Monetary Economics I:

Basic Ideas and Models

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Reading:

1. Lagos et al (2017) "Liquidity: A New Monetarist Perspective," *JEL*.

2. Rocheteau and Nosal (2017) *Money, Payments, and Liquidity*, 2nd ed.

3. Wright el al (2021) "Directed Search and Competitive Search Equilibria: A Guided Tour," *JEL*.

4. Azariadis (1993) *Intertemporal Macroeconomics*.

Watching: Videos on *Markets With Frictions*.

TA will send you more information.

Grading: Homework.

Objective:

Discuss the principles and practices of recent research on money, credit, banking, asset markets and liquidity.

Since the financial crisis, economists agree liquidity is important; in this course we aim to model it rigorously.

Goals:

- Discuss a few methodological issues.
- Study simple stylized models to make conceptual points.
- Study models more suited to policy and data analysis.

Note: Some call this research New Monetarist Economics – that is not the same as Modern Monetary Theory – it's the opposite!

Principle

Kareken and Wallace (1980): Progress can be made in monetary theory and policy analysis only by modeling monetary arrangements explicitly.

This is not the belief of many macroeconomists.

In NME agents trade with each other, as *in search theory,* and not merely with budget lines, as in classical GE theory or sloppy macro.

Trading process is hindered by *frictions*, like spatial or temporal separation, limited commitment, and imperfect info.

We then ask *how* agents trade and study institutions meant to ameliorate frictions, like money, banks, reputation, collateral...

Methodology

From the purpose of studying the exchange process, short-cut models like CIA or MUF are pretty much useless.

NKE models are no better (they are actually worse).

Picking a method is critical for many policy issues:

- NKE says printing money stimulates the economy.
- NKE says gov't deficits stimulate the economy.
- NKE says low nominal interest rates are bad.
- NKE says inflation reduces unemployment.

NME theories generate can very different implications.

What's in a Name?

We agree with many (not all) traditional Monetarist tenets:

- LR is more important than SR (e.g., growth matters more than cycles);
- Phillips curves do not determine (ℓ, ω), except ...;
- central banks control π and i, but not r, except ....

We consider misguided the way Keynesians:

- handle microfoundations in general;
- ignore (frictions in) the exchange/payment process;
- fixate on sticky prices as the *key* and indeed often the *only* factor in all theory, empirical work and policy analysis.

A Brief History of Thought

Back in the 1960s, it was a healthy situation when Friedman and other Monetarists constantly challenged the Keynesian consensus.

Progress in the '70s and '80s seemed to render Keynesian macro obsolete:

- New Classical Macro
- Rational Expectations
- Real Business Cycle Theory

At some point there arose a New Keynesian consensus, more technical but otherwise much like Old Keynesian macro.

Too much consensus is unhealthy; it should be challenged; we try to communicate an alternative approach.

Generation 1: A Simple Model of Liquidity

The Environment (based on Kiyotaki-Wright 91,93)

1. Time is discrete and continues forever*.
2. A large* set of agents, say $[0,1]$, prod and cons specialized* goods that for now are indivisible* and nonstorable*.
3. Preferences: utility of cons $u > 0$, disutility cost of prod $c \in [0, u)$, discount factor $\beta = 1/(1+r)$.
4. Agents meet bilaterally* and at random* at rate α.
5. Specialization:* $prob(\text{SC}) = \sigma$ and $prob(\text{DC}) = \delta$, where SC is a single-coincidence and DC a double-coincidence meeting.

Each feature with a * merits extended discussion!

Regime 1: Barter

Let $V^A = 0$ and V^B be the value functions (life-time, expected, discounted payoffs) under autarky and barter.

As barter requires a DC meeting, the standard DP eqn is

$$V^B = \beta\left[\alpha\delta(u - c + V^B) + (1 - \alpha\delta)V^B\right]$$

where u, c and continuation value are all discounted wlog.

Simplification yields the flow DP eqn, which is nice since it actually holds in discrete or continuous time (see Tech App below):

$$rV^B = \alpha\delta\,(u - c)$$

- Barter captures some social gains from trade if $\delta > 0$, but also misses some if $\sigma > 0$.

Technical Appendix : The Flow DP Eqn, page 1 of 2

One way to derive the flow DP eqn is to go from discrete to continuous time. Let time proceed in increments $dt > 0$, so $t = 1, 1 + dt, 1 + 2dt...$ Assume meetings occur according to a Poisson process with arrival rate α. This means that $\forall t$, independent of history, the prob of 1 arrival between t and $t + dt$ is approximately αdt, written $\alpha dt + o(dt)$, where $o(dt)$ is a function such that $\lim_{dt \to 0} o(dt)/dt = 0$. Hence, multple meetings are possible, but very unlikely when dt is small. Writing the discount rate as $\beta = 1/(1 + rdt)$, we clearly have

$$V_t^B = \frac{\alpha dt \delta \left(u - c + V_{t+dt}^B\right) + (1 - \alpha dt \delta) V_{t+dt}^B + o(dt)}{1 + rdt}$$

with $o(dt)$ capturing Poisson approximation error for $dt > 0$.

Technical Appendix : The Flow DP Eqn, page 2 of 2

Multiplying by $1 + rdt$ and subtracting V_t^B from BS, we get

$$rdtV_t^B = \alpha dt\delta\left(u - c\right) + V_{t+dt}^B - V_t^B + o\left(dt\right).$$

Dividing BS by dt and taking $dt \to 0$, we get

$$rV_t^B = \alpha\delta\left(u - c\right) + \dot{V}_t^B,$$

where $\dot{V}_t^B = dV_t^B / dt$ (the time derivative). Imposing stationarity, $\dot{V}^B = 0$, and ignoring t subscripts, we get $rV^B = \alpha\delta\left(u - c\right)$. Continuous time is elegant, but we can get the same expression in discrete time: simply let $dt = 1$ and directly assume the prob of 1 meeting per period is α while the prob of more than 1 is 0, rather than having that a result from Poisson arrivals. ■

Regime 2: A Stylized Credit System

Let V^C be the value function under *perfect credit*, where agents produce whenever asked. Then

$$\begin{aligned} rV^C &= \alpha\delta(u-c) + \alpha\sigma u - \alpha\sigma c \\ &= \alpha\left(\delta+\sigma\right)\left(u-c\right) \end{aligned}$$

- Notice $\sigma > 0 \Rightarrow V^C > V^B$ so credit beats barter.
- So *if* agents can commit they would commit to perfect credit.
- That captures all gains from trade, given the search and matching frictions, parameterized by α, δ and σ.

Credit Without Commitment?

Following Kehoe-Levine, if agents can renege on promises, credit works iff it satisfies the IC condition:

$$-c + V^C \geq \mu V^D + (1 - \mu) V^C$$

- V^D = deviation payoff
- μ = prob a deviation is detected.

Note: $\mu < 1$ can be interpreted as imperfect "memory" reflecting frictions in *monitoring, communication, or record keeping*.

Note: $\mu < 1$ hinders credit which may give a role to liquid assets.

Note: In principle there are IC conditions for other trades, e.g., barter, but they won't bind.

Punishing Bad Behavior

To encourage good behavior (produce when asked) we must punish deviants by either:

1. denied them future trade, $V^D = V^A$;
2. denied them credit trade, $V^D = V^B$.

In cases 1 and 2, IC reduces to

$$r \leq \frac{\mu\alpha\sigma\,(u-c)}{c} \equiv \hat{r}_C \text{ and } r \leq \frac{\mu\alpha\sigma\,(\delta+\sigma)\,(u-c)}{c} \equiv \tilde{r}_C$$

- note $\tilde{r}_C > \hat{r}_C$ because harsher punishment is better
- but case 1 requires barter cannot be hidden
- either way credit requires (r, c) small and (α, σ, u, μ) big.

A Role for Money

If μ is low, naturally, credit is not viable, in which case let us consider *money*.

Money is a storable and transferable asset produced by society at low cost, let's say 0, but not by individuals.

As it is useful later, we endow the asset with return ρ:

- $\rho > 0 \Rightarrow$ the standard "Lucas tree" from financial economics;
- $\rho < 0 \Rightarrow$ a bad asset, e.g. one with a storage cost;
- $\rho = 0 \Rightarrow$ the theoretically pure case of fiat money.

Note: The pure fiat case may never have existed – until recently, with the advent of e-money, like bitcoin or CBDC.

Properties of Money

Traditionally, money is said to be a store of value, unit of account, and medium of exchange, but clearly its salient role is as a medium of exchange, also called a means of payment.

A good money should have (or tends to have?) properties like storability, portability, transferability, recognizability, and divisibility.

For now it has all but the last – i.e., money is indivisible and agents can only store $m \in \{0, 1\}$.

- even with this restriction we can derive important results.
- note it is an assumption on the storage technology, not on behavior; also, in some models it can be endogenized.

If M is the money supply, a fraction M of agents, called buyers, have $m = 1$, while $1 - M$, called sellers, have $m = 0$.

When is Monetary Exchange Viable?

Assuming ρ, like the payoffs from trade and continuation values, is discounted, we have

$$\begin{aligned} rV_0 &= \alpha\delta(u-c) + \alpha\sigma M\left(-c + V_1 - V_0\right) \\ rV_1 &= \alpha\delta(u-c) + \alpha\sigma(1-M)\left(u + V_0 - V_1\right) + \rho \end{aligned}$$

Assuming $|\rho|$ is not too big, the key IC condition is for sellers to produce in exchange for money, $-c + V_1 \geq V_0$, or

$$r \leq \frac{(1-M)\,\alpha\sigma\,(u-c) + \rho}{c} \equiv \hat{r}_M .$$

Consider $\rho = 0$ (fiat currency) and ask, when is money essential?

Here *essential* means *socially useful* – i.e., we can support better outcomes with money than without it.

When is Monetary Exchange Essential?

Here are versions of important results (e.g., Kocherlakota 1998):

Prop: If $\mu = 1$ fiat money cannot be essential.

Proof: It is easy to check $\hat{r}_M < \hat{r}_c$. Hence, if we can can support monetary exchange we can also support perfect credit, and the latter is better – i.e., it delivers higher payoffs. ■

- Why is credit is better than money? In a monetary regime, trade fails in single-coincidence meetings if the (potential) consumer has $m = 0$ or producer has $m = 1$.
- In some specifications (e.g., directed rather than random search), for some parameters money may do as well as credit, but it cannot do better.

Necessary Conditions for Essentiality

Prop: Assume $\mu < \mu^*$, where $\mu^* \in (0, 1)$; then if fiat money is viable it is essential.

Proof: If μ is low then credit is not viable. Money is viable if $r \leq \hat{r}_M$, independent of μ. So for some parameters credit cannot work, but money can, and while not as good as credit it beats barter. ■

- Some people interpret this to say that money is a substitute, albeit an imperfect substitute, for memory.
- These results are easy here because the environment is simple – but the ideas are robust.
- This can be a model of e-money although it abstracts from mining (for Bitcoin, and also for monies like gold), and uses $m \in \{0, 1\}$, which we relax later.

An Aside

Is Bitcoin, or related instruments, money or credit?

They require significant record keeping, like credit, but consider the following:

- With credit you consume now and work off debt later.
- With money you must work first and then consume.
- This is especially relevant with search and matching frictions, because it is uncertain when you get to spend money.

A related point is that you can run out of money

- In the model above, that happens after one purchase, whence you must produce to get more before consuming again.
- And you cannot run out of credit (in other models you might).

Summary of the Approach and Results

So far we studied the role of money in implementing good outcomes, given incentive problems, in the spirit of mechanism design, to endogenize institution of monetary exchange.

We learned several elements are key to making money essential:

1. a single-coincidence problem;
2. limited commitment;
3. imperfect information (memory, monitoring, etc.).

Some – e.g. Wallace – argue we should only analyze monetary issues and policy in environments where money is essential.

This suggests any good model must contain elements 1-2 above, but we can relax auxiliary assumptions, like indivisibility.

Equilibrium Analysis

Before relaxing assumptions, in the same environment consider equilibrium outcomes, in the spirit of noncooperative game theory.

Let τ denote a trading strategy, which here is simply

$$\tau = \textit{prob}(\text{seller will produce to get } m)$$

Taking as given others' $\bar{\tau}$, an individual chooses a best response τ, and we look for Nash equilibrium, $\tau = \bar{\tau}$.

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Note: As in the previous analysis, in principle we should ask if buyers' are willing to go from $m = 1$ to $m = 0$ to get consumption, but that is a dominant strategy if ρ is not too big.

When to Accept Money?

$$
\begin{aligned}
rV_0 &= \alpha\delta(u-c) + \alpha\sigma M \max_{\tau} \tau\,(-c+\Delta) \\
rV_1 &= \alpha\delta(u-c) + \alpha\sigma(1-M)\bar{\tau}\,(u-\Delta) + \rho
\end{aligned}
$$

where $\Delta \equiv V_1 - V_0$, and: $c < \Delta \Rightarrow \tau = 1$; $c > \Delta \Rightarrow \tau = 0$; and $c = \Delta \Rightarrow \tau = [0,1]$.

Lemma: $\exists \tau^*$such that: $\bar{\tau} > \tau^* \Rightarrow \tau = 1$; $\bar{\tau} < \tau^* \Rightarrow \tau = 0$; and $\bar{\tau} = \tau^* \Rightarrow \tau = [0,1]$.

Exercise: Find τ^* in terms of parameters, give conditions such that $0 < \tau^* < 1$, and compare them to the conditions derived above that make monetary exchange viable.

Nash Equilibrium Acceptability

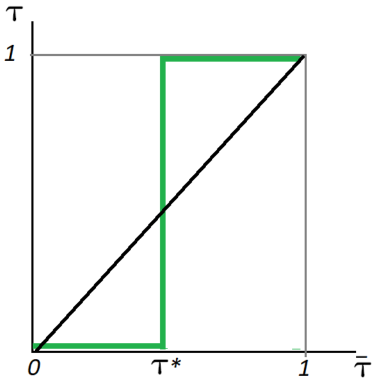

Figure: Best Response Correspondence with 3 Equilibria

Monetary Equilibrium

Prop: Consider $\rho = 0$ and $0 < \tau^* < 1$. There are three Nash equilibria – a nonmonetary equil $\tau = 0$; a pure monetary equil $\tau = 1$; and a mixed monetary equilibrium $\tau = \tau^*$.

Homework: What happens if $\tau^* > 1$ or $\tau^* < 0$? What if $\rho \neq 0$?

Remarks:

- The mixed-strategy equilibrium $\tau = \tau^*$ is not robust; it is an artifact of indivisibility.
- The two pure-strategy equilibria are more robust.
- Multiple equilibria are natural in models like this.
- Importantly, this is not just about fiat currency; it applies to any asset that facilitates trade.

Big Ideas

- Existence: There are equil where an asset is valued – people work to get it – even if $\rho = 0$, contrary to standard finance.
- Robustness: Such equilibria survive even if $\rho < 0$, at least as long as $|\rho|$ is not too big.
- Tenuousness: If $\rho \leq 0$, including the case of fiat money, there exists an equil where the asset is not valued.
- Volatility: Going beyond stationary outcomes, there are also equil where the asset value varies over time.
- Efficiency: When there are multiple equilibria they can be ranked in terms of welfare.

More Big Ideas

Assets can be valued even if $\rho \leq 0$ because they convey liquidity.

- You work to get m because you believe others will work to get m from you, just like in the real word.
- The self-referential aspect of such beliefs explains why models of liquidity tend to have multiple equil.

Liquidity $\Rightarrow$ asset can be valued above its fundamental ρ/r.

- This is obviously true for $\rho \leq 0$, but also true for $\rho > 0$.
- So we can have asset bubbles, and they are good for welfare.
- It is of course bad if a bubble bursts – e.g., τ goes from 1 to 0 – but nothing says it must.

Still More Big Ideas

Contrary to CIA and NKE models, here money is not a *problem*, its a *solution*:

- It ameliorates problems related to spatial/temporal separation, limited commitment and incomplete information.

But it is an imperfect solution:

- Credit would be better, if it were viable;
- Monetary equil can be fragile, tenuous or volatile.

There are policy implications – e.g., related to ρ – but discussion is postponed until we relax some assumptions.

Also, above we considered money or credit, but it is interesting to have models with both.

Application: Inside vs Ouside Money (Cavalcanti and Wallace)

- Fraction n of agents are type B (bankers) while the rest are type A (anonymous):
  - type A are never monitored;
  - type B are monitored in all meetings.
- Agents can issue *notes*, pieces of paper with their names.
- Notes issued by A are never accepted – why produce to get a note when you can print your own?
- Notes issued by B might be – thus they resemble banks.
- For illustration, suppose B never hold outside money (not crucial).

Inside vs Ouside Money (Cavalcanti and Wallace)

- We can support outcome where B produces whenever asked, with autarky punishment, if

$$r \leq \alpha(1-n)(1-M)\left(u-c\right)/c.$$

- With no inside money B only consumes when meeting another B, so $rV^B = \alpha\left(nu-c\right)$, while for A

$$\begin{aligned} rV_0^A &= \alpha nu + \alpha\left(1-n\right)\left(1-M\right)\left(u+V_1-V_0\right) \\ rV_1^A &= \alpha nu + \alpha\left(1-n\right)M\left(-c+V_0-V_1\right). \end{aligned}$$

- With inside money, B pay A by issuing notes so W is higher.
- Not surprising – but at least we can discuss relative merits of different arrangements.

Extension: Victor Li (IER,JME)

Add search intensity, gov't taxes and transfers:

- at rate γ take away m; at rate $\gamma M / (1-M)$ give it back
- buyers choose $\alpha_1 = \alpha$, so sellers get $\alpha_0 = \alpha M/(1-M)$

$$\begin{aligned} rV_1 &= \gamma (V_0 - V_1) + \alpha (u + V_0 - V_1) - k(\alpha). \\ rV_0 &= \frac{\gamma M}{1-M} (V_1 - V_0) + \frac{\alpha M}{1-M}(-c + V_1 - V_0) \end{aligned}$$

- Combine FOC $k'(\alpha) = u + V_0 - V_1$ with (V_0, V_1) to get

$$\begin{aligned} T(\alpha) = [r(1-M) + \gamma + M\alpha]\, u - M\alpha c + (1-M)k(\alpha) \\ - [r(1-M) + \gamma + \alpha]\, k'(\alpha) = 0 \end{aligned}$$

where reasonable conditions imply T has nice properties.

Policy Implications of Li?

- Under these conditions $\exists!$ equil $\forall \gamma \leq \bar{\gamma}$ where $\bar{\gamma}$ is the tax that makes $c + V_1 - V_0 = 0$.
- In equil $\partial \alpha / \partial \gamma > 0$, so inflation-like taxes raise velocity.
  - hot potato effect: spending money faster to avoid inflation
- Optimal α is $k'(\alpha^*) = u - c$ and equil is efficient iff $\gamma = \bar{\gamma}$
  - Hosios condition: buyers equate MC of search to their MB, which is below social MB unless they get the whole suplus.
- Key result is inflation-like taxes raise search effort and welfare.
- It is tricky to generalize to divisible goods/money, but still it's a nice example of a classic issue in monetary econ *requiring* search theory – how else do we model *spending money faster*?

Extension: Equilibria for Any ρ

Above we assumed $|\rho|$ is not too big; here is the equil set for the general case where we endogenize:

$\tau_0 = pr$(seller trades good for m) and $\tau_1 = pr$(buyer trades m for good)

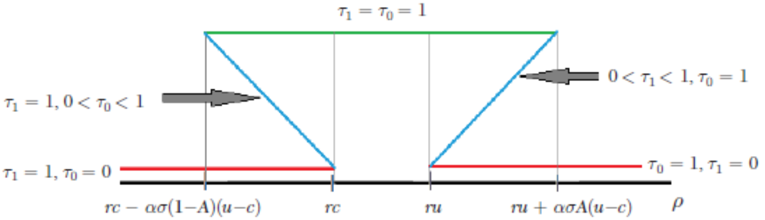

Figure 2. Equilibria with Assets as Money

Extension: Partial Acceptability

- Above $\tau^* \in (0,1)$ has partial acceptability but it's not robust
  - not an equil if goods or assets divisible or we allow lotteries
  - unstable wrt trembles, evolution, etc.
- Shevchenko and Wright (ET) assume individuals are heterog wrt (r_i, u_i, c_i, ρ_i), $i \in \mathbb{I}$.
- Let $\tau_i = pr\,(i \text{ accepts } M)$, $m_i = pr(i \text{ has } M)$, $\tilde{n} = \int_I \tau_i$.

$$
\begin{aligned}
r_i V_{i0} &= B + \alpha \int_{\mathbb{I}} m_j \tau_i \left(\Delta_i - c_i\right) dj \\
r_i V_{i1} &= B + \alpha \int_{\mathbb{I}} \left(1 - m_j\right) \tau_j \left(u_i - \Delta_i\right) dj + \rho_i .
\end{aligned}
$$

- In ss $m_i = M/\tilde{n}\,\forall i$, assuming $M < \tilde{n}$ (this maybe can endogenized by allowing agents to throw away money).

Equilibrium Acceptability

$$r_i \Delta_i = \rho_i + \alpha \left(1 - M/\tilde{n}\right)\left(u_i - \Delta_i\right)\tilde{n} - \alpha \left(M/\tilde{n}\right)\tau_i \left(\Delta_i - c_i\right)\tilde{n}$$

- It is a BR for i to set $\tau_i = 1$ iff

$$\Delta_i - c_i = \frac{\rho_i + \alpha \left(\tilde{n} - M\right) u_i - \left(r_i + \alpha \tilde{n} - \alpha M\right) c_i}{r_i + \alpha \tilde{n}} \geq 0$$

- Rearranging, $\tau_i = 1$ is a BR for i iff $\tilde{n} \geq \xi_i$ where

$$\xi_i \equiv \frac{\alpha M \left(u_i - c_i\right) + r_i c_i - \rho_i}{\alpha \left(u_i - c_i\right)}.$$

- If CDF of ξ_i is $F\left(\xi\right) = prob\left(\xi_i \leq \xi\right)$, then the measure who accept M is $\tilde{n} = F\left(\tilde{n}\right)$.

Endogenous Acceptability

- Equil is a fixed point of $\tilde{n} = F(\tilde{n})$.
- Existence follows from Tarski's Thm.
  - might not need such a powerful tool here;
  - but we may as well use it once we've done the work to describe equil in terms of a threshold $\tilde{n} \geq \xi_i$.
- Easy to get various types of equil and multiplicity.
  - the fraction of agents who accept M depends on the fraction who accept M!
- And now we can experiment with parameter changes:
  - that is not very interesting in baseline model since the robust equil involve conrner solutions, $\tau = 0$ or $\tau = 1$.

Endogenous Acceptability

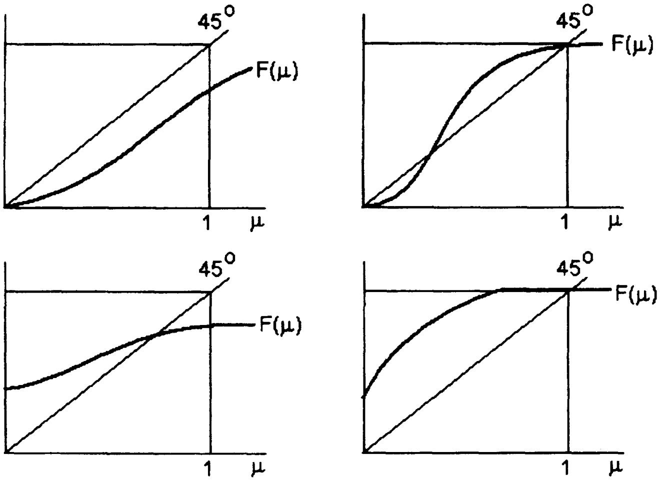

Figure 1. Some possible outcomes

Kiyotaki-Wright (JET): Barter

- Goods come in varieties, agents have favorites, and utility of a good is $u(z)$ where z is distance from favorite, $u' < 0$.
  - before trading agent produces a good he doesn't like.
  - prod opportunities arrive at rate α_0 and random cost c.
- In meetings the z's are independent and $U[0,1]$.
- Good is only storable by its producer (not critical).
- Given $X = prob$(random agent wants to trade), choose res. cost k and res. trade x, $k = u(x) = V_g - V_0$:

$$
\begin{aligned}
rV_0 &= \alpha_0 \int_0^k (k - c)\, dF(c) \\
rVg &= \alpha_1 X \int_0^x [u(z) - u(x)]\, dz
\end{aligned}
$$

Kiyotaki-Wright (JET): Money

Prop: $\exists!$ barter equil where $X = x$, making $\delta = x^2$ *endogenous*.
Small x implies very small x^2, motivating a role for money:

$$
\begin{aligned}
rV_0 &= \alpha_0 \int_0^k (k - c)\, dF(c) \\
rVg &= \alpha_1 (1 - \hat{n}) X \int_0^x [u(z) - u(x)]\, dz + \alpha_1 \hat{n} Y \max_{\tau} \tau (V_m - V_g) \\
rV_m &= \alpha_1 (1 - \hat{n}) \bar{\tau} \int_0^y [u(z) - u(y)]\, dz
\end{aligned}
$$

where in equil $\hat{n} = n_m / (n_m + n_g)$, $x = X$, $y = Y$ and $\tau = \bar{\tau}$.

Prop: Given any M there $\exists$ $\hat{n}$ consistent with ss, and given $\hat{n}$, $\exists!$ equil with $\tau = 1$. In equil $x^2 < y < x$.

Simplified Kiyotaki-Wright (AER)

Following Lucas' suggestion, consider $u(z) = U$ if $z \leq x$ and 0 otherwise, where U and x are parameters.

Then res. trade x is fixed, instead of sol'n to $u(x) = V_g - V_0$.

If we also make c nonrandom, and small, then

$$\begin{aligned} rV_0 &= \alpha_0 (V_g - V_0 - c)\, dF(c) \\ rVg &= \alpha_1 x^2 (1 - \hat{n})(U + V_0 - V_g) + \alpha_1 \hat{n} x \tau (V_m - V_g) \\ rV_m &= \alpha_1 (1 - \hat{n}) x \bar{\tau} (U + V_0 - V_m) \end{aligned}$$

Prop: Given M there are exactly three equil: $\tau = 1$; $\tau = 0$; and $\tau = x$. Under reasonable conditions, equil can be ranked in terms of welfare.

Specialization and Efficiency: Kiyotaki-Wright (AER)

Adam Smith's connection between specialization and efficiency: you choose output per unit time α_0, but high α_0 means low prob x that your produce is desired by others.

$$\begin{aligned} rV_0 &= \max_x \alpha_0(x)\left[V_g(x) - V_0 - c\right] dF(c) \\ rVg(x) &= \alpha_1 x X (1-\hat{n})(U + V_0 - V_g) + \alpha_1 \hat{n} x \tau \left[V_m(x) - V_g\right] \\ rV_m &= \alpha_1 (1-\hat{n}) X \bar{\tau} (U + V_0 - V_m) \end{aligned}$$

Adam Smith's idea that specialization is limited by the extent of the market, and that's where money really helps:

Prop: There are three equil, $\tau = 1$, $\tau = 0$ and $\tau = x$, with high, low and medium α_0.

Prop: When $\tau = 1$ and $\alpha_1 \to \infty$, we get complete specialization $x = 0$, and all trade is monetary, $\alpha_1 x > 0$ while $\alpha_1 x^2 = 0$.