before discussing prices, write the continuous time DP eqs as follows:

$$\begin{aligned} rV_0 &= \alpha\delta\tilde{S} + \alpha\sigma M\tau\left[-c\left(q\right) + V_1 - V_0\right] + \dot{V}_0 \\ rV_1 &= \alpha\delta\tilde{S} + \alpha\sigma(1-M)\tau\left[u\left(q\right) + V_0 - V_1\right] + \rho + \dot{V}_1 \end{aligned}$$

where $\tilde{S} = u\left(\tilde{q}\right) - c\left(\tilde{q}\right)$ is the surplus from bartering $\tilde{q}$, and q is quantity traded for assets, and $\tau = 1$ with one exception below.

- Note that we include the time derivatives $\dot{V} = dV/dt$.
- To ease the presentation set $\delta = 0$, so we can ignore $\tilde{q}$ and focus on q; we can bring back barter below

An Important Equation

Subtracting and grouping terms in $\Delta = V_1 - V_0$, we get

$$\dot{\Delta} = (r + \alpha_0\tau + \alpha_1\tau)\Delta - \alpha_1\tau u(q) - \alpha_0\tau c(q) - \rho$$

- where we use $\dot{\Delta} = \dot{V}_1 - \dot{V}_0$.
- and write $\alpha_0 = \alpha\sigma M$ and $\alpha_1 = \alpha\sigma(1 - M)$.

It is useful below to define $F(\Delta, q)$ and write this as

$$\dot{\Delta} = F(\Delta, q) \equiv (r + \alpha_0\tau + \alpha_1\tau)\Delta - \alpha_1\tau u(q) - \alpha_0\tau c(q) - \rho$$

Equilibrium in the Generalized Model

DP eqns determine (V_0, V_1) given q; we now to determine q given (V_0, V_1).

Notice $p = 1/q$ is the price of goods in terms of assets, so if m is fiat money p is the nominal price level.

Options?

- Bargaining: seems natural (Shi, Trejos-Wright);
- Price posting: maybe even more natural (Curtis-Wright, Burdett et al, Julien et al);
- Auctions: natural in version where multiple buyers can meet same seller (Julien et al);
- Abstract mechanism design (Wallace et al).

Bargaining

In general, we use a generic bargaining solution $v(q)$, where $v(\cdot)$ is some function with the following interpretation:

> *To get q, a buyer must transfer to a seller surplus $v(q)$.*

Given $m \in \{0, 1\}$ we have $V_1 - V_0 = v(q)$.

Natural properties: $v(0) = 0$ and $v'(q) > 0$.

Also natural to impose $c(q) \leq v(q) \leq u(q)$ for voluntary trade and we usually want $v(q) < u(q)$.

An example is Walrasian pricing $v(q) = Pq$, where in equil $P = c'(q)$.

More Interesting Examples

Kalai bargaining where θ is the buyer's bargaining power:

$$v(q) = \theta c(q) + (1-\theta) u(q)$$

Nash bargaining:

$$v(q) = \frac{\theta u'(q) c(q) + (1-\theta) c'(q) u(q)}{\theta u'(q) + (1-\theta) c'(q)}$$

These are the same at efficient q^* but that does not necessarily occur with liquidity considerations.

Kalai seems easier and has some advantages (Aruoba et al).

Of course they are the same if $\theta = 1$ or $\theta = 0$.

Equilibrium

In steady state $F(\Delta, q) = 0$ which means

$$(r + \alpha_1\tau + \alpha_0\tau)\Delta = \alpha_1\tau u(q) + \alpha_0\tau c(q) + \rho,$$

where $\tau = 1$ except in one case discussed below.

Combine with bargaining $\Delta = v(q)$ to get $f(q) = 0$ where

$$f(q) \equiv (r + \alpha_1\tau + \alpha_0\tau) v(q) - \alpha_1\tau u(q) - \alpha_0\tau c(q) - \rho.$$

A stationary equil, or steady state, is given by $q \in [0, \bar{q}]$ solving $f(q) = 0$, plus $\tau \in [0, 1]$ satisfying the BR condition:

$$\Delta > c(q) \Rightarrow \tau = 1; \Delta < c(q) \Rightarrow \tau = 0; \Delta = c(q) \Rightarrow \tau \in [0, 1]$$

Nonmonetary Equilibrium

A monetary equil (ME) has $q > 0$ and $\tau > 0$; a nonmonetary equil (NE) has $q = 0$ or $\tau = 0$.

- If $\rho < 0$, there is always a NE, since $\tau = 0$ is a best response to itself (for any q). In this equil agents dispose of the asset.

From now on we set $\tau = 1$.

- If $\rho = 0$, there is always a NE where $q = 0$.
- If $\rho > 0$, NE does not exist.

From now one we focus on monetary equil.

Monetary Equilibrium

Existence and properties of ME have been studied extensively, with He and Wright a recent example going through all the details.

Simple case: If we assume $v(q) = c(q)$ (bargaining with $\theta = 1$) and $|\rho|$ not too big then $f(q) = 0$ iff

$$c(q) = \frac{\alpha_1 u(q) + \rho}{r + \alpha_1}.$$

- If $\rho \geq 0$ there is a unique ME $q \in [0, \bar{q}]$.
- If $\rho < 0$ there are two ME $q_H \in (0, \bar{q})$ and $q_L \in (0, q_H)$.

Other bargaining solutions give similar if not the same results.

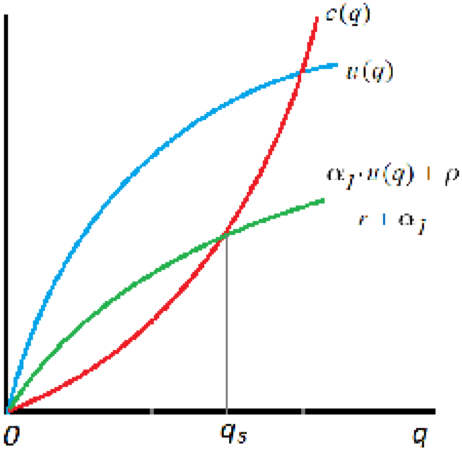


Figure: Equilibria with $\theta = 1$

Interesting Result

Consider Nash bargaining, and notice various factors influence q:

- bargaining power, $\partial q / \partial \theta > 0$;
- market tightness, $\partial q / \partial M < 0$;
- asset returns, $\partial q / \partial \rho > 0$.

Let's "neutralize" all that by setting $\theta = M = 1/2$ and $\rho = 0$.

Prop: $\theta = M = 1/2$ and $\rho = 0 \Rightarrow q < q^*$ but $q \rightarrow q^*$ as $r \rightarrow 0$.

This says all prices are too high in equil (intuitively).

It also gives rise to a tradeoff between liquidity and prices – what is optimal M?

Some Microfoundations

The above formulae are not the definitions of Nash or Kalai bargaining – they are results.

Nash specifies some axioms on solutions to bargaining problems.

He proves there exists a unique outcome satisfying the axioms, and it can be found by maximizing the Nash product (in our notation)

$$\max_q \left[u(q) - \Delta\right]^\theta \left[-c(q) + \Delta\right]^{1-\theta},$$

where $\theta = 1/2$ in his original (symmetric) specification, but more generally $\theta \in [0, 1]$ captures buyer bargaining power.

The FOC for max yields $\Delta = v(q)$, with $v(\cdot)$ as given above.

More Microfoundations

Kalai noticed the following issue with Nash bargaining in general – when the total surplus goes up, one party can be worse off.

He replaced one of Nash's axioms with the axiom that when the total surplus goes up both parties must be better off.

He proves the outcome solves (again in our notation)

$$\max_q \{u(q) - \Delta\} \text{ st } u(q) - \Delta = \theta [u(q) - c(q)]$$

where the constraint says buyers get fraction θ of the total surplus.

Since there is only one choice here, q, the constraint yields $\Delta = v(q)$, with $v(\cdot)$ as given above.

Still More Microfoundations

The *Nash Program* seeks strategic bargaining games that deliver the same outcomes as axiomatic solutions.

Example: A random-offer game (Rubinstein; Binmore). In the unique SPE, offers are always accepted, and $q = q_1$ or q_0 depending on who goes first.
As the time between offers gets small, $q_1, q_0 \rightarrow q$ where q solves

$$\max_q \left[u(q) + V_0 - T_1\right]^{\theta} \left[-c(q) + V_1 - T_0\right]^{1-\theta},$$

$\theta = \mathit{prob}$(buyer makes offer), and T_1, T_0 are threat points.

Result: $T_1 = V_1$ and $T_0 = V_0$, so q is the Nash solution given above, if agents can search between offers (else $T_1 = T_0 = 0$).

More Bargaining Games

Random-offer games in general involve risk.

To avoid this, let one agent for sure make the first offer, which is accepted, but if (off the equil path) it were rejected they enter a random offer game.

If seller goes first, it looks like price posting, but the posted price is disciplined by the threat of rejecting and bargaining.

Conveniently, the game ends in finite time on and off the equil path.

Equil in this game can be represented $v(q) = \Delta$ (see Zhu).

Continuous Time Dynamics

Setting $\tau = 1$ we have

$$\dot{\Delta} = F(\Delta, q) = (r + \alpha_0 + \alpha_1)\Delta - \alpha_1 u(q) - \alpha_0 c(q) - \rho$$

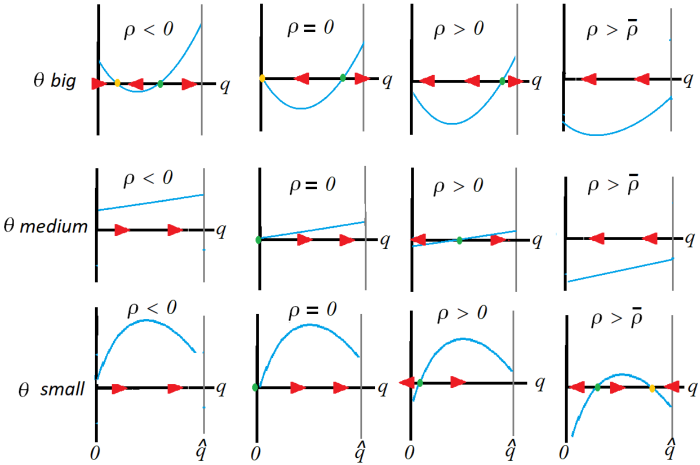
or, inserting $\Delta = v(q)$,

$$v'(q)\,\dot{q} = f(q) = (r + \alpha_0 + \alpha_1)v(q) - \alpha_1 u(q) - \alpha_0 c(q) - \rho$$

- Equilibrium is path for q solving this d.e. with $q \in [0, \hat{q}]$. A monetary equilibrium has $q > 0$.
- A MSS is a monetary equil with $\dot{q} = 0$ (same as before).
- Equivalently we could use

$$\dot{\Delta} = g(\Delta) = (r + \alpha_0 + \alpha_1)\Delta - \alpha_1 u \circ v^{-1}(\Delta) - \alpha_0 c \circ v^{-1}(\Delta) - \rho$$

Equilibria with Kalai Bargaining: Existence, Uniqueness, Dynamics



Issues with Dynamic Bargaining

- It is one thing to impose $v(q) = \Delta$ as an equil condition; it is something else to claim microfoundations coming from Binmore, Rubinstein...
- The latter is valid *if* we restrict attention to steady state.
- Coles-Wright: in nonstationary settings the limit of the game is not Nash, but $v'(q)\dot{q} = f(q)$
  - except in special cases, eg linear utility or $\theta = 1$
  - of course in steady state $f(q) = 0$ gives Nash
  - imposing Nash out of steady state is like agents playing the game with myopic expectations.
- One can use the correct limit $v'(q)\dot{q} = f(q)$ but it is complicated.
- And this matters for the results.

Cyclic Equilibria with Coles-Wright Bargaining and Fixed Cost

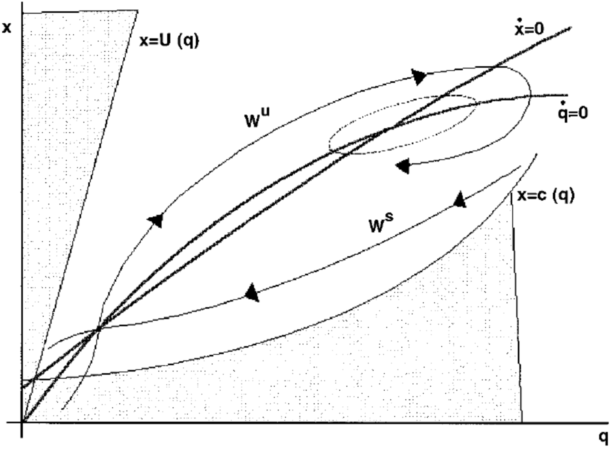


FIGURE 4

Alternative 1: Julien-Kennes-King Posting with Directed Search

Set $\sigma = 1$ as one simple implication of directed search.

Given $n_1 = M$ buyers and $n_0 = 1 - M$ sellers, $N = n_1 / n_0$ is aggregate market tightness.

Each seller posts a price p, or equivalently $q = 1/p$, and we define a submarket as the set of sellers posting the same q.

Buyers see all q's and decide where to search, giving tightness n in each submarket.

In submarket (q, n), agents meet in pairs at rates $\alpha_0 = \alpha(n)$ and $\alpha_1 = \alpha(n)/n$.

We get these from a CRS meeting technology $A = A(n_1, n_0)$ by noticing $\alpha_0 = A/n_0$ and $\alpha_1 = A/n_1$.

More on Meeting Technologies

The models analyzed above implicitly used a special case

$$A(n_1, n_0) = \alpha n_1 n_0 / (n_1 + n_0),$$

so $\alpha_0 = \alpha n / (1 + n) = \alpha M$ and $\alpha_1 = \alpha / (1 + n) = \alpha (1 - M)$.
An alternative is Cobb-Douglas:

$$A(n_1, n_0) = a n_1^{\varepsilon} n_0^{1-\varepsilon}.$$

Another is the urn-ball technology (can be endogenized):

$$A(n_1, n_0) = n_0 \left[1 - (1 - 1/n_0)^{n_1}\right].$$

Frictionless matching, short side always meet:

$$A(n_1, n_0) = \min\{n_1, n_0\}$$

Competitive Search Equilibrium

CSE can be found by maximizing seller flow payoff subject to buyers getting the market payoff

$$\begin{aligned} rV_0 &= \max_{(q,n)} \{\alpha(n)[-c(q)+\Delta]\} \\ \text{st } rV_1 &= \frac{\alpha(n)}{n}[u(q)-\Delta]+\rho \end{aligned}$$

where rV_1 is taken as given by sellers, but determined below in equil (like prices in Walrasian markets).

To solve this, form the Lagrangian

$$\mathcal{L} = \alpha(n)[-c(q)+\Delta] + \eta\left\{\frac{\alpha(n)}{n}[u(q)-\Delta] - rV_1\right\}$$

where η is the multiplier.

First-Order Conditions

$$\begin{aligned}
\mathcal{L}_q &= -\alpha(n)\,c'(q) + \eta\frac{\alpha(n)}{n}u'(q) \\
\mathcal{L}_n &= \alpha'(n)\left[-c(q)+\Delta\right] + \eta\frac{n\alpha'(n)-\alpha(n)}{n^2}\left[u(q)-\Delta\right] \\
\mathcal{L}_\eta &= \frac{\alpha(n)}{n}\left[u(q)-\Delta\right] - rV_1
\end{aligned}$$

Solving $\mathcal{L}_q = 0$ for η and inserting it into $\mathcal{L}_n = 0$, after simplification, we get

$$\Delta = \frac{\varepsilon u'(q)\,c(q) + (1-\varepsilon)\,c'(q)\,u(q)}{\varepsilon u'(q) + (1-\varepsilon)\,c'(q)},$$

where $\varepsilon \equiv n\alpha'(n)/\alpha(n)$ is the elasticity of $\alpha(\cdot)$.

An Amazing Result

- Hence CSE $\Rightarrow \Delta = v(q)$ where $v(\cdot)$ is the same as Nash bargaining if $\theta = \varepsilon$.
- Now $\varepsilon = \varepsilon(n)$ is not constant except for $A = an_1^{\varepsilon} n_0^{1-\varepsilon}$.
- An elegant contribution to the Nash program!
- Useful in nonstationary analysis, where convergence of q to Nash in random-offer games does not work (Coles-Wright):
  - It delivers $\dot{q} = f(q)$
  - in steady state $f(q) = 0$ is Nash, but not out of steady state, except in very special cases;
- CSE is a better microfoundation for Nash than game theory.

Equilibrium

In any case, since there is a unique solution to the sellers problem, all sellers and submarkets have the same (q, n)

- Equivalently we can have just one, by CRS

If $n_1 = M$ and $n_0 = 1 - M$, $N = M/(1 - M)$ is fixed:

- "Market clearing" implies $n = N$;
- then we know $\varepsilon = \varepsilon(N)$, and q solves $\Delta = v(q)$;
- where $\Delta = V_1 - V_0$ comes from

$$\begin{aligned} rV_0 &= \alpha(n)\left[-c(q) + \Delta\right] \\ rV_1 &= \frac{\alpha(n)}{n}\left[u(q) - \Delta\right] + \rho. \end{aligned}$$

Nice Properties of CSE

We can add entry by sellers or buyers to make n endogenous; results are similar.

We can switch who posts and who searches; results are the same.

Consider the usual specification $A = \alpha n_1 n_0 / (n_1 + n_0)$

- With bargaining and $\rho = 0$, we found $q < q^*$ but $q \to q^*$ as $r \to 0$, if $M = \theta = 1/2$.
- In CSE with the same specification, $M = 1/2 \Rightarrow \varepsilon = 1/2$ automatically, so the result holds with fewer assumptions.

CSE avoids the "black box" of bargaining & gives stronger results.

Some versions avoid the "black box" of the meeting technology.

Simple Example to Build Intuition

Consider a one-period model.

Assume transferable utility, and hence no restriction on payments p buyers can make.

Also let q be fixed (indivisible goods), but that's easy to relax.

So sellers post (p, n) to solve:

$$V_0 = \max_{(p,n)} \{\alpha(n)(p - c)\} \text{ st } V_1 = \frac{\alpha(n)}{n}(u - p)$$

One can use a Lagrangian, as above, or simply use the constraint to eliminate p and solve

$$\max_n \{\alpha(n)(u - c) - nV_1\}$$

Equilibrium in the Simple Example

FOC $\alpha'(n)(u-c) = V_1$ implies all sellers again choose same n.

If the measures of buyers and sellers, and hence aggregate tightness N, are fixed, then $n = N$ and FOC $\Rightarrow V_1$.

Then constraint $\Rightarrow p = \varepsilon c + (1-\varepsilon) u$, with $\varepsilon = \varepsilon(N)$ as above.

Then payoffs are

$$\begin{aligned} V_0 &= \alpha(N)(p-c) = \alpha(N)\left[1-\varepsilon(N)\right](u-c) \\ V_1 &= \frac{\alpha(N)}{N}(u-p) = \frac{\alpha(N)}{N}\varepsilon(N)(u-c). \end{aligned}$$

Notice: your payoff V is your trading prob, times your share, times the total surplus $u-c$.

Entry in the Simple Example

If sellers can enter at cost k, we still get

$$p = \varepsilon c + (1 - \varepsilon) u.$$

But now n solves the entry condition $k = V_0$, or

$$k = \alpha(n) [1 - \varepsilon(n)] (u - c).$$

And then

$$V_1 = \frac{\alpha(n)}{n} \varepsilon(n) (u - c).$$

Again we solve for (p, n, V_1, V_0), but with entry V_0 is fixed and n is endogenous, while previously n was fixed and V_0 was endogenous.

Equilibrium Properties in the Example

If buyers post to maximize V_1 st sellers their getting market payoff V_0, results are the same.

When there is entry, it is efficient – not true under bargaining unless $\theta = \varepsilon(n)$ (Hosios condition).

With no entry $\partial V_0/\partial N > 0$ and $\partial V_1/\partial N < 0$, but $\partial p/\partial N$ is in general ambiguous.

- Similarly, with entry, $\partial p/\partial k$ is ambiguous.
- Hence higher tightness (more demand) can reduce price – why?
- In CSE resources are allocated through both price and prob.

Alternative 2: Burdett-Judd Pricing with Noisy Search

- For now indivisible good, transferable utility, noisy search:
  - sellers post p implying dist'n $F(p)$
  - w/prob α_s buyer sees s draws from F and picks the min
  - w/prob $\tilde{\alpha}_s$ seller gets buyers that see s sellers' p (including his)
- Identity: $\tilde{\alpha}_s = n\alpha_s s$ where n is market buyer/seller ratio.

Lemma:

1. $\alpha_s = 0 \, \forall s > 1 \Rightarrow p = u$ (Diamond monopoly)
2. $\alpha_s = 0 \, \forall s < 2 \Rightarrow p = c$ (Bertrand competition)
3. otherwise $p \in \left[\underline{p}, \overline{p}\right]$ with no gaps or mass points.

BJ Algebra

- For any $p < u$ seller's value is

$$rV(p) = (p - c)\, n \sum_s \alpha_s s \left[1 - F(p)\right]^{s-1}$$

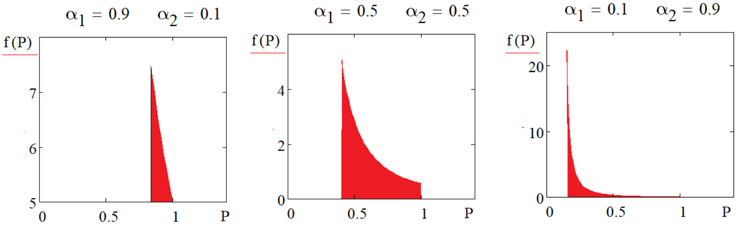
- Clearly $\overline{p} = u$ and $rV(\overline{p}) = (u - c)\, n\alpha_1$.
- Then EP (equal profit) implies

$$\sum_s \alpha_s s \left[1 - F(p)\right]^{s-1} = \alpha_1 \frac{u - c}{p - c}$$

- Also $rV(\underline{p}) = (\underline{p} - c) n \sum_s \alpha_s s = (\underline{p} - c) n\bar{s}$.
- So $V(\underline{p}) = V(u) \Rightarrow \underline{p} = \frac{\alpha_1}{\bar{s}} u + \left(1 - \frac{\alpha_1}{\bar{s}}\right) c$

A Nice Special Case: $\alpha_s = 0 \, \forall s > 2 \Rightarrow$

$$F(p) = 1 - \frac{\alpha_1(u-p)}{2\alpha_2(p-c)}, \; f(p) = \frac{\alpha_1(u-c)}{2\alpha_2(p-c)^2}, \; \underline{p} = \frac{\alpha_1 u + 2\alpha_2 c}{\alpha_1 + 2\alpha_2}$$



close to monopoly *lots of dispersion* *close to competitive*

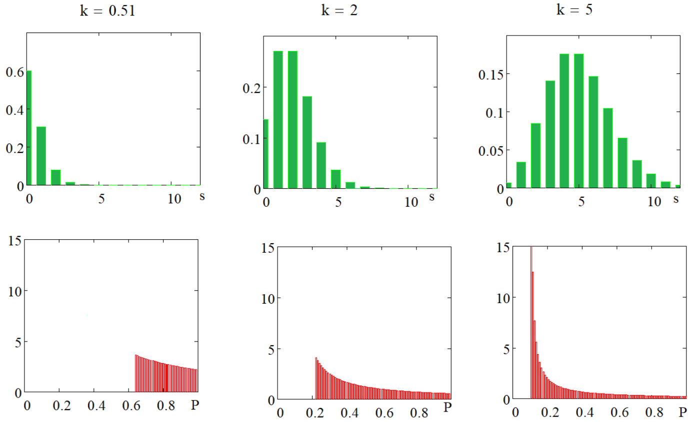
Note: $\alpha_s = 0 \, \forall s > 2$ is endogenous if agents choose s with a cost.
Note: p dispersion is **not** monotone decreasing as frictions fall.

Another Nice Case: s Poisson w/param $k = \mathbb{E}s$, $\alpha_s = e^{-k}k^s/s! \Rightarrow$

$$\begin{aligned}
\textstyle\sum_s \alpha_s s\left[1-F(p)\right]^{s-1} &= \alpha_1\left(\frac{u-c}{p-c}\right) \\
\textstyle\sum_s \frac{e^{-k}k^s}{s!} s\left[1-F(p)\right]^{s-1} &= e^{-k}k\left(\frac{u-c}{p-c}\right) \\
\textstyle\sum_s \frac{k^{s-1}\left[1-F(p)\right]^{s-1}}{(s-1)!} &= \frac{u-c}{p-c} \\
e^{k[1-F(p)]} &= \frac{u-c}{p-c}
\end{aligned}$$

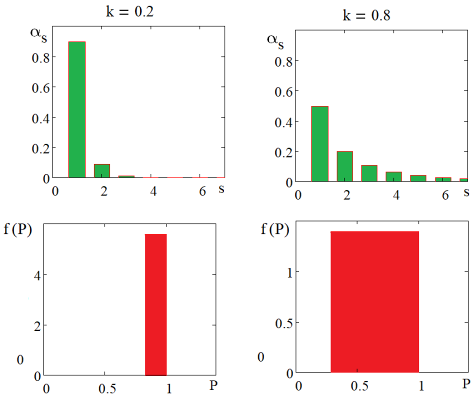
$$\Rightarrow F(p) = 1 - \frac{1}{k}\log\left(\frac{u-c}{p-c}\right), f(p) = \frac{1}{k(p-c)}$$

Distribution of s and implied BJ density when s is Poisson for different k, where $\underline{p} = e^{-k}u + (1 - e^{-k})c$.



And: s logarithmic $\alpha_s = \dfrac{-k^s}{s\log(1-k)}$ for $k \in (0,1) \Rightarrow$

$$F(p) = 1 - \frac{u-p}{k(u-c)},\ f(p) = \frac{1}{k(u-c)},\ \underline{p} = kc + (1-k)u$$



BJ Pricing in STW

Sellers post $q \Rightarrow F(q) \Rightarrow G(p) = 1 - F(1/p)$ and

$$rV_1 = \sum_s \alpha_s \int_{\underline{q}}^{\bar{q}} [u(q) - \Delta]\, dF(q)^s + \rho + \dot{V}_1 \tag{1}$$

$$rV_0(q) = n[\Delta - c(q)] \sum_s \alpha_s s F(q)^{s-1} + \dot{V}_0(q) \tag{2}$$

In particular, at lowest $\underline{q}$,

$$rV_0(\underline{q}) = n\alpha_1 \left[\Delta - c(\underline{q})\right] + \dot{V}_0(\underline{q}) \tag{3}$$

Using $\Delta = u(\underline{q})$, (2) = (3) (equal profit) implies

$$\sum_s \alpha_s s F(q)^{s-1} = \alpha_1 \frac{u(\underline{q}) - c(\underline{q})}{u(\underline{q}) - c(q)} \tag{4}$$

BJ Equilibrium in STW

Since $F(\bar{q}) = 1$,

$$c(\bar{q}) = \frac{\alpha_1 c(\underline{q}) + (\bar{s} - \alpha_1) u(\underline{q})}{\bar{s}}$$

giving $\bar{q} = Q(\underline{q})$, but now $\underline{q}$ depends on *endogenous* $\Delta = V_1 - V_0$.

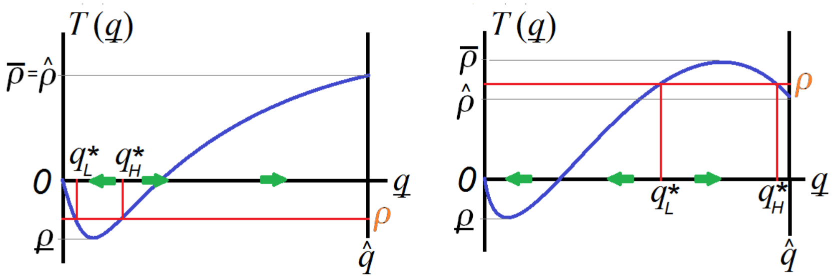
Subtract (1)-(3) to get $u'(\underline{q})\dot{\underline{q}} = T(\underline{q}) - \rho$ where

$$T(\underline{q}) \equiv \psi u(\underline{q}) - n\alpha_1 c(\underline{q}) - \alpha_1 \left[u(\underline{q}) - c(\underline{q}) \right] \int_{\underline{q}}^{Q(\underline{q})} \frac{u(q) F'(q)\, dq}{u(\underline{q}) - c(q)}$$

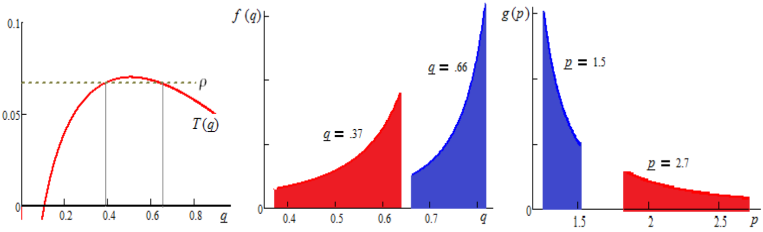
with $\psi \equiv r + (1 - \alpha_0) + n\alpha_1$, simplified using integration by parts.

A monetary equil is a path for $\underline{q}$ solving $u'(\underline{q})\dot{\underline{q}} = T(\underline{q}) - \rho$, with $\underline{q} \in (0, \hat{q}]$ so $[\underline{q}, \bar{q}] \subset (0, \hat{q}]$; from $\underline{q}$ we get $\bar{q} = Q(\underline{q})$ and $F(q)$.

In dynamic equil the *dist'n* $F(q)$ varies, but we can focus on $\underline{q}$.



Example: $u(q) = q^{0.2}$, $\alpha_1 = 0.48$, $\alpha_2 = 0.36$, $M = 0.3$, $r = 0.04$. When $\rho = 0.067$ $\exists$ two MSS $\underline{q}_L^* = 0.37$ and $\underline{q}_H^* = 0.66$ with very different dist'n $f(q)$ and $g(p)$.



Application: Rational Inattention.

Buyers see $s = 1$ for free and $s \geq 1$ at cost $(s-1)k$.

Choose s to maximize $\hat{B}(s) - (s-1)k$ where

$$\hat{B}(s) = \int_{\underline{q}}^{\bar{q}} [u(q) - \Delta]\, sF(q)^{s-1}\, dF(q)$$

$n < \infty \Rightarrow$ rational to not solicit all possible information.

There is always an equil at $s = 1$ for all (if everyone chooses $s = 1$ there us only one q so why pay for $s = 2$?)

In any other equil a fraction $\chi \in (0,1)$ of buyers choose $s = 1$ and the rest $1 - \chi$ choose $s = 2$.

There can be multple equil with different χ, so endogenous info creates additional multiplicity.

Other Applications

Burdett et al (JET) use the model to discuss

- Cyclic and chaotic equil
  - these exist with posting, but not with standard bargaining
- Sunspot equil
  - as with cyclic and chaotic equil posting avoids serious technical issues that arise with bargaining
- Efficiency
- Fundamental Shocks
- News Shocks
- Sticky Prices

Sunspots

Two states $S \in \{A, B\}$ with Poisson switching rates $(\varepsilon_A, \varepsilon_B)$

S does not affect fundamentals – but could it affect behavior?

$$
\begin{aligned}
rV_0^A &= \alpha_0 \left[-c\left(q^A\right) + \Delta^A\right] + \dot{V}_0^A + \varepsilon_A \left(V_0^B - V_0^A\right) \\
rV_1^A &= \rho + \alpha_1 \left[u\left(q^A\right) - \Delta^A\right] + \dot{V}_1^A + \varepsilon_A \left(V_1^B - V_1^A\right)
\end{aligned}
$$

$$
\dot{\Delta}^A = (r + \alpha_1 + \alpha_0)\,\Delta^A - \alpha_1 u\left(q^A\right) - \alpha_0 c\left(q^A\right) - \rho - \varepsilon_A(\Delta^B - \Delta^A)
$$

$$
\begin{aligned}
v'\left(q^A\right)\dot{q}^A &= (r + \alpha_1 + \alpha_0)\, v\left(q^A\right) - \alpha_1 u\left(q^A\right) - \alpha_0 c\left(q^A\right) - \rho \\
&\quad - \varepsilon_A \left[v\left(q^B\right) - v\left(q^A\right)\right]
\end{aligned}
$$

Solving for Sunspots (Azariadis 1981)

Write as $v'\left(q^A\right)\dot{q}^A = f\left(q^A\right) + \varepsilon_A\left[v\left(q^B\right) - v\left(q^A\right)\right]$.

Suppose $\dot{q}^S = 0$ and write this for both states as

$$\begin{aligned} 0 &= f\left(q^A\right) - \varepsilon_A\left[v\left(q^B\right) - v\left(q^A\right)\right] \\ 0 &= f\left(q^B\right) - \varepsilon_B\left[v\left(q^A\right) - v\left(q^B\right)\right] \end{aligned}$$

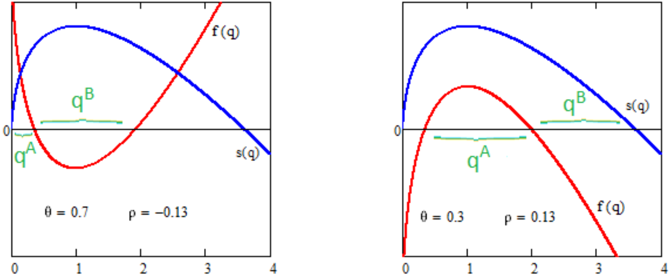
We seek sol'n $\left(q^A, q^B\right)$ for some $\left(\varepsilon_A, \varepsilon_B\right)$ with $\hat{q} > q^B > q^A > 0$.

The trick: solve for $\left(\varepsilon_A, \varepsilon_B\right)$ and check $\varepsilon_A, \varepsilon_B > 0$

$$\varepsilon_A = \frac{f\left(q^A\right)}{v\left(q^B\right) - v\left(q^A\right)} \text{ and } \varepsilon_B = \frac{-f\left(q^B\right)}{v\left(q^B\right) - v\left(q^A\right)}.$$

Clearly $\varepsilon_A, \varepsilon_B > 0$ iff $f\left(q^A\right) > 0 > f\left(q^B\right)$ iff $q^A < q^s < q^B$ around stable q^s, the low (high) one in left (right) panel.

Also notice $0 < q^A, q^B < \hat{q}$ since $s(q) = u(q) - c(q) > 0$.



$r := .001 \quad \alpha_0 := .5 \quad \alpha_1 := .5 \quad a := .5 \quad b := .01 \quad u(q) := \dfrac{(q+b)^{1-a} - b^{1-a}}{1-a} \quad c(q) := q \quad v(q) := \theta \cdot c(q) + (1-\theta) \cdot u(q)$