Payments on Digital Platforms: Resiliency, Interoperability and Welfare *

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Abstract

Digital platforms, such as Amazon and Alibaba, operate an online marketplace to facilitate transactions. This paper studies a platform's business model choice between accepting cash and issuing its own tokens, as well as the implications for welfare, resiliency and interoperability. By issuing tokens, the platform insulates transaction activities from the liquidity costs of using cash and maximizes consumer surplus extraction without levying transaction fees. We characterize the equilibrium business model, which is not necessarily socially optimal because the platform ignores the full impacts on on-platform and off-platform activities. When cyber security is exogenous, it is optimal for a token platform to minimize transactions fees and reserves, and reserve requirement is welfare reducing. When cyber security is endogenous and subject to moral hazard, however, keeping reserves allows the platform to relax its incentive constraint. Imposing a minimum reserve requirement can sometimes improve welfare.

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1 Introduction

Recent years have witnessed the emergence of private digital tokens issued by social and economic platforms to facilitate transactions among users in a variety of contexts. Early digital tokens were issued by social media networks to allow platform users to conduct peer-to-peer transfers.[1] In the past few years, many blockchain-based platforms have launched utility coins circulating in their distributed networks as the only acceptable means of payments.[2] In China, WeChat's and Alipay's digital wallets, which allow holders to accumulate redeemable, interest-bearing balances for online and point-of-sale transactions, have dominated the retail payments system. In the United States, retail giants Walmart and Amazon have also recently applied for patents related to cryptocurrencies.

Digital platforms, such as Amazon and Alibaba, operate an online network to facilitate transactions between consumers and merchants. These platforms can often control the payment options available to their users and sometimes issue their own digital tokens for transactions. Private digital currencies, especially those issued by major platforms with a large user base, could reshape the retail payment landscape and influence the functioning of the monetary system. Against this context, our paper studies the business model choice between running a *cash platform* and a *token platform*, as well as the welfare and policy implications of this choice. Specifically, the paper investigates the following questions: What trade-offs does a platform face when choosing between accepting cash and issuing its own tokens? Is the choice of business model socially optimal? What are the implications for payment resiliency and interoperability? What is the scope for regulation?

Let us briefly describe the model and give the basic intuition behind our findings. In our model, there is a monopolist running an online platform for goods trading. The resilience of the platform is captured by the probability that it collapses after a cyber attack (which is exogenous for now). Information frictions give rise to a cash-in-advance (CIA) constraint, which implies that platform

[1] Early examples are Tencent QQ, Second Life Linden and Facebook Credits. In June 2019, Facebook announced its intention to introduce Libra, a blockchain-based stable coin.

[2] Examples are Ethereum tokens, used in the Ethereum system, as well as Filecoin and Sia, used in cloud storage networks.

users need to pre-acquire a means of payment in order to make a purchase. The monopolist can choose to set up a cash platform that allows consumers to pay merchants on the platform by transferring traditional payment balances ("cash"). An alternative business model is a token platform, which issues its own redeemable balances ("tokens") for consumers to make payments to merchants.

The following is the main trade-off between the two business models. A cash platform makes profits by charging fees on platform transactions. Due to the payment frictions, the platform's ability to extract surplus can be constrained by consumers' cash holdings. In particular, the platform cannot charge the profit-maximizing fee rate when consumers economize on their cash holdings due to high inflation (or any other costs associated with using cash). By issuing tokens, a platform can insulate platform activities from the costs associated with cash.[3] Issuing tokens also provides the platform with an option to use token sales as an alternative source of revenue. However, tokens can lose transaction value when the platform ceases to operate (e.g., due to cyber attacks or under-investment by the platform), and hence tokens, unlike cash, are subject to default risk. To support the transaction value of tokens, the platform may need to hold costly cash reserves for consumers' redemption.

When the resilience of the platform is exogenous, we find that the optimal platform design features zero reserves and zero transaction fees. It is optimal to keep zero reserves because a token platform can support the transaction value of tokens by simply engineering a sufficiently fast appreciation rate of tokens. It is optimal to collect a zero transaction fee because fee payments tighten the "tokens-in-advance" constraint of consumers, leading to consumption distortion on the platform.

Given the above tradeoff, when would a platform choose to issue tokens instead of accepting cash? When inflation is not too high, using cash is optimal, especially when setting up an independent token system involves an additional cost. A platform chooses to issue tokens when inflation

[3]For example. Alibaba launched its own payments product, Alipay, in 2004 partly due to the inefficiency of the "cash-on-delivery" payment method on its Taobao and Tmall platforms.

is sufficiently high (so that the CIA constraint is tight on the platform), when the platform is sufficiently resilient to cyber attacks (so that tokens are expected to circulate longer) and when its market share is sufficiently large (so that transaction fee is not a good way to collect revenue). Interestingly, the platform's choice of business model is not necessarily socially optimal because it ignores the impact on consumers' welfare. Depending on parameter values, both cash and token platforms can be suboptimally adopted. For example, over-adoption of token platforms happens when the platform does not internalize the negative impacts of rejecting cash on non-platform consumption.

Could policy intervention improve social welfare? When the platform's business model choice is suboptimal, a simple tax/subsidy scheme, if feasible, can induce the platform to make the optimal decision. However, conditional on the adoption of a token system, its design can still be suboptimal relative to the first-best allocation. Could imposing reserve requirements be welfare improving?[4] Our model suggests that the answer depends on the nature of the risk associated with the platform. When cyber security is exogenous, it is optimal for a token platform to minimize its reserve holdings, and hence a reserve regulation can be welfare reducing. When cyber security is endogenous and subject to moral hazard, however, keeping reserves allows the platform to relax its incentive constraint, and imposing a minimum reserve requirement can sometimes improve welfare.

Our paper contributes to the young but fast-growing literature exploring the economics of digital currencies. A large amount of research focuses on understanding the design and functioning of cryptocurrencies such as Bitcoin. This literature can be broadly divided into three lines of research. The first line aims at understanding the pricing of cryptocurrencies (Biais et al. (2018), Choi and Rocheteau (2019), Schilling and Uhlig (2018)). The second line examines the reliability and robustness of the blockchain (e.g., Biais et al. (2017), Eyal and Sirer (2013), Saleh (2019), Chiu and Koeppl (2019), Pagnotta (2018)). The third line focuses on mining activities and the

[4]For example, in China, the PBoC recently imposed 100 percent reserve requirements on third-party payment groups. According to a report by FT.com, Alipay and Tencent were poised to lose around $1 billion in combined annual revenue.

determination of fees (e.g., Huberman et al. (2017) and Easley et al. (2019)). Unlike general-purpose cryptocurrencies, platform-based tokens often entitle bearers to redeem the balances for national currency or platform goods. Therefore, the value of platform-based tokens depends on the functioning of the issuing platform as well as the issuer's commitment to future actions (e.g., redemption, investment, reserve holdings).[5]

A more closely related research area focuses on the design of utility tokens issued by platforms and the value of initial coin offerings as a mode of financing (e.g., Catalini and Gans (2018)), Cong, Li, and Wang (2018), Garratt and van Oordt (2019), You and Rogoff (2019)). Gans and Halaburda (2015) study the design of private digital currencies from a platform-management perspective and argue why a profit-maximizing platform may choose to limit the functionality of digital currencies. This stream of research typically focuses on partial-equilibrium analysis and thus does not study macro implications of platform-based tokens. Our paper complements by highlighting the interaction between on-platform and off-platform activities in a general equilibrium setting.

Some recent studies develop general equilibrium models to investigate the macro implications of issuing central bank digital currencies (e.g., Andolfatto (2018), Barrdear and Kumhof (2016), Chiu, et. al. (2020), Davoodalhosseini (2018), Fernández-Villaverde et al. (2020), Keister and Sanches (2018) and Williamson (2019)) Zhu and Hendry (2019) study the competition between a national currency and a general-purpose electronic money. To the best of our knowledge, this paper is the first one that studies the welfare and policy implications of issuing platform-based digital tokens in a general equilibrium setting.

Security issues have been investigated in the payment literature, for example, Kahn and Roberds (2008) examine identity theft, and Kahn et al (2020) study the trade-off between security and convenience. We explore how platform security influences payment design in a general equilibrium environment.

Our model is based on Lagos and Wright (2005), which provides a tractable framework for

[5] In other words, government-issued fiat currencies and cryptocurrencies, like bitcoin, are outside money, while redeemable platform-based digital tokens are inside money.

us to study the joint determination of the transaction values of cash and tokens and the general equilibrium effects of a platform's payment choice. Recent models of payment systems building on the Lagos-Wright framework include Li (2011) and Monnet and Roberds (2008). See also the survey in Kahn and Roberds (2009).

The rest of the paper is organized as follows. Section 2 describes the basic environment. Sections 3 and 4 examine, respectively, a cash platform and a token platform. Section 5 studies the equilibrium business model and its welfare implications. Section 6 discusses the effects of token platform regulation. Sections 7 investigate the case with endogenous resiliency. Section 8 considers an extension with endogenous interoperability and provides examples calibrated to Amazon and Alibaba. Section 9 concludes. All proofs are provided in the Appendix.

2 Environment

Our model is based on the alternating market formulation of Lagos and Wright (2005). Time is indexed by $t \in \mathbb{N}_0$. Each period is divided into two stages. In stage 1, competitive markets for retail goods open and consumption opportunities arise. In stage 2, agents trade assets and numeraires. The economy starts in stage 2 of period 0. There are three types of agents: *merchants* who can produce retail goods in stage 1 and consume numeraires in stage 2, *consumers* who consume and produce numeraires in stage 2 and consume retail goods if opportunities arise in stage 1, and a *platform* that produces nothing but facilitates meetings between consumers and merchants in stage 1 as explained below. The population of consumers is normalized to one. When the consumption opportunities arise, there is a chance that the consumer can find a suitable merchant on or off the platform. Consumers and the platform discount across periods with a factor $\beta = 1/(1+r)$, where $r > 0$. Merchants live for a period with new merchants entering at the beginning of stage 1. A cyber shock hits the platform with probability γ in stage 2, after which the platform will cease to exist in the next period.

The preferences of merchants in period t are given by

$$-q_t + c_t,$$

where q_t is the retail goods produced in stage 1, and c_t is the numeraire consumed in stage 2. The preferences of consumers are given by

$$\mathbb{E}\sum_{t=0}^{\infty}\beta^{-t}\left\{I_t U\left(q_t\right) - h_t + c_t\right\},$$

where $U(q)$ is a CRRA utility function with the relative risk-aversion parameter $\eta \in (0,1)$; h_t is the consumer's production of numeraires in stage 2; and I_t is an indicator variable capturing the iid consumption opportunity such that with probability $1-\sigma$, it does not occur; with probability $\sigma\alpha$, it arises on the platform; with probability $\sigma(1-\alpha)$, it arises off the platform. Thus, a higher α means the consumption opportunities are more likely to be met via the platform. The preferences of the platform are given by

$$\mathbb{E}\sum_{t=0}^{\infty}\left[(1-\gamma)\beta\right]^{-t} c_t.$$

Agents can only observe the actions and outcomes of their trades and are anonymous. Consumers cannot commit, and there is no enforcement or record-keeping technology. As a result, consumers cannot borrow to finance their consumption because they would renege on their promise to repay their debt. This generates the need for a medium of exchange. In this economy, there are perfectly divisible storable objects called cash that can be used as a medium of exchange. The supply of cash is maintained exogenously by a central bank's lump-sum transfers to consumers in stage 2, such that the real price of cash in period t is ϕ_t. We assume an exogenous monetary policy that fixes the inflation rate at $\bar{\mu} = \phi_t/\phi_{t+1}$.

We interpret the platform as an online marketplace, such as Amazon and Alibaba, where the platform does not act as a buyer/seller but intermediates goods and payments between consumers and merchants who do not have direct contacts among themselves. This role allows the platform

to dictate the acceptable means of payments on the platform. Also, note that the amount paid by the consumer is not necessarily equal to the amount received by the merchant. The difference is the transaction fee charged by the platform. In the next section, we will focus on a *cash platform* which allows its users to use cash to transact on the platform. To trade on the cash platform, in stage 1, consumers need to acquire cash balances in the previous stage 2 by selling their production of numeraires. On the platform, consumers are subject to a CIA constraint. In addition, the platform can charge a proportional transaction fee that is proportional to the transaction value on the platform and is paid in cash.[6] Later we will extend the model to study a *token platform*, which issues an additional medium of exchange called tokens. We will then study the equilibrium business model of the platform and its welfare implications.

3 Cash platform

A cash platform allows consumers to use cash as the medium of exchange and makes profit solely from charging the transaction fee τ_t on the (post-fee) transaction value on the platform. We solve the merchant's and consumer's problems backward by first considering stage 1 of period t and then stage 2 of period $t-1$.

3.1 Merchant's problem

In the competitive market of stage 1, merchants choose q_t units of retail goods to be sold on the platform and y_t units of retail goods to be sold off the platform, taking as given the nominal price P_t of retail goods, which is the same on or off the platform. The zero-profit condition implies $-q_t + \phi_t P_t q_t = 0 = -y_t + \phi_t P_t y_t$, where $-q_t$ is the cost of producing retail goods, and $\phi_t P_t q_t$ is the real value of cash receipt. Hence, the equilibrium price of retail goods is $P_t = 1/\phi_t$.

[6] Unlike lump-sum fees, proportional fees are not subject to side-trading; see Chiu and Wong (2015) for details. It is now well-known in the IO literature that proportional fees are more profitable to lump-sum fees in general environments with heterogeneity, see Shy and Wang (2011). Thus, we do not pursue the analysis of lump-sum fee in this paper.

3.2 Consumer's problem

At the beginning of stage 1 in period t, the value function of a consumer holding M units of cash is

$$V_t(M) = \max_{q_t, y_t} \left\{ \begin{array}{l} \sigma(1-\alpha)\left[U(q_t) + W_t(M - P_t q_t)\right] \\ +\sigma\alpha\left\{U(y_t) + W_t\left(M - \frac{P_t y_t}{1-\tau_t}\right)\right\} \\ +(1-\sigma) W_t(M) \end{array} \right\} \text{ s.t.} \tag{1}$$

$$M \geq P_t q_t, \tag{2}$$

$$M \geq \frac{P_t y_t}{1-\tau_t}, \tag{3}$$

where q_t is the quantity bought off the platform, y_t is the quantity of retail goods bought on the cash platform, W_t is the continuation value function in stage 2, and (2) and (3) are the associated CIA constraints for non-platform and platform transactions respectively. With probability $1-\sigma$, the consumer does not need anything in stage 1, and he brings M units of cash to stage 2. With probability $\sigma(1-\alpha)$, the consumer pays some merchants off the platform $P_t q_t$ units of cash for q_t units of retail goods, bringing the remaining $M - P_t q_t$ units of cash to stage 2. With probability $\sigma\alpha$, the consumer pays on the cash platforms $P_t y_t / (1-\tau_t)$ units of cash for y_t units of retail goods, of which $P_t y_t$ is paid to the merchants and $\tau_t P_t y_t / (1-\tau_t)$ to the platform as the transaction fees.[7]

At the beginning of stage 2 of period $t-1$, the value function of a consumer holding M units of cash is

$$W_{t-1}(M) = \max_{\{c_t, h_t, m_t\} \in \mathbb{R}^3_+} \{c_t - h_t + \beta V_t(M_t)\}, \text{ s.t.} \tag{4}$$

$$c_t + \phi_{t-1} M_t = h_t + \phi_{t-1} M + \Upsilon_t,$$

where Υ_t is the real lump-sum transfer from the central bank needed to implement the price path

[7] Consistent with off-the-platform trades, we use $P_t y_t$ to denote the amount that on-platform merchants receive. Denote D_t as the total payment the consumer pays, then the on-platform merchant receives $(1-\tau_t) D_t = P_t y_t$. Thus, the transaction fee the platform receives is $\tau_t D_t = \tau_t P_t y_t / (1-\tau_t)$.

$\phi_{t-1} = \bar{\mu}\phi_t$. Applying the envelope theorem to (4) and (1), we obtain

$$\frac{dW_t(M)}{dM} = \phi_t, \ \frac{dV_t(M)}{dM} = \phi_t \left[\begin{array}{c} 1 + \sigma(1-\alpha)\{U'(q_t) - 1\}_+ \\ +\sigma\alpha\{(1-\tau_t)U'(y_t) - 1\}_+ \end{array} \right], \tag{5}$$

where $\{X\}_+ \equiv \max\{X, 0\}$.

The first-order condition of (4) with respect to M_t implies that

$$i = \sigma(1-\alpha)\left\{m_t^{-\eta} - 1\right\}_+ + \sigma\alpha\left\{(1-\tau_t)^{1-\eta} m_t^{-\eta} - 1\right\}_+, \tag{6}$$

where $i \equiv (1+r)\phi_{t-1}/\phi_t - 1 = (1+r)\bar{\mu} - 1 > 0$ is the nominal interest rate and $m_t = \phi_t M_t$ is the real balances. The rate i on the LHS of (6) captures the marginal cost of holding cash. The first and second terms on the RHS of (6) capture the marginal returns of spending cash on off-platform and on-platform transactions, respectively. These terms are positive whenever holding cash helps relax the binding CIA constraints, (2) and (3). We first derive the consumer's choices (q_t, y_t, m_t) as a function of (τ_t, i), which is summarized by Lemma 1.

Lemma 1 *Define*

$$\begin{aligned} \tau_C(i) &\equiv 1 - \left[1 + \frac{i}{\sigma(1-\alpha)}\right]^{-1/(1-\eta)}, \\ m(\tau_t) &\equiv \left(1 + \frac{i}{\sigma}\right)^{-1/\eta} \left[1 - \alpha + \alpha(1-\tau_t)^{1-\eta}\right]^{1/\eta}. \end{aligned}$$

Under a cash platform, the consumer's solution has two regimes, depending on i and τ_t:

(i) If $\tau_t \geq \tau_C(i)$, then $q_t = m_t = [1 - \tau_C(i)]^{1/\eta - 1}$, and $y_t = (1-\tau_t)^{1/\eta} \leq (1-\tau_t)m_t$, where equality holds when $\tau_t = \tau_C(i)$;

(ii) if $\tau_t \leq \tau_C(i)$, then $q_t = m_t = m(\tau_t)$, and $y_t = (1-\tau_t)m_t$.

The proof is provided in the Appendix. When τ is sufficiently high, the consumer's on-platform CIA constraint is not binding (case i). Inflation reduces off-platform transactions but has no effects

on on-platform transactions. Transaction fees lower on-platform transactions but have no effects on off-platform transactions. When τ is sufficiently low, both CIA constraints are binding (case ii). Inflation and transaction fees reduce consumption on both platforms. Note that case (ii) is more likely when i is high because $\tau_C(i)$ is an increasing function. The idea is that the consumer economizes on cash holdings when inflation is high, implying that the CIA constraint is more likely to be binding on the platform.

3.3 Platform's problem

The previous results suggest that, in setting the transaction fee, the platform has to take into account not only the negative impacts on platform consumption y_t given the money holding m_t, but also the potential effects on the consumer's choice of m_t, which happens only in case (ii). The platform maximizes the discounted sum of profit flow given the consumer's solution:

$$\Pi^C \equiv \max_{\tau_t} \sum_{t=1}^{\infty} (1-\gamma)^{t-1} \beta^t \alpha\sigma \frac{\tau_t \phi_t P_t y_t}{1-\tau_t}. \tag{7}$$

There is a measure $\alpha\sigma$ of consumers trading on the platform. Each consumer pays $\phi_t P_t y_t / (1-\tau_t)$ units of real balances of which the platform collects a fraction of τ_t. Notice that (7) is a repeated static problem so its solution is time invariant. Using the two regimes of the consumer solution characterized by Lemma 1, we can rewrite the repeated static problem as

$$\begin{aligned}
\Pi^C &= \frac{\alpha\sigma}{r+\gamma} \max\left\{\Pi_1^C, \Pi_2^C\right\}, \text{ where} \\
\Pi_1^C &\equiv \max_{\tau_t} G_1(\tau_t) \text{ s.t. } \tau_t \geq \tau_C(i); \\
\Pi_2^C &\equiv \max_{\tau_t} G_2(\tau_t) \text{ s.t. } \tau_t \leq \tau_C(i);
\end{aligned}$$

where

$$\begin{aligned} G_1(\tau) &\equiv \tau(1-\tau)^{1/\eta-1}, \\ G_2(\tau) &\equiv \tau m(\tau). \end{aligned}$$

Here, the Π_1^C function corresponds to case (i) in Lemma 1, where τ is sufficiently high to make the CIA constraint non-binding on the platform. Similarly, the Π_2^C function corresponds to case (ii), where τ is sufficiently low to make the CIA constraint binding. To proceed, we first define a level of transaction fee

$$\bar{\tau} \equiv \arg\max \left\{ \tau_t \left[\alpha (1-\tau_t)^{1-\eta} + (1-\alpha) \right]^{\frac{1}{\eta}} \right\},$$

which uniquely solves the first-order condition

$$(1-\bar{\tau})^{\eta} = \frac{\alpha(\bar{\tau}-\eta)}{\eta(1-\alpha)}.$$

In setting the optimal fee, the platform compares the values in the unconstrained and the constrained regions, Π_1^C and Π_2^C, respectively.

Given these results, we can now derive the optimal fee τ chosen by the cash platform and the equilibrium quantities (q, y) and platform profits Π^C, summarized in Proposition 1.

Proposition 1 *Define i^* and $\hat{\imath}$ as*

$$\begin{aligned} i^* &\equiv \sigma(1-\alpha)\left[(1-\eta)^{-(1-\eta)} - 1\right], \\ \hat{\imath} &\equiv \sigma(1-\alpha)\left[(1-\bar{\tau})^{-(1-\eta)} - 1\right] > i^*. \end{aligned}$$

Under a cash platform, the platform's solution has three regimes, depending on i:

	τ	q_t	y_t	Π^C
(a). $i < i^*$	η	$[1-\tau_C(i)]^{1/\eta-1}$	$(1-\eta)^{1/\eta}$	$\frac{\alpha\sigma}{r+\gamma}G_1(\eta)$
(b). $i \in [i^*, \hat{\imath}]$	$\tau_C(i)$	$[1-\tau_C(i)]^{1/\eta-1}$	$[1-\tau_C(i)]^{1/\eta}$	$\frac{\alpha\sigma}{r+\gamma}G_1[\tau_C(i)]$
(c). $i > \hat{\imath}$	$\bar{\tau}$	$m(\bar{\tau})$	$(1-\bar{\tau})q_t$	$\frac{\alpha\sigma}{r+\gamma}G_2(\bar{\tau}; i)$

The optimal fee is $\tau = \eta$ when the constraint (3) is non-binding, while $\bar{\tau} > \eta$ is the optimal fee when the constraint is binding. When the CIA constraint is non-binding on the platform, it is optimal to set a lower fee to maximize fee revenue. Setting $\tau = \eta$ is not feasible when the CIA constraint is binding, and the platform needs to raise τ to $\bar{\tau}$.

There are three cases. In case (a), as $i < i^*$, consumers hold sufficiently high cash balances so that the platform can set the fee at the unconstrained level $\tau = \eta$. The on-platform consumption y is not affected by inflation, while the off-platform consumption q and money holding m are not affected by the fee rate.

As i rises and enters the range $[i^*, \hat{\imath}]$ in case (b), the CIA starts to bind on the platform. The platform optimally raises the fee to $\tau = \tau_C(i) \in [\eta, \bar{\tau}]$, which induces a level of platform spending such that the CIA constraint is just binding. In this case, the marginal valuation of cash is not determined by platform activities. The on-platform consumption y decreases with inflation, while the off-platform consumption q and money holding m are still independent of the fee rate.

When $i > \hat{\imath}$ in case (c), the consumers hold so little cash that the platform chooses to set a high $\tau = \bar{\tau}$, which implies the CIA is binding on the platform. In this case, the marginal valuation of cash is determined by platform activities. The fee rate and the inflation rate affect the money balances, the on-platform and the off-platform consumption.

To finish the study of the cash platform, we report some comparative statics in the following proposition.

Proposition 2 *Under a cash platform, the comparative statics are given by*

	$\frac{\partial \tau_t}{\partial i}$	$\frac{\partial q_t}{\partial i}$	$\frac{\partial y_t}{\partial i}$	$\frac{\partial \Pi^C}{\partial i}$	$\frac{\partial \tau_t}{\partial \alpha}$	$\frac{\partial q_t}{\partial \alpha}$	$\frac{\partial y_t}{\partial \alpha}$	$\frac{\partial \Pi^C}{\partial \alpha}$	$\frac{\partial \tau_t}{\partial \gamma}, \frac{\partial q_t}{\partial \gamma}, \frac{\partial y_t}{\partial \gamma}$	$\frac{\partial \Pi^C}{\partial \gamma}$
(a). $i < i^*$	0	$-$	0	0	0	$-$	0	$+$	0	$-$
(b). $i \in [i^*, \hat{\imath}]$	$+$	$-$	$-$	$-$	$+$	$-$	$-$	$\pm$	0	$-$
(c). $i > \hat{\imath}$	0	$-$	$-$	$-$	$-$	$\pm$	$\pm$	$\pm$	0	$-$

Inflation generally has a negative effect on consumption and platform profits, with the exception of case (a), where on-platform activities are unconstrained by cash holdings. The effects of market share α is more complicated. In case (a), α reduces the off-platform marginal value of cash, while the on-platform marginal value of cash is not affected. This implies a lower q but a fixed y. In case (b), the platform needs to raise τ to τ_C, which increases with α. Hence, the on-platform consumption y declines with α, too. In case (c), the platform raises the fee to $\bar{\tau}$. There are two opposite forces with respect to α. First, the negative impact of the fee on cash holdings increases with α. Second, the optimal fee $\bar{\tau}$ decreases with α. As a result, the overall effects on quantities and profits are ambiguous. The effects of the resiliency parameter γ are also intuitive. It reduces the (discounted) profits of the platform but has no effects on the fee rate or the quantities because the platform has a repeated static problem.

4 Token platform

We now consider a different business model in which the platform issues private IOUs, called tokens, in stage 2. In stage 1, consumers can then use these tokens to buy goods from merchants as the only acceptable medium of exchange on the platform. A token platform can make profit from charging transaction fees as well as selling new tokens. Issuing tokens incurs a setup cost $\kappa\alpha$, which is increasing in the market share α of platform trades. This captures the expenses of arranging the initial coin offering as well as building the new payment infrastructure. Define p_t as the real price of tokens (in numeraire) if the cyber shock does not hit the platform. The nominal exchange rate of tokens is simply $e_t \equiv p_t/\phi_t$ (the amount of cash traded for each unit of tokens).

Notice that, unlike cash, the platform can fix the price of tokens p_t. How? One simple implementation is that the platform announces a price p_t and stands ready to trade (redeem and issue) tokens in stage 2 at this targeted price. Tokens can keep circulating without redemption, which could happen when the redemption price is set below the stage 2 market price. Obviously, in an equilibrium with positive redemption, the market price is equal to the redemption price.[8]

Recall that the cyber shock hits with probability γ in stage 2. When this happens, the token can no longer be used for future platform transactions. Hence, the exchange value of tokens in stage 1 needs to price in the cyber risk in stage 2. To mitigate the token holder's exposure to the cyber risk, the platform can hold cash reserves to finance token redemption in case the platform collapses. Specifically, bearers can redeem each token for B_t units of cash in stage 2 so that $b_t \equiv B_t/e_t$ is the real redemption rate. If $b_t = 1,$ then the token is fully backed and hence risk-free. To deliver on this promise, the platform issuing K_t units of tokens needs to hold $B_t K_t$ units of cash reserves.

Before formally analyzing the token platform, we can briefly compare it to a cash platform to highlight some differences. Under a cash platform, inflation reduces platform transactions and profits whenever the CIA is binding on the platform. Using tokens to facilitate platform transactions can avoid the inflation tax. There are, however, costs associated with the token platform. First, issuing tokens incurs a setup cost. Second, token returns are risky due to the cyber risk, while cash is always safe. Third, the platform is subject to the opportunity cost of holding cash reserves to back the redemption price.

We now solve the equilibrium under a token platform. The merchant's problem is the same as that on the cash platform. To buy a unit of retail goods, the consumer needs to pay $P_t = 1/\phi_t$ units of cash. Similarly, to buy retail goods using tokens, a consumer needs to pay $P_t/[(1-\gamma+\gamma b_t)\, e_t]$ units of tokens, where the denominator captures the associated risk and the exchange rate. We can

[8] An alternative way, which does not involve trading with consumers, is that the platform can allow token holders to invest in a "digital" asset that pays an interest rate ρ_t in terms of tokens. As the interest payment is financed by newly minted tokens, it does not have any direct financial impact on the platform. For consumers, this digital asset works like money market mutual funds. One real-world example is interesting-bearing Yu'ebao accounts offered by Alipay. As we will show, by administering the interest rate ρ_t, the platform can target p_t as well.

then define $P_t^T = P_t / (1 - \gamma + \gamma b_t)$ as the cash price of goods for a token user.

4.1 Consumer's problem

At the beginning of stage 1 in period t, the value function of a consumer holding M units of cash and K units of tokens is

$$V_t(M, K) = \max_{q_t, y_t} \left\{ \begin{array}{l} \sigma\alpha \left\{ U(y_t) + W_t \left(M + e_t K - \frac{P_t^T y_t}{1-\tau_t} \right) \right\} \\ +\sigma(1-\alpha) \left[U(q_t) + W_t \left[M - P_t q_t + e_t K \right] \right] \\ + (1-\sigma) W_t (M + e_t K) \end{array} \right\} \text{ s.t.} \tag{8}$$

$$e_t K \geq \frac{P_t^T y_t}{1 - \tau_t}, \tag{9}$$

$$M \geq P_t q_t, \tag{10}$$

where y_t is the quantity of retail goods bought on the token platform. In stage 1, the nominal value of the tokens held by the consumer is $e_t K$. The token-in-advance constraint is given by (9). With probability $\sigma\alpha$, the consumer pays on the token platform $P_t^T y_t / (1 - \tau_t)$ units of tokens for y_t units of retail goods, of which $P_t^T y_t$ is paid to the merchant and $\tau_t P_t^T y_t / (1 - \tau_t)$ to the platform as the transaction fees. The expected nominal value of balances (cash and tokens) brought to stage 2 is $M + e_t K - P_t^T y_t / (1 - \tau_t)$.

At the beginning of stage 2 in period $t-1$, the value function of a consumer holding A units of nominal balances (cash and tokens) is

$$W_{t-1}(A) = \max_{\{c_t, h_t, M_t, K_t\} \in \mathbb{R}_+^4} \{ c_t - h_t + \beta V_t (M_t, K_t) \}, \text{ s.t.} \tag{11}$$

$$c_t + \phi_{t-1} M_t + p_{t-1} K_t = h_t + \phi_{t-1} A + \Upsilon_t.$$

Applying the envelope theorem to (11) and (8), we obtain

$$\begin{aligned}\frac{\partial V_t(M,K)}{\partial M} &= \phi_t\left[1+\sigma(1-\alpha)\left\{U'(q_t)-1\right\}_+\right],\\ \frac{\partial V_t(M,K)}{\partial K} &= p_t(1-\gamma+\gamma b_t)\left[1+\sigma\alpha\left\{(1-\tau_t)U'(y_t)-1\right\}_+\right].\end{aligned}$$

When $b_t = 1$, the full real value of tokens is always guaranteed. The first-order conditions of (11) with respect to M_t and K_t are

$$i = \sigma(1-\alpha)\left\{m_t^{-\eta}-1\right\}_+, \tag{12}$$

$$\rho_t = -\gamma(1-b_t)+\sigma\alpha[1-\gamma(1-b_t)]\left\{(1-\tau_t)^{1-\eta}[[1-\gamma(1-b_t)]k_t]^{-\eta}-1\right\}_+, \tag{13}$$

where $k_t \equiv p_t K_t$ denotes the real token balances and $\rho_t \equiv (1+r)p_{t-1}/p_t - 1 > 0$ is the (shadow) interest rate of holding tokens – the interest rate that a token bearer forgoes. The LHS of (13) is the marginal (opportunity) costs of holding tokens. The RHS is the marginal value of holding tokens, consisting of the expected loss of token value $[-\gamma(1-b_t)]$ and the liquidity value of using tokens on the platform. Under a token platform, the consumer's solution is given by

$$q_t = m_t = \left[1+\frac{i}{\sigma(1-\alpha)}\right]^{-1/\eta}, \tag{14}$$

$$y_t = (1-\tau_t)[1-\gamma(1-b_t)]k_t = (1-\tau_t)^{1/\eta-1}\left[1+\frac{\rho_t+\gamma(1-b_t)}{\sigma\alpha[1-\gamma(1-b_t)]}\right]^{-1/\eta}. \tag{15}$$

Intuitively, the on-platform consumption decreases with ρ, γ, τ and increases with b.

4.2 Platform's problem

The initial profit flow of the platform in stage 2 is

$$c_0 = p_0K_1 - \kappa\alpha - \bar{\mu}b_1p_1K_1 = \beta[1+\rho_1-(1+i)b_1]k_1 - \kappa\alpha,$$

which consists of the revenue from the initial issuance of K_1 units of tokens each worth p_0 units of numeraire net of the setup cost and the amount of cash reserves the platform holds to back up its redemption in the following period. Each token is redeemable for $B_1 = b_1 p_1$ units of cash in period 1, and each unit of period-1 cash is worth $\bar{\mu} = \phi_0/\phi_1$ units of numeraire in period 0. The expected profit flow in any subsequent period is

$$\begin{aligned} c_t &= (1-\gamma)p_t\left(b_t K_t + K_{t+1} - K_t - \frac{\bar{\mu}}{p_t} b_{t+1} p_{t+1} K_{t+1}\right) + (1-\gamma)\,\sigma\alpha\frac{\tau_t \phi_t P_t^T y_t}{1-\tau_t} + \gamma\sigma\alpha\tau_t b_t k_t \\ &= (1-\gamma)\left\{\left[\sigma\alpha\tau_t - (1-b_t)\right]k_t + \beta\left[1+\rho_{t+1} - (1+i)\,b_{t+1}\right]k_{t+1}\right\} + \gamma\sigma\alpha\tau_t b_t k_t. \end{aligned}$$

If the cyber shock does not hit in stage 2, then the profit flow consists of the reserves brought forward ($p_t b_t K_t$), the sale of new tokens, $[p_t\,(K_{t+1} - K_t)]$, the receipt of transaction fees in tokens $[\sigma\alpha\tau_t\phi_t P_t^T y_t/\,(1-\tau_t)]$, subtracting the expenses on new cash reserves ($\bar{\mu} b_{t+1} p_{t+1} K_{t+1}$). If the cyber shock hits in stage 1, then the platform saves the tokens taxed by the transaction fees from redemption, which leaves the platform with some cash reserves ($\gamma\sigma\alpha\tau_t b_t k_t$).

The platform's profit is given by $\Pi^T = \max\left\{c_0 + \sum_{t=1}^{\infty}(1-\gamma)^{t-1}\beta^t c_t\right\}$. By collecting terms with the same t across periods, we can express the platform's problem as

$$\Pi^T = -\kappa\alpha + \max_{b_t, k_t, \tau_t \geq 0}\left\{\sum_{t=1}^{\infty}(1-\gamma)^{t-1}\beta^t\left[\gamma + \rho_t - (\gamma+i)\,b_t + (1-\gamma)\,\sigma\alpha\tau_t + \gamma\sigma\alpha\tau_t b_t\right]k_t\right\}.$$

The optimal design of a token platform is summarized by the following proposition.

Proposition 3 *Under a token platform, $b_t = \tau_t = 0$ solve the platform's problem, and*

$$\begin{aligned} \rho_t &= \frac{\sigma\alpha\,(1-\gamma)\,\eta}{1-\eta} - \gamma, \\ q_t &= \left[1 - \tau_C\,(i)\right]^{1/\eta - 1}, \\ y_t &= (1-\eta)^{1/\eta}, \\ \Pi^T &= -\kappa\alpha + \frac{\sigma\alpha\eta}{r+\gamma}\,(1-\eta)^{1/\eta-1}. \end{aligned}$$

The platform monopoly designs the features $\{\rho_t, \tau_t, b_t\}$ to extract the consumer's surplus of the platform trades. Rearranging terms, the expected profit flow in t is proportional to

$$\sigma\alpha\left(y_t^{-\eta} - 1\right) y_t - ib_t k_t$$

In this economy, it is the purchasing power of tokens, $y_t = (1-\tau_t)\left[1-\gamma\left(1-b_t\right)\right] k_t$, that matters for the consumer surplus, which is proportional to the first term above, $\left(y_t^{-\eta} - 1\right)\sigma\alpha y_t$, as the marginal utility of retail consumption net of real price (which is unity) times the total retail consumption purchased by the tokens. The platform can implement any y_t by varying the token interest rate ρ_t, via the pricing kernel (13). The platform faces the cost $ib_t k_t$ to maintain the purchasing power of token, which is the opportunity cost forgone by holding reserves.

There are two reasons why charging a transaction fee is no longer optimal to the token platform. First, the revenue generated by charging a higher transaction fee can always be replicated by selling token at a higher price without the transaction fee. Second, given y_t and $b_t > 0$, a higher transaction fee means the consumers need to hold more real tokens. Charging a higher transaction fee increases the total reserves held, which is costly to the platform. Notice that the latter channel implies that the platform is indifferent to the level of transaction fee when $b_t = 0$.

Define the growth rate of the nominal token price as $g_t \equiv e_t/e_{t-1} = (1+i)/(1+\rho_t)$. The following proposition reports some comparative statics results.

Proposition 4 *Under a token platform, the comparative statics are given by*

	∂g_t	∂q_t	∂y_t	$\partial \Pi^T$
∂i	$+$	$-$	0	0
$\partial \alpha$	$-$	$-$	0	$+$
$\partial \gamma$	$+$	0	0	$-$

Propositions 1 and 2 suggest that, under a cash platform, platform consumption y and profits Π^C are both maximized when $i < i^*$. Higher i tightens the CIA constraint on the platform

and reduces platform transactions and profits. By issuing tokens, the platform can replicate the maximum consumption and profits under a cash platform as the usage of tokens helps avoid inflation tax. The platform can maintain a fixed consumption level y by adjusting the nominal price paths of tokens (through g_t) in response to changes in i, α and γ. Interestingly, while a higher γ increases the risk of holding tokens, the platform can simply compensate users by appreciating token prices faster without the need to hold costly cash reserves.

4.3 Acceptance of cash on token platforms

We have implicitly assumed that a token platform does not accept cash. We here consider the general setting and show that accepting both cash and tokens are not optimal. Suppose the platform has an additional option of running a dual payments business model where both cash and tokens can be used on the platform. The consumer's problem becomes

$$V_t(M,K) = \max_{q_t, y_t} \left\{ \begin{array}{l} \sigma(1-\alpha)\{U(q_t) + W_t(M + S_t K - P_t q_t)\} \\ +\sigma\alpha\left[U\left(y_t^M + y_t\right) + W_t\left[M - \frac{P_t y_t}{1-\tau_t} + S_t K - \frac{P_t^T y_t}{1-\tau_t}\right]\right] \\ +(1-\sigma) W_t(M + S_t K) \end{array} \right\} \text{ s.t.} \tag{16}$$

$$M \geq P_t q_t, \tag{17}$$

$$M \geq \frac{P_t y_t^M}{1-\tau_t} \tag{18}$$

$$S_t K \geq \frac{P_t^T y_t}{1-\tau_t}, \tag{19}$$

where q_t denotes off-platform consumption paid with cash, y_t^M denotes on-platform consumption paid with cash, and y_t denotes on-platform consumption paid with tokens. Note that all on-platform transactions are subject to transaction fees.

The shadow interest rates for cash and tokens become

$$
\begin{aligned}
i &= \sigma(1-\alpha)\left\{m_t^{-\eta}-1\right\}_+ + \sigma\alpha\left\{(1-\tau_t)\,y_t'^{-\eta}-1\right\}_+, && (20)\\
\rho_t &\geq -\gamma(1-b_t)+\sigma\alpha\left[1-\gamma(1-b_t)\right]\left\{(1-\tau_t)\,y_t'^{-\eta}-1\right\}_+, \text{ equality when } y_t>0 && (21)
\end{aligned}
$$

where $y_t' = (1-\tau_t)\,m_t + (1-\tau_t)\left[1-\gamma(1-b)\right]p_t K_t$ is the total on-platform consumption purchased by both cash and tokens.

Lemma 2 *Define*

$$
\begin{aligned}
\rho_t' &\equiv \frac{\rho_t+\gamma(1-b_t)}{1-\gamma(1-b_t)},\\
\bar{\rho}(\tau_t,i) &\equiv \sigma\alpha\frac{(1-\alpha)\left[1-(1-\tau_t)^{\eta-1}\right]+\frac{i}{\sigma}}{(1-\alpha)(1-\tau_t)^{\eta-1}+\alpha}
\end{aligned}
$$

Under a dual platform, the consumer's solution, conditional on i and ρ_t, is given by:

	q_t	y_t'	y_t
(i). $\rho_t' < \bar{\rho}(\tau_t,i)$	$\left[1+\frac{i-\rho_t'}{\sigma(1-\alpha)}\right]^{-1/\eta}$	$(1-\tau_t)^{1/\eta}\left(1+\frac{\rho_t'}{\sigma\alpha}\right)^{-1/\eta}$	$y_t'-(1-\tau_t)\,q_t$
(ii). $\rho_t' \geq \bar{\rho}(\tau_t,i)$			
a. $\tau_t \geq \tau_C(i)$	$\left[1+\frac{i}{\sigma(1-\alpha)}\right]^{-1/\eta}$	$(1-\tau_t)^{1/\eta}$	0
b. $\tau_t < \tau_C(i)$	$\left(1+\frac{i}{\sigma}\right)^{-1/\eta}\left[1-\alpha+\alpha(1-\tau_t)^{1-\eta}\right]^{1/\eta}$	$(1-\tau_t)\,q_t$	0

In a dual system, tokens need to compete with cash on the platform. When the opportunity cost of holding ρ_t' is too high (case ii), tokens are not used and hence the problem reduces to that of a cash platform. Otherwise, both tokens and cash are used on the platform (case i). The first case is more likely when the fee rate is high and inflation is low because $\bar{\rho}(\tau_t,i)$ is a function increasing in i and decreasing in τ_t.

If the platform does not issue tokens, then the system becomes a cash system and the profit is

simply Π^C. If the platform issues tokens (i.e., $y_t > 0$), then the platform problem is

$$\Pi^D = -\kappa\alpha + \max\left\{\frac{1}{r+\gamma}\left\{\frac{[\gamma+\rho_t-(\gamma+i)\,b_t+(1-\gamma)\,\sigma\alpha\tau_t+\gamma\sigma\alpha\tau_t b_t]}{[1-\gamma\,(1-b_t)]\,(1-\tau_t)}y_t+\tau_t\frac{y_t'-y_t}{1-\tau_t}\right\}\right\},\ \text{s.t.}$$

$$\rho_t' \leq \sigma\alpha\frac{(1-\alpha)\left[1-(1-\tau_t)^{\eta-1}\right]+\frac{i}{\sigma}}{(1-\alpha)\,(1-\tau_t)^{\eta-1}+\alpha}.$$

which can be reduced to

$$\Pi^D = -\kappa\alpha + \frac{\sigma\alpha}{r+\gamma}\max\left\{\left(y_t'^{-\eta}-1\right)y_t - \frac{i}{\sigma\alpha}\frac{b_t y_t}{[1-\gamma\,(1-b_t)]\,(1-\tau_t)}+\tau_t\frac{y_t'-y_t}{1-\tau_t}\right\}\ \text{s.t.} \tag{22}$$

$$(1-\tau_t)\,y_t'^{-\eta} \leq \frac{1+\frac{i}{\sigma}}{(1-\alpha)\,(1-\tau_t)^{\eta-1}+\alpha}, \tag{23}$$

$$y_t = y_t' - (1-\tau_t)\left[1+\frac{i-\sigma\alpha\left[(1-\tau_t)\,y_t'^{-\eta}-1\right]}{\sigma\,(1-\alpha)}\right]^{-1/\eta}. \tag{24}$$

The plaftorm will allow both cash and token on the platform if

$$\Pi^D \geq \Pi^C.$$

We now show that the platform does not accept cash whenever tokens are issued. To understand this result, we consider the two cases discussed above. In case (i), tokens are used in the dual system. The choice between a dual system and a pure token system involves a trade-off between the token sales revenue and the transaction fee revenue collected from on-platform transactions purchased with cash. On the one hand, the acceptance of cash on platform crowds out the demand for tokens, reducing token sales revenue. On the other hand, the additional cash purchase brings in additional fee revenue. We show in the lemma that the token sales loss always dominates because

collecting transaction fees is a less efficient way of surplus extraction than selling tokens.[9] In case (ii), tokens are not used so this is just a simple cash platform.

Proposition 5 *The platform never accepts cash when tokens are issued.*

This result helps us understand why blockchain-based platforms issue utility tokens as the only means of payments. At the same time, there do exist dual systems where both tokens and cash are accepted on the platform. The model can potentially be extended to generate this as an equilibrium outcome. For example, if consumers are heterogeneous in terms of their costs of adopting tokens, then the platform might choose to adopt a dual system where cash is used by high adoption cost consumers (e.g. the elderlies) and tokens used by others. We leave this for future research.

5 Equilibrium business model and welfare

We now study the platform's choice to accept cash or issue tokens and the welfare implications of that choice.

5.1 Platform's business model choice

Obviously, the platform chooses to issue tokens, instead of using cash, whenever $\Pi^T > \Pi^C$. The three key parameters are i, γ, and α. Lemma 3 completely characterizes the parameter space such that the platform issues tokens, instead of using cash.

Lemma 3 *There exists a threshold interest rate $\iota^{\Pi}(\alpha, \gamma) > i^*$ such that $\Pi^T > \Pi^C$ if and only if $i > \iota^{\Pi}(\alpha, \gamma)$. Moreover, $\iota^{\Pi}(\alpha, \gamma)$ is a function decreasing in α but increasing in γ.*

[9] In a pure token platform studied earlier, collecting transaction fees (from transactions paid with tokens) and selling tokens are equally efficient because any adjustment in τ can be undone by modifying ρ. This is different in a dual system where some transactions are paid with cash, and changes in τ are not mitigated by an adjustment in μ (which is exogenous). The cost of taxing token trades can be mitigated by lowering ρ by the platform so that tax and token sale are equally effective. Taxing cash trades will not be mitigated (as mu is exogenous), so that it is more costly.

As discussed above, when $i < i^*$, the tax collected on the cash platform is the same as the token sale revenue collected from the token platform because the CIA constraint is not binding on the cash platform. Hence, the token platform is dominated due to the additional cost. As i increases, the CIA constraint on the cash platform starts to bind. The token platform dominates for high i because the usage of tokens insulates the platform from the inflation tax.

The token platform dominates the cash platform when the share of platform trade (α) is sufficiently high. There are two channels to explain why this happens. First, a higher α increases the extensive margin of platform trades, which raises the revenues of both the token and cash platform proportionally. Second, a higher α will have potential effects on the intensive margin of platform trades. A higher α means there is less off-platform trade, implying a lower cash holding. When the CIA constraint is not binding, the fee revenue per trade is unaffected. When the CIA constraint is binding, the fee revenue per trade falls in proportion to the cash holding. To counteract that effect, the cash platform reduces τ_t. The overall effect of α is thus ambiguous on a cash platform. However, a higher α does not change the intensive margin under a token platform because it does not alter the consumer surplus per platform trade. Hence, the token platform does not want to change the distortion. In sum, when α increases, the profit increases more under a token model than a cash model. When the premium in profit justifies the fixed cost of issuing tokens, the token platform dominates. Finally, as the platform becomes less resilient (higher γ), the platform is less willing to issue tokens because γ reduces the expected returns to pay a set-up cost to start a token platform.

We now consider the general-equilibrium effects of the platform's decision to issue tokens. First, to examine the effects on trading activities, we define the off-platform social surplus as $U(q) - q$ and the on-platform social surplus as $U(y) - y$. To study how the platform's business model affects

the central bank's seigniorage revenue, we define the equilibrium seigniorage from cash issuance as

$$\begin{aligned}
\mathcal{S} &= \phi_0 M_1 + \bar{\mu} b_1 p_1 K_1 + \sum_{t=1} (1-\gamma)^t \beta^t \left[\phi_t \left(M_{t+1} - M_t\right) - p_t b_t K_t + \bar{\mu} b_{t+1} p_{t+1} K_{t+1}\right] \\
&\quad + \sum_{t=1} (1-\gamma)^{t-1} \gamma \beta^t \left[-p_t b_t K_t + \bar{\mathcal{S}}\right], \\
&= \frac{i+\gamma}{r+\gamma} m + \frac{i}{r+\gamma} bk + \frac{\gamma}{r+\gamma} \bar{\mathcal{S}},
\end{aligned}$$

where $\bar{\mathcal{S}}$ is the continuation present value of the seigniorage revenue after the cyber shock hits. We can use the above findings to obtain the following proposition.

Proposition 6 *When the platform chooses to issue tokens instead of accepting cash,*

1. *on-platform consumption and social surplus go up,*
2. *off-platform consumption and social surplus go down,*
3. *seigniorage revenue goes down.*

When the platform issues tokens instead of accepting cash, it insulates the platform transactions from the inflation tax. This will not only increase profits but also increase the total trade surplus on the platform. However, this has a spillover effect on off-platform transactions. The switch to a token platform lowers the demand for cash, and hence the off-platform transactions and surplus go down as the CIA constraint becomes more binding. Finally, since the token platform does not hold cash reserves, the central bank's seigniorage revenue will decline too.

5.2 Welfare implications

Having analyzed the platform's private incentives to issue tokens, we now investigate whether the business model chosen by the platform also maximizes social welfare. The welfare of this economy is defined as the discounted sum of utilities of consumers, merchants and the platform, which are

given by

$$\begin{aligned}
\mathcal{W}^C &= \frac{1}{r+\gamma}\left\{\sigma\alpha\left[U\left(y^C\right)-y^C\right]+\sigma\left(1-\alpha\right)\left[U\left(q^C\right)-q^C\right]\right\}+\frac{\gamma\beta}{r+\gamma}\bar{\mathcal{W}};\\
\mathcal{W}^T &= -\kappa\alpha+\frac{1}{r+\gamma}\left\{\sigma\alpha\left[U\left(y^T\right)-y^T\right]+\sigma\left(1-\alpha\right)\left[U\left(q^T\right)-q^T\right]\right\}+\frac{\gamma\beta}{r+\gamma}\bar{\mathcal{W}},
\end{aligned}$$

where $\mathcal{W}^C$ is the welfare under a cash platform, $\mathcal{W}^T$ is the welfare under a token platform, and $\bar{\mathcal{W}}$ is the continuation welfare after the cyber shock. The following lemma describes when issuing tokens maximizes social welfare.

Lemma 4 *There exists a threshold rate $\iota^{\mathcal{W}}(\alpha,\gamma) > i^*$ such that $\mathcal{W}^T < \mathcal{W}^C$ if and only if $i < \iota^{\mathcal{W}}(\alpha,\gamma)$. Moreover, $\iota^{\mathcal{W}}(\alpha,\gamma)$ is a function decreasing in α if $i \leq \hat{\imath}$, but increasing in γ.*

The effects of i and γ on welfare are similar to those on the platform's profits. When $i < i^*$, a token platform and a cash platform support the same consumption allocations and profits. Therefore, the token platform is dominated due to the additional setup cost. As i increases, both platform profits and consumption drop when a cash platform is chosen. Hence, a token platform is preferred when the inflation rate is sufficiently high because tokens insulate the platform consumption from inflation tax. Furthermore, an increase in γ reduces the expected benefits of setting up a token platform.

The effects of α on welfare can be different from those on the platform's profits when $i > \hat{\imath}$. In particular, for $i > \hat{\imath}$, it could well be that $\mathcal{W}^C$ overtakes $\mathcal{W}^T$ when α becomes higher. This is due to the spillover effects on off-platform surplus discussed in the previous proposition. On the other hand, Π^C never overtakes Π^T. This is because the cash demand decreases when α becomes higher. Since the cash platform relies on cash trades for revenue, it responds by cutting the transaction fee to mitigate the reduction in cash holding. Doing so reduces the distortion on the platform trades, thus the welfare under the cash platform can be higher than it is under the token platform.

The equilibrium choice of the platform is not socially optimal when the welfare could increased by making the alternative choice. This happens when $\Pi^T > \Pi^C$ but $\mathcal{W}^T < \mathcal{W}^C$ or when $\Pi^T < \Pi^C$

but $\mathcal{W}^T > \mathcal{W}^C$. In the former, a token platform is chosen in equilibrium but a cash platform yields higher welfare. By using Lemmas 3 and 4, we obtain the following corollary, which summarizes the welfare properties of the equilibrium business model.

Corollary 1 *A cash platform is a socially optimal business model if and only if* $i \leq \min\left\{\iota^{\Pi}(\alpha,\gamma), \iota^{\mathcal{W}}(\alpha,\gamma)\right\}$. *The measure of this parameter subspace decreases with* α *if* $i \leq \hat{\imath}$ *but increases with* γ. *A token platform is a socially optimal business model if and only if* $i \geq \max\left\{\iota^{\Pi}(\alpha,\gamma), \iota^{\mathcal{W}}(\alpha,\gamma)\right\}$. *The measure of this parameter subspace increases with* α *if* $i \leq \hat{\imath}$ *but decreases with* γ.

The equilibrium choice of business model can be suboptimal because the platform only considers its profits and does not internalize the effects on the consumers' surplus (merchants have zero surplus). In particular, a token platform is suboptimally adopted when $i \in (\iota^{\Pi}(\alpha,\gamma), \iota^{\mathcal{W}}(\alpha,\gamma))$ and a cash platform is suboptimally adopted when $i \in (\iota^{\mathcal{W}}(\alpha,\gamma), \iota^{\Pi}(\alpha,\gamma))$. A simple way to correct these suboptimal outcomes is to impose a tax T on token platform adoption (subsidy when negative).

Corollary 2 *The government can induce the platform to choose the socially optimal business model by imposing a tax on the token platform:*

$$T = (\mathcal{W}^C - \Pi^C) - (\mathcal{W}^T - \Pi^T).$$

5.3 Numerical examples

We study a numerical example with the following parameter values: $\beta = 0.97$, $\sigma = 1$, $\eta = 0.08$, $\kappa = 0.001$. Figure 1 compares the relative ranking of a cash platform and a token platform in terms of social welfare and the platform's profits for different levels of interest rate i and platform fragility α. As implied by Corollary 1, the platform's choice of its business model (i.e., cash vs. token platforms) can be socially optimal or not. In particular, there are altogether four regions in the (i, α) parameter space, depending on whether the platform's choice and the social planner's

choice are aligned or not. Similarly, Figure 2 shows the disagreement between maximizing welfare and maximizing profits for different combinations of (i, γ) and (γ, α).

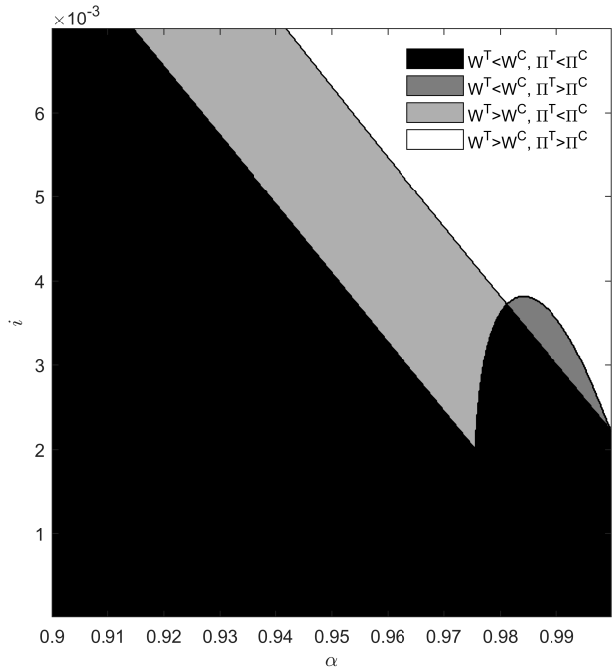


Figure 1: Profits vs welfare ($\gamma = 0.8$)

Figure 3 uses two snapshots to highlight the divergence between welfare and profits under different i and α. To understand the intuitions, in tables (1) and (2), we decompose welfare into profits and consumers' surplus (denoted by $\bar{U}$) under the cash and token platforms. In the left panel of figure 3 ($\alpha = 0.96$), there is a range of i under which the platform suboptimally chooses not to issue tokens. As shown in example 1 in Table 1, switching from a cash platform to a token platform increases consumers' surplus, which is big enough to offset the platform's profit loss. In contrast, in the right panel ($\alpha = 0.99$), there is a range of i under which the platform suboptimally chooses to issue tokens. As shown in example 2 in Table 2, switching from a token platform to a cash platform can significantly increase off-platform consumption and hence increase welfare.

To understand these findings, note that in choosing its business model, the platform considers

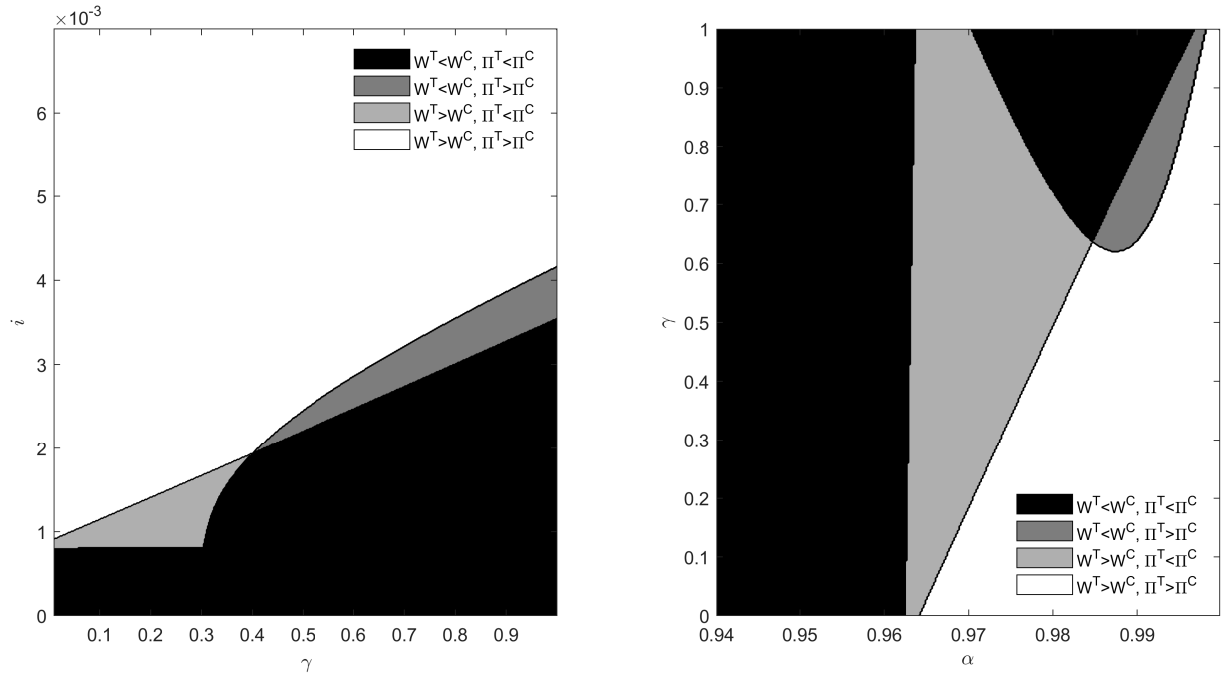


Figure 2: Profits vs welfare (left: $\alpha = 0.99$, right: $i = 0.3\%$)

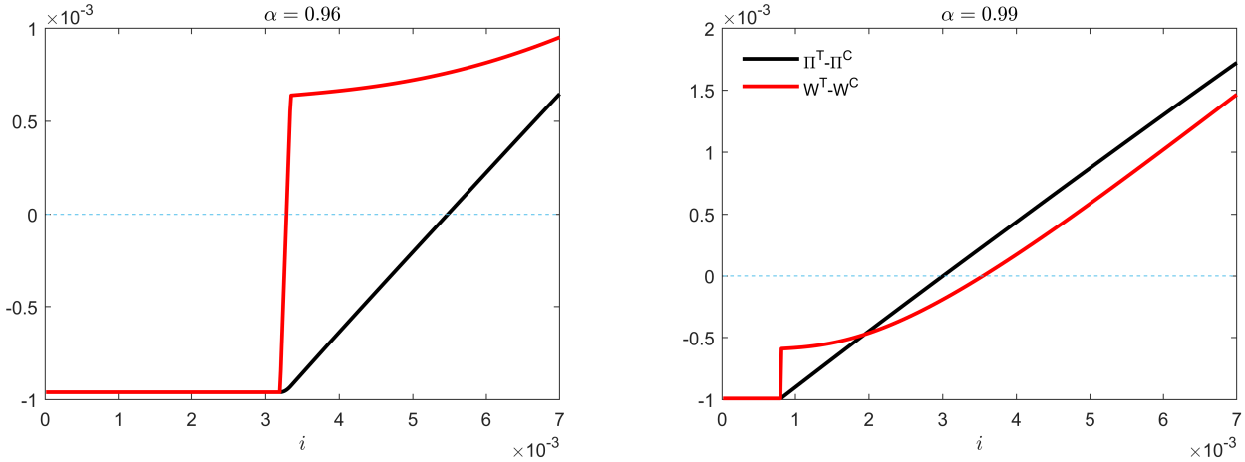


Figure 3: Divergences between profits and welfare ($\gamma = 0.8$)

Table 1: Example 1 ($\alpha = 0.96$, $i = 0.34, \gamma = 0.8$)

	non-platform	platform	welfare	profits	buyers' surplus		
	q	y	W	Π	$\bar{U}^q$	$\bar{U}^y$	$\bar{U}$
Cash platform	0.3674	0.3368	0.0755	0.0354	0.0031	0.0369	0.0401
Token platform	0.3607	0.3527	0.0761	0.0345	0.0031	0.0385	0.0416

Table 2: Example 2 ($\alpha = 0.99$, $i = 0.34, \gamma = 0.8$)

	non-platform	platform	welfare	profits	buyers' surplus		
	q	y	W	Π	$\bar{U}^q$	$\bar{U}^y$	$\bar{U}$
Cash platform	0.3674	0.3377	0.0755	0.0354	0.0008	0.0393	0.0401
Token platform	0.0258	0.3527	0.0754	0.0355	0.0001	0.0397	0.0399

only the effects on its profits:

$$\Pi^T - \Pi^C = -\kappa\alpha + \frac{\sigma\alpha}{r+\gamma}\left[\frac{\eta}{1-\eta}y^T - \frac{\tau}{1-\tau}y^C\right].$$

However, the platform does not fully internalize the effects on the consumers' surplus, which is given by

$$\frac{\sigma}{r+\gamma}\left\{\alpha\underbrace{\left[\left(U\left(y^T\right) - U(y^C)\right) - \left(\frac{y^T}{1-\eta} - \frac{y^C}{1-\tau}\right)\right]}_{\text{change in on-platform surplus } \bar{U}^y} + (1-\alpha)\underbrace{\left[U\left(q^T\right) - U(q^C) - (q^T - q^C)\right]}_{\text{change in off-platform surplus } \bar{U}^q}\right\}$$

This is a weighed sum of the effects on consumers' on-platform surplus $\bar{U}^y$ and off-platform surplus $\bar{U}^q$ with the weight given by α. As α goes up, the on-platform surplus goes up, while the on-platform surplus goes down. In the first (second) example where α is lower (higher), the on-platform (off-platform) surplus change dominates. Since the platform fails to fully internalize the social benefits (costs) of issuing tokens in the first (second) example, the platform tends to under-issue (over-issue) tokens.

6 Token platform regulation

The above discussion concerns the optimality of adopting a token platform. Conditional on its adoption, the allocation under a token platform can still be suboptimal. Relative to the first-best, there is under-consumption of platform goods when the purchasing power of tokens is so low that $y_t \leq 1$. Recall that the purchasing power of tokens is $y_t = (1-\tau_t)\left[1-\gamma\left(1-b_t\right)\right]k_t$. From (15), we know that the demand for tokens is low when the transaction fee is high, or the shadow rate is high (tokens depreciate quickly), or the cyber risk is high, or the reserve backing is low. Since the token platform charges the minimal transaction fee, $\tau_t = 0$, the other three are the potential sources of welfare loss. A natural question is whether the government can impose certain policies to reduce welfare loss. In reality, widely circulated priavte currencies are often subject to a minimum reserve requirement (e.g., PBoC's regulation on Alipay and WeChat). We consider below whether regulating the reserve holding of the platform is welfare-improving. In the Appendix, we also study whether providing deposit insurance can reduce the impacts of the cyber risk.

Suppose the central bank introduces a minimum reserve requirement on the token platform such that at least a fraction $\bar{b}$ of the token value is backed by cash reserves, i.e., $b_t \geq \bar{b}$. The platform problem becomes

$$\Pi^R = -\kappa\alpha + \max_{k_t,\tau_t\geq 0}\left\{\sum_{t=1}^{\infty}(1-\gamma)^{t-1}\beta^t\left[\rho_t - i + (1-\gamma)\sigma\alpha\tau_t\right]k_t\right\}.$$

The equilibrium quantities and profits are given by the following proposition.

Proposition 7 *Under a reserve requirement, the solution of a token platform is* $b_t = \bar{b}$ *and* $\tau_t = 0$

and

$$
\begin{aligned}
\rho_t &= -\gamma\left(1-\bar{b}\right)+\frac{\sigma\alpha\eta\left[1-\gamma\left(1-\bar{b}\right)\right]-i\bar{b}}{1-\eta}, \\
q_t &= \left[1-\tau_C\left(i\right)\right]^{1/\eta-1}, \\
y_t &= \left[\frac{1-\eta}{1+\frac{i}{\sigma\alpha}\frac{\bar{b}}{1-\gamma\left(1-\bar{b}\right)}}\right]^{1/\eta}, \\
\Pi^R &= -\kappa\alpha+\frac{\sigma\alpha\eta}{r+\gamma}\left[\frac{1-\eta}{1+\frac{i}{\sigma\alpha}\frac{\bar{b}}{1-\gamma\left(1-\bar{b}\right)}}\right]^{1/\eta-1}.
\end{aligned}
$$

The token platform under a higher reserve requirement features lower trading activities y_t. This is not due to a higher transaction fee as the platform still sets τ_t at 0. Recall that from (15), the demand for tokens is lower if the reserve backing is lower or the shadow rate is higher. The reserve requirement increases cash reserves and actually increases the demand for tokens. However, the platform takes advantage of the higher demand by charging a far higher shadow rate ρ, which ultimately pushes down the token demand. But this does not mean the platform makes more profit because holding reserves is costly and the higher shadow rate is to compensate that. Overall, both the consumer surplus and platform profit fall after a higher reserve requirement. Hence, conditional on the usage of a token platform, a higher reserve requirement only reduces welfare. Nevertheless, a sufficiently high reserve requirement could improve welfare when the platform's choice of token issuance is suboptimal (see Corollary 1) because it induces the platform to use cash instead of issuing tokens. The following corollary summarizes these findings.

Corollary 3 *The token platform under a higher reserve requirement features a slower bubble growth (lower g_t), a lower platform trade volume and a lower platform profit. Reserve requirement potentially results in a higher welfare only if the platform ceases to issue tokens.*

To deal with the default risk of a deposit-taking institution, a commonly used regulation is deposit insurance. In the appendix, we study and show that the provision of deposit insurance

does not affect welfare because its effect is offset one-to-one by the corresponding decrease in g_t, while the profits and allocation stay unchanged.

7 Endogenous resilience and moral hazard

Our analysis so far suggests that reserve holdings do not play any role on a token platform because keeping zero reserves maximizes both profits and welfare. This section shows that positive reserves and minimum reserve requirement could be optimal when the resilience of the platform is endogenous.

Now we extend the model to endogenize the resilience of the platform. Suppose the platform is still subject to cyber attacks, but whether the platform will collapse depends on the platform's investment. Instead of a one-time setup cost $\kappa\alpha$, we assume that a token platform can invest in cyber security by incurring a flow cost $\bar{\kappa} \equiv (r+\gamma)\,\kappa\alpha/\,(1+r)$ each period to contain the cyber risk.[10] After paying the flow cost, the platform will fail in the following period with probability γ. If the cost is not paid, then the platform will fail with certainty in the following period. Importantly, the cyber security investment is private information. As a result, the token platform is subject to the problem of moral hazard. To simplify the analysis, we assume that the token platform cannot charge transaction fees ($\tau_t = 0$). In period $t-1$, the platform issues K_t units of tokens in stage 2 and holds reserves b_t before making the investment in cyber security. The platform cannot commit to other future actions, and thus it takes as given the continuation future value Π^T.

If the platform in period $t-1$ invests in cyber security, then its expected profit flow involving K_{t+1} and b_{t+1} in stage 2 is

$$\begin{aligned} c'_{t-1} &= -\bar{\kappa} + (p_{t-1}K_t - \bar{\mu} b_t p_t K_t) + \beta\,(1-\gamma)\,p_t\,(-K_t + b_t K_t)\,, \\ &= -\bar{\kappa} + \beta\,[\gamma + \rho_t - (\gamma + i)\,b_t]\,k_t. \end{aligned}$$

[10] We normalize $\bar{\kappa}$ so that the problem is equivalent to the earlier one with exogenous resilience when the incentive constraint is not binding.

If there is no investment in cyber security, then the expected profit flow involving K_{t+1} and b_{t+1} in stage 2 of period $t-1$ is

$$c'_{t-1} = p_{t-1}K_t - \bar{\mu} b_t p_t K_t = \beta \left[1 + \rho_t - (1+i)\, b_t\right] k_t.$$

The incentive compatibility constraint guaranteeing that the platform at $t-1$ will invest $\bar{\kappa}$ in cyber security after announcing k_t and b_t is given by

$$\beta \left[1 + \rho_t - (1+i)\, b_t\right] k_t \leq -\bar{\kappa} + \beta \left[\gamma + \rho_t - (\gamma + i)\, b_t\right] k_t + (1-\gamma)\, \beta \Pi^T. \tag{25}$$

Rearranging terms the incentive compatibility constraint (25) reduces to

$$(1 - b_t)\, k_t \leq -(r+\gamma)\, \kappa\alpha + (1-\gamma)\, \Pi^T. \tag{26}$$

This constraint ensures that the platform will invest in cyber security to protect the platform. The RHS suggests that the constraint is less binding when the investment cost $\kappa\alpha$ is low, the investment is effective (γ is low), and the continuation value of the platform Π^T is high. The LHS suggests that the constraint imposes a limit on the amount of unbacked tokens. Since the platform loses the reserves when it fails, a higher b implies that the platform has more skin in the game and helps relax the incentive constraint. This implies that, unlike in the case of exogenous resilience, positive reserve holdings can be optimal.

In the (steady-state) Markov perfect equilibrium, we have $c'_t = c'$ for all t, and the platform's profit is given by

$$\Pi^T = c' + (1-\gamma)\, \beta \Pi^T.$$

The problem of the token platform at $t-1$ is thus

$$\max \left\{ -\bar{\kappa} + \beta \left[\gamma + \rho_t - (\gamma + i)\, b_t\right] k_t + (1-\gamma)\, \beta \Pi^T \right\} \text{ s.t. } (26)$$

Our results in the baseline model remain valid if the incentive compatibility constraint (26) is not binding. The following proposition provides a condition under which the incentive compatibility constraint (26) is binding.

Proposition 8 *The incentive compatibility constraint (26) is binding if γ is sufficiently high such that*

$$\sigma\alpha\eta < \frac{r+\gamma}{(1-\gamma)^2}. \tag{27}$$

Suppose (27) is satisfied such that the incentive compatibility constraint is binding. The platform's profit in the Markov perfect equilibrium becomes

$$\Pi^T = -\kappa\alpha + \frac{1}{r+\gamma}\left(\rho_t k_t - i k_t\right) - (\gamma+i)(r+\gamma)\kappa\alpha + (\gamma+i)(1-\gamma)\Pi^T,$$

where

$$\begin{aligned}\rho_t k_t &= \gamma\left[(r+\gamma)\kappa\alpha - (1-\gamma)\Pi^T\right] \\ &\quad -\sigma\alpha\left[k_t + \gamma\left[(r+\gamma)\kappa\alpha - (1-\gamma)\Pi^T\right]\right] \\ &\quad +\sigma\alpha\left[k_t + \gamma\left[(r+\gamma)\kappa\alpha - (1-\gamma)\Pi^T\right]\right]^{1-\eta}.\end{aligned}$$

While the model with binding incentive constraint is not analytically solvable, we will use the following numerical examples to illustrate the design of the platform and the effects of regulation.

7.1 Numerical examples

Figures (4)-(5) consider some numerical examples under moral hazard. The first panel of figure (4) reports the equilibrium outcomes for different combinations of α and γ. First, notice that the platform is unconstrained only for sufficiently low γ and sufficiently high α (in the blue area). This is consistent with Proposition 8. For intermediate values of α and γ, the platform is constrained and needs to increase reserves b and reduce balances k (in the green area). For γ too low and/or

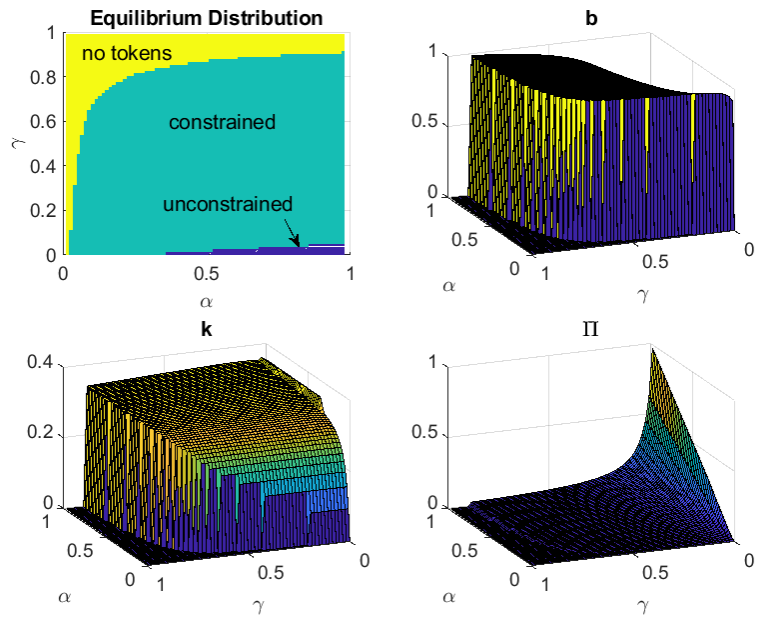


Figure 4: Token platform design as a function of α and γ

α too low, it is no longer profitable for the token platform to operate (in the yellow area).

We now study the optimal reserve b and token supply k of the platform. Figure (5) suggests that an increase in α and a decrease in γ tend to relax the platform's incentive constraint. Hence, b increases with γ and decreases with α, while k decreases with γ and increases with α. As a result, profits decrease with γ and increase with α.

We now examine how the moral hazard problem affects the platform's business model. Figure (6) studies the choice between a cash platform and a token platform under moral hazard. The first panel just reproduces the distribution of equilibria in figure (4). The second, third and fourth panels indicate the parameter subset (in yellow) in which using a token platform maximizes, respectively, the welfare, the profits and the central bank's seigniorage.

Without moral hazard, a token platform maximizes welfare/profits for high α and low γ. Under moral hazard, this still holds for sufficiently high α and sufficiently low γ such that the token platform is unconstrained. For higher γ and/or lower α, however, the token platform is constrained

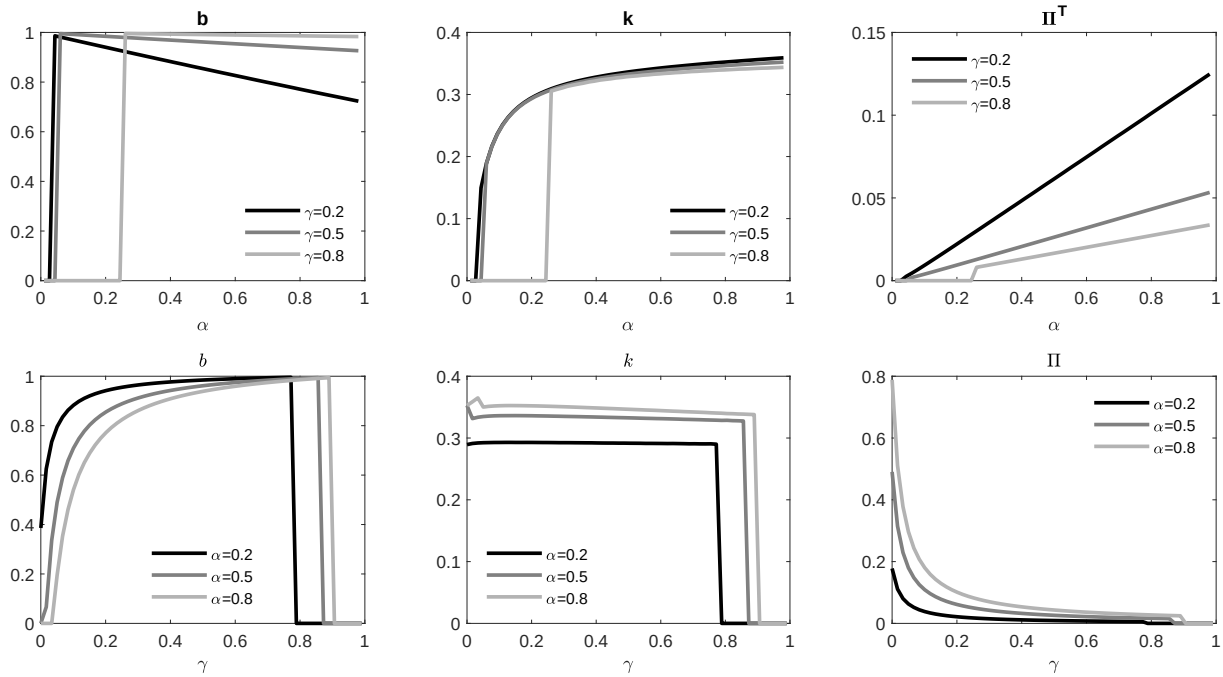


Figure 5: Token platform design as a function of α (top) and γ (bottom)

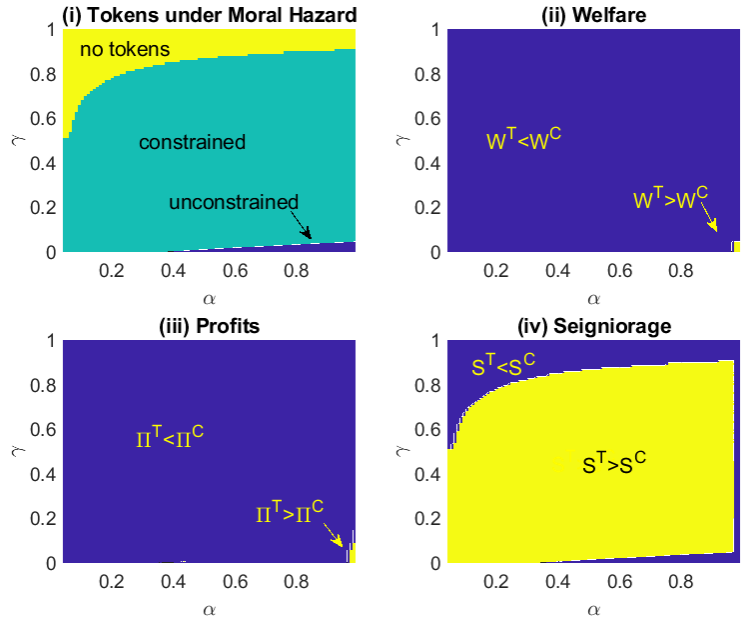
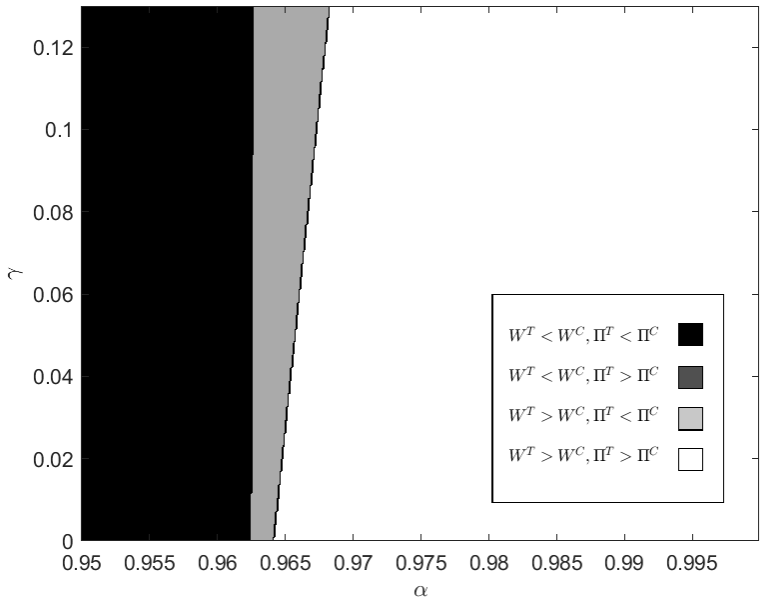


Figure 6: Platform's choice of business model

Figure 7: Platform choice (without moral hazard)



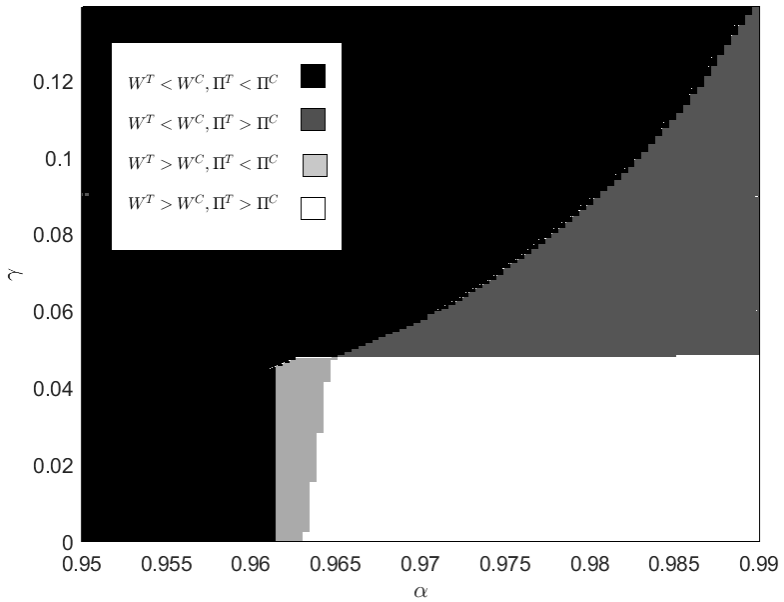


Figure 8: Platform choice (with moral hazard)

and hence becomes a less attractive option for the platform and the planner. In general, the region where a token platform maximizes profits/welfare becomes smaller.

Without moral hazard problems, a cash platform always generates higher seigniorage because a token platform keeps no reserves. With moral hazard, this is no longer true. In the fourth panel, a token platform tends to maximize seigniorage for intermediate values of γ and α. If γ is too high and/or α too small, then a token platform is not viable. If α is too high and/or γ too low, then the platform does not need to hold (too much) reserves. In addition, under a token platform, cash demand for off-platform transactions converges to zero as α approaches 1. Hence, a seigniorage-maximizing central bank prefers a cash platform when α is too high and γ is too low.

7.2 Optimal reserve requirements under moral hazard

In Figure (6), the token platform maximizes profits/welfare only for a small region with high α and low γ. Figure 7 and 8 take a more careful look at this region to compare the outcomes with and without moral hazard. Interestingly, under moral hazard, the platform over-adopts a token platform for high α and low γ. As discussed before, this could also happen without moral hazard because of the spillover effects of the platform choice on non-platform consumption. Here, the reason is different because over-adoption can happen even when $\alpha = 1$. Notice that while moral hazard does not impact the efficiency of a cash platform (because tax revenue is earned only if the platform survives), it tightens the incentive constraint of a token platform (because the revenue from token issuance is earned even if the platform collapses) and leads to efficiency loss in the usage of tokens. However, the platform only internalizes part of the efficiency loss, with consumers bearing the remaining loss. Hence, under moral hazard, the platform can choose to issue tokens even when it is dominated by a cash platform from the social welfare perspective. As a result, imposing a minimum reserve requirement $b_{\min}$ may be welfare-improving, as illustrated by the following three examples.

- Example 1: $\alpha = 0.99, \gamma = 0.02$

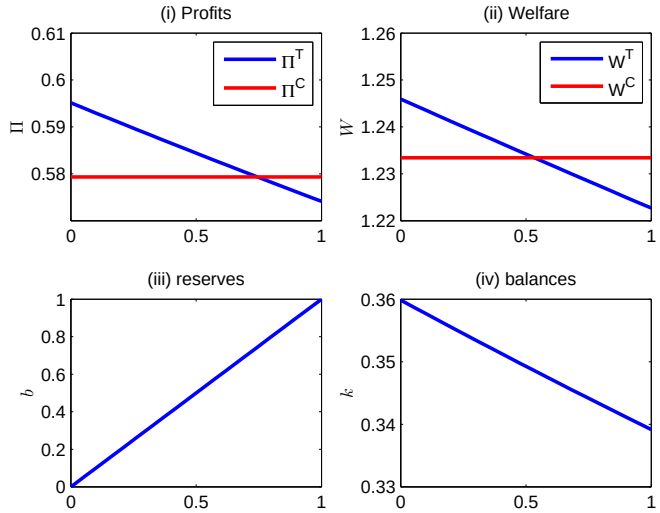


Figure 9: Effects of reserve requirements (Example 1: $\alpha = 0.99, \gamma = 0.02$)

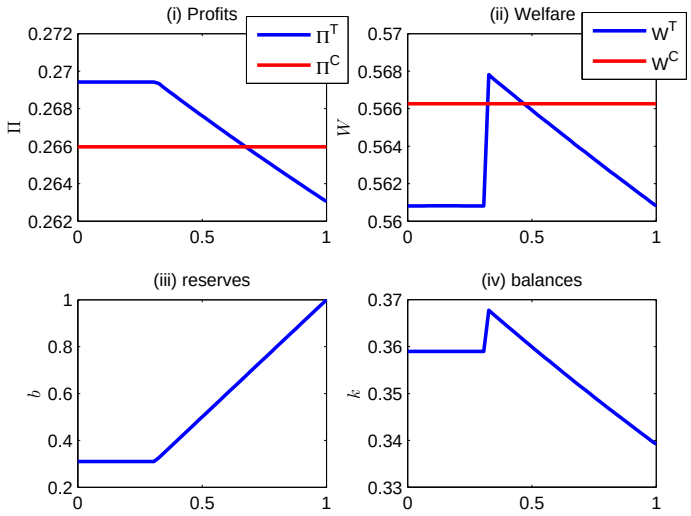


Figure 10: Effects of reserve requirements (Example 2: $\alpha = 0.99, \gamma = 0.08$)

– Figure 9 illustrates an example where not imposing a reserve requirement is optimal. Without regulation, a token platform is optimally adopted with $b = 0$ and $k = 0.3599$ because the incentive constraint of the platform is not binding. With any minimum

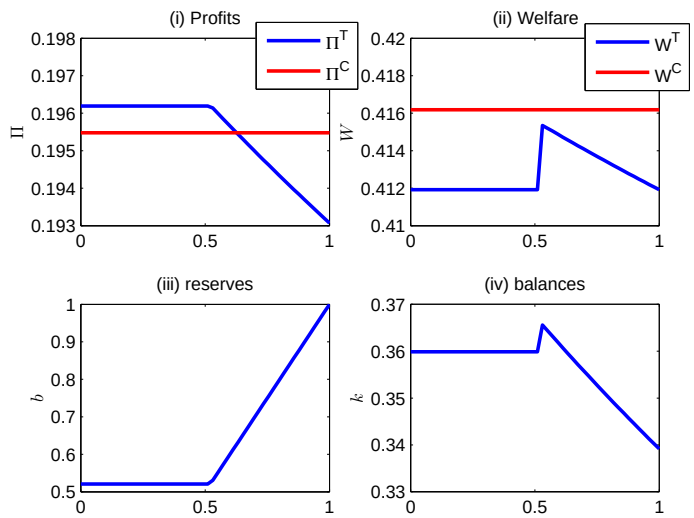


Figure 11: Effects of reserve requirements (Example 3: $\alpha = 0.99, \gamma = 0.12$)

reserve requirement $b_{min} > 0$, the platform is forced to hold $b = b_{min} > 0$. As a result, the platform reduces token supply k, implying lower profits and welfare. In this case, it is optimal to set $b_{\min} = 0$ as the original platform choice is optimal.

- Example 2: $\alpha = 0.99, \gamma = 0.08$
  - Figure 10 illustrates an example where imposing a fractional reserve requirement is optimal. Without regulation, a token platform is suboptimally adopted with $b = 0.3098$ and $k = 0.3590$, and the incentive constraint is binding. Any reserve requirement $b_{\min} < 0.3098$ has no effects. When $b_{\min} \in (0.3098, 0.3265)$, reserve b is forced to go up. This will increase k, too, as the incentive constraint is relaxed. When $b_{\min} > 0.3265$, however, k drops as b goes up further because the incentive constraint is no longer binding. In this case, it is optimal to set $b_{\min} = 0.3265$ to induce the platform to supply more tokens to increase welfare.

- Example 3: $\alpha = 0.99, \gamma = 0.12$

– Figure 11 illustrates an example where imposing a 100% reserve requirement can be optimal. Without regulation, a token platform is sub-optimally adopted with $b = 0.5207$ and $k = 0.3599$, and the incentive constraint is binding. Any reserve requirement $b_{\min} < 0.5207$ has no effects. When $b_{\min} \in (0.5207, 0.5306)$, reserve b is forced to increase. This will increase k, too, as the incentive constraint is relaxed. When $b_{\min} > 0.5306$, k drops because b goes up as the incentive constraint is no longer binding. While $W^T < W^C$ for all $b_{\min}$, $\Pi^T > \Pi^C$ only for $b_{\min} \geq 0.6327$. In this case, it is optimal to set $b_{\min} \in [0.6327, 1]$ to induce the platform not to issue tokens. Hence, 100% reserve requirement is also optimal.

8 Endogenous interoperability

One concern about private digital currencies is that the lack of payment interoperability across platforms can increase the fragmentation of the payment system (Brunnermeier, et al, 2019). We have already shown that a token platform has no incentives to accept cash. We will now examine whether the platform wants to promote the circulation of tokens outside the platform.

Suppose the platform has an option to circulate tokens and provide payment services outside the platform. We call these "circulatable tokens". In reality, private currencies that are widely accessible tend to be subject to regulation due to concerns about financial risk and consumer protection. For example, China's central bank requires payment firms such as Alipay and WeChat to deposit reserves of client funds with a centralized custodian. To capture this real-world regulation, we assume that the token platform has to satisfy a minimum reserve ratio b. To simplify the analysis simple, we assume $\tau_t = 0$ in this case. The consumer problem becomes

$$V_t(M, K) = \max_{q_t, y_t} \left\{ \begin{array}{l} \sigma\alpha \left\{ U(y_t) + W_t \left(M + S_t K - P_t^T y_t \right) \right\} \\ +\sigma(1-\alpha) \left[U\left(q_t + q_t^K\right) + W_t \left[M - P_t q_t + S_t K - P_t^T q_t^K \right] \right] \\ +(1-\sigma) W_t (M + S_t K) \end{array} \right\} \text{ s.t.} \tag{28}$$

$$
\begin{aligned}
S_t K &\geq P_t^T y_t, && (29)\\
M &\geq P_t q_t, && (30)\\
S_t K &\geq P_t^T q_t^K, && (31)
\end{aligned}
$$

where q_t^K is the off-platform consumption purchased with tokens. The shadow interest rates are given by

$$
\begin{aligned}
i &\geq \sigma(1-\alpha)\left\{\left(m_t + k_t'\right)^{-\eta} - 1\right\}_+, \text{ equal when } m_t > 0, && (32)\\
\rho_t &= -\gamma(1-b) + \sigma[1-\gamma(1-b)]\left\{\begin{array}{l} \alpha\left\{k_t'^{-\eta} - 1\right\}_+ + \\ (1-\alpha)\left\{\min\left\{k_t'^{-\eta}, 1 + \frac{i}{\sigma(1-\alpha)}\right\} - 1\right\}_+ \end{array}\right\} && (33)
\end{aligned}
$$

where $k_t' = [1-\gamma(1-b)]\, p_t K_t$.

Lemma 5 *Under a circulatable token platform, the consumer's solution has two regimes, depending on i and ρ_t:*

	$q_t = m_t$	$q_t^K = y_t = k_t'$
(i). $i \leq (1-\alpha)\frac{\rho_t+\gamma(1-b)}{1-\gamma(1-b)}$	$\left[1 + \frac{i}{\sigma(1-\alpha)}\right]^{-1/\eta} - k_t'$	$\left[1 + \frac{1}{\sigma\alpha}\left[\frac{\rho_t+\gamma(1-b)}{1-\gamma(1-b)} - i\right]\right]^{-1/\eta}$
(ii). $i > (1-\alpha)\frac{\rho_t+\gamma(1-b)}{1-\gamma(1-b)}$	0	$\left[1 + \frac{1}{\sigma}\frac{\rho_t+\gamma(1-b)}{1-\gamma(1-b)}\right]^{-1/\eta}$

Circulatable tokens and cash are in direct competition off-platform. When i is sufficiently high (case ii), tokens can drive out cash and become the only payment instrument in the retail sector (even though there is still demand for cash as a reserve asset). When i is not too high (case i), cash is still used off-platform.

The platform problem is

$$
\Pi^{CT} = -\kappa\alpha + \max_{b_t, k_t}\left\{\sum_{t=1}^{\infty}\frac{(1-\gamma)^{t-1}\beta^t}{1-\gamma(1-b)}\left[\gamma + \rho_t - (\gamma+i)\, b\right] k_t'\right\}
$$

Rearranging term, the platform problem becomes

$$\begin{aligned}
\Pi^{CT} &= -\kappa\alpha + \frac{1}{r+\gamma}\max\left\{\Pi_1^{CT}, \Pi_2^{CT}\right\}, \text{ where} \\
\Pi_1^{CT} &\equiv \max_{\rho_t'} G_1^{CT}\left(\rho_t'\right) \text{ s.t. } i \leq (1-\alpha)\,\rho_t'; \\
\Pi_2^{CT} &\equiv \max_{\rho_t'} G_2^{CT}\left(\rho_t'\right) \text{ s.t. } i > (1-\alpha)\,\rho_t',
\end{aligned}$$

where

$$\begin{aligned}
G_1^{CT}\left(\rho'\right) &\equiv \left[\rho' - \frac{ib}{1-\gamma\,(1-b)}\right]\left(1+\frac{\rho'-i}{\sigma\alpha}\right)^{-1/\eta}, \\
G_2^{CT}\left(\rho'\right) &\equiv \left[\rho' - \frac{ib}{1-\gamma\,(1-b)}\right]\left(1+\frac{\rho'}{\sigma}\right)^{-1/\eta}.
\end{aligned}$$

where $\rho_t' \equiv \frac{\rho_t+\gamma(1-b)}{1-\gamma(1-b)}$.

We use a numerical example to illustrate the incentives of a platform to make tokens circulatable. Figure 12 plots the supply of circulatable tokens k' and profits Π^{CT} as a function of i and b. Whenever $b > 0$, the platform needs to keep cash reserves which involve a marginal cost i. As a result, both token issuance and profits go down as the reserve ratio b increases. The interest rate has non-linear effects. When i is low, increasing i makes cash a worse payment alternative outside the platform, allowing the platform to issue more tokens and make a higher profit. When i is too high, further increasing i will lower token issuance as platform's profits decline with i. Figure 13 shows the platform's business model choice between issuing non-circulatable tokens, circulatable tokens and using cash. Under this parameterization, we find that a cash platform is profit-maximizing only for sufficiently low i and sufficiently high b. The platform issues non-circulatable tokens when i or b are sufficiently high. In other cases issuing circulatable tokens maximizes profits.

8.1 Calibrated examples

We finish the study with two calibrated examples.

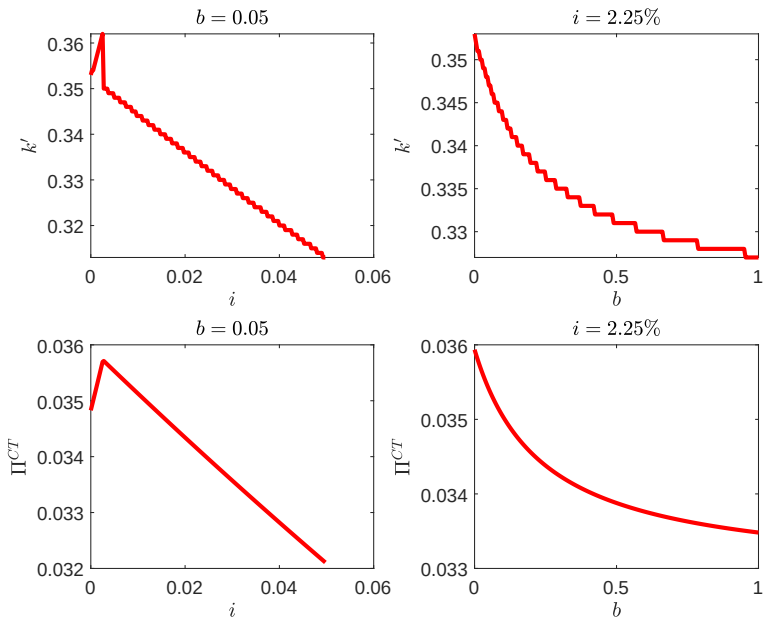


Figure 12: Supply of Circulatable Tokens

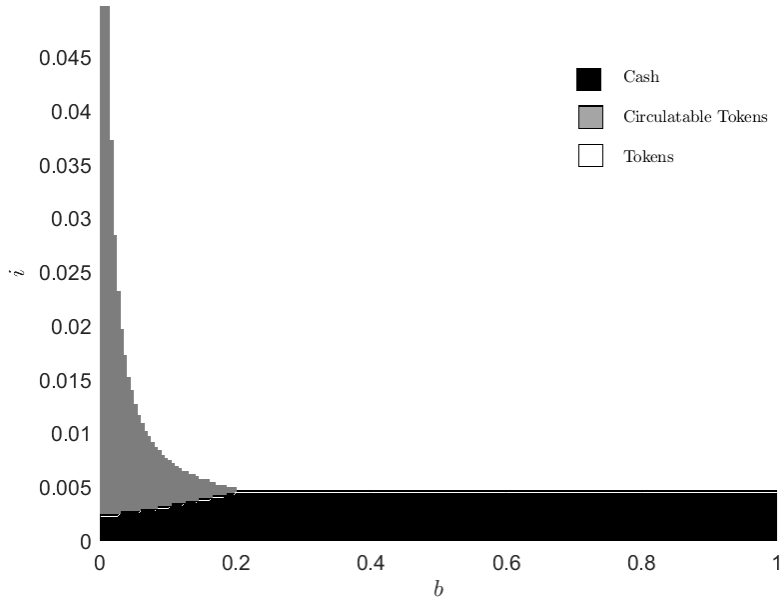


Figure 13: Platform Choice

Example 1: Cash platform: Amazon

We consider an annual model and set $\beta = 0.97$. We pick μ to generate $i = 1.5\%$ which is the 3-month T-bill rate in 2020 Q1. We set $\sigma = 1$ to minimize matching frictions. With Amazon exemplifying the modeled cash platform in our model, we calibrate the trading parameter α and preference parameter η to match the market share and trading fees of this e-commerce platform. First, for each item sold on their platform, Amazon charges the seller a referral fee which is a flat percentage typically about 15%.[11] Second, according to the Department of Commerce, the e-commerce share of retail in 2020 Q1 is 12%. Since Amazon accounts for about 50% of the US e-commerce market[12], its market share is about 6%. We set $\alpha = 0.12$ and $\eta = 0.13$ to minimize the sum of squared percentage differences.

Parameters	Notation	Value	Notes
Discount factor	β	0.97	Standard in literature
Nominal interest rate	i	1.5%	3-month T-bill rate in 2020
Prob. of trading	σ	1	Minimal search fractions
Prob. of platform trades	α	0.12	Amazon share of retail share
Preference parameter	η	0.13	Amazon fee 15%

Under our parameterization, $i^* = 11\%$. Since $i < i^*$, we are in case (i) considered in Proposition 3. This implies that it is not profitable for the platform to issue tokens for any κ and γ. Suppose the setup cost of the token system is 1% of y^T (approximately 750 millions USD) and the probability of collapse is 1%. Our model then suggests that it is socially optimal not to issue tokens for any γ. However, when i goes above 12% or when α goes above 90%, issuing tokens maximizes both profits and social welfare. As shown in Section 5, this is because high inflation constrains a cash platform's ability to charge fees especially when its market share is large.

[11] Source: https://www.junglescout.com/blog/amazon-fba-fees/

[12] Source: https://techcrunch.com/2018/07/13/amazons-share-of-the-us-e-commerce-market-is-now-49-or-5-of-all-retail-spend/

Example 2: Circulatable tokens: Alibaba

With Alibaba exemplifying the circulable token platform in our model, we set a small $\gamma = 1\%$ to capture the reliability of the platform. We pick μ to generate $i = 3.5\%$ which is the average 3-month T-bill rate in China in 2017.[13] We set $\sigma = 1$ to minimize the matching frictions. In 2017, the reserve requirement was $b = 50\%$. We calibrate the trading parameter α and preference parameter η to match the market share and token returns of this e-commerce platform. First, Alibaba accounts for about 11% of China's total retail market. Second, the annual return rate of Alipay balances is about 4% in 2017.[14] We set $\alpha = 0.11$ and $\eta = 0.15$ to minimize the sum of squared percentage differences.

Parameter Values

Parameters	Notation	Value	Notes
Discount factor	β	0.97	Standard in literature
Nominal interest rate	i	3.5%	3-month Tresaury bond rate in 2017
Prob. of trading	σ	1	Minimal search fractions
Prob. of platform trades	α	0.11	Alibaba accounts for 11% of total retail in China
Preference parameter	η	0.05	Alipay annual return rate is about 4% in 2017
Minimum reserve	b	0.5	Regulatory requirement

Under our parameterization, $q^{CT} = 0.2583$ and $y^T = 0.3580$. The maximum setup cost required to justify the choice of a token platform is $\kappa_{\max} = 1.75$. Suppose the setup cost of the token system is 1% of y^T, then issuing circulatable tokens is also welfare maximizing.[15] Our model predicts that the platform has strong incentives to issue circulatable tokens. The plaform chooses to issue circulable tokens even when the reserve requirement is raised to 99% or the interest rate is lowered to 1 basis point.

[13] The T-bond rate is about 2.25% in 2019 (https://tinyurl.com/ycgfk8l4)

[14] Alibaba's market share in China is about 11% of the total retail market in China (https://tinyurl.com/y37lxgfz). In 2017, Tianhong Yu'e Bao fund's seven-day annualized yield was about 4% (https://tinyurl.com/yy7br66m).

[15] In 2019, Alibaba's China commerce retial is about 35bn USD, the equivalent setup cost is then 350 million USD.

9 Conclusion

We develop a general equilibrium model to study a platform's business model choice between running a cash platform and a token platform, as well as the welfare and policy implications of this choice. We show that the equilibrium business model choice is not necessarily socially optimal because the private platform does not internalize its impacts on the economy, especially the spillover effects on non-platform transactions. We also extend the model to study endogenize resiliency under moral hazard and endogenize payment interoperability. The model suggests that the need to impose a reserve requirement depends on nature of the risk associated with the platform, as well as the platform's market share and the central bank monetary policy.

While our paper has provided novel economic and policy insights, we have abstracted from some interesting aspects. One could extend the model to study the interaction between multiple private platforms. It would also be interesting to explore other policy tools such as the monetary policy of the central bank. We leave these extensions for future research.

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Appendix

A Proof of Lemma 1

Proof. Note first that, given $\tau_t \leq 1$ and $\eta < 1$, constraint (2) is always binding, implying that we must have $q_t = m_t$. We then consider the two possible cases for equation (6): (i) $\left\{(1-\tau_t)^{1-\eta} m_t^{-\eta} - 1\right\}_+ = 0$, and (ii) $\left\{(1-\tau_t)^{1-\eta} m_t^{-\eta} - 1\right\}_+ > 0$.

In case (i), equation (6) becomes $i = \sigma(1-\alpha)\left(m_t^{-\eta} - 1\right)$, so we have $m_t = [1 + i/[\sigma(1-\alpha)]]^{-1/\eta} = [1 - \tau_C(i)]^{1/\eta - 1}$. Since constraint (3) is not binding in case (i), using equation (5) the first order condition of (4) with respect to y_t implies that $y_t = (1-\tau_t)^{1/\eta} \leq (1-\tau_t) m_t$.

In case (ii), equation (6) becomes $i = \sigma(1-\alpha)\left(m_t^{-\eta} - 1\right) + \sigma\alpha\left[(1-\tau_t)^{1-\eta} m_t^{-\eta} - 1\right]$, so we have $m_t = m(\tau_t)$. Since constraint (3) is binding in case (ii), we have $y_t = (1-\tau_t) q_t$. ■

B Proof of Proposition 1

We first establish the following lemma, which derives some properties of the objective functions in these two regions.

Lemma 6 *We have*
(i) $G_1(\tau) \leq G_2(\tau)$ *if and only if* $\tau \geq \tau_C(i)$;
(ii) $G_1(\tau)$ *is single peaked at* $\tau = \eta$; $G_2(\tau)$ *is single peaked at* $\tau = \bar{\tau}$, *where* $\bar{\tau} > \eta$.

Proof. Part (i) is obvious after arranging terms of $G_1(\tau)$ and $G_2(\tau)$. To show part (ii), differentiating $\log G_1(\tau)$ we have

$$\frac{d}{d\tau}\log G_1(\tau) = \frac{1}{\tau} - \left(\frac{1}{\eta} - 1\right)\frac{1}{1-\tau},$$

which is strictly decreasing from infinity to negative infinity when τ goes from 0 to 1, attaining zero at $\tau = \eta$. So $G_1(\tau)$ is single peaked at $\tau = \eta$. Differentiating $\log G_2(\tau; i)$ we have

$$\frac{d}{d\tau}\log G_2(\tau; i) = \frac{1}{\tau} - \frac{1-\eta}{\eta}\frac{\alpha(1-\tau)^{-\eta}}{1-\alpha+\alpha(1-\tau)^{1-\eta}},$$

which is strictly decreasing from infinity to negative infinity when τ goes from 0 to 1, attaining zero at $\tau = \bar{\tau}$. So $G_2(\tau)$ is single peaked at $\tau = \bar{\tau}$. Finally, notice that

$$\left.\frac{d}{d\tau}\log G_2(\tau)\right|_{\tau=\eta} = \frac{1}{\eta}\frac{1-\alpha}{1-\alpha+\alpha(1-\eta)^{1-\eta}} > 0 = \left.\frac{d}{d\tau}\log G_2(\tau)\right|_{\tau=\bar{\tau}}.$$

Since $\frac{d}{d\tau}\log G_2(\tau)$ is strictly decreasing, we must have $\bar{\tau} > \eta$. ■

Given the results of Lemma 6, now we are ready to prove Proposition 1.

Proof. Consider the following three cases: (a) $\tau_C(i) < \eta < \bar{\tau}$; (b) $\tau_C(i) \in [\eta, \bar{\tau}]$; (c) $\tau_C(i) > \bar{\tau} > \eta$.

Consider case (a). Since $G_2(\tau; i)$ is single peaked at $\tau = \bar{\tau}$ but $\bar{\tau} > \tau_C(i)$, we must have $\max_{\tau_t \leq \tau_C(i)} G_2(\tau_t; i) = G_2[\tau_C(i)_t; i]$. Notice that

$$\Pi_2^C = \max_{\tau_t \leq \tau_C(i)} G_2(\tau_t; i) = G_2[\tau_C(i); i] = G_1[\tau_C(i)] < \max_{\tau_t \geq \tau_C(i)} G_1(\tau_t) = \Pi_1^C.$$

Thus we have $\Pi^C = \frac{\alpha\sigma}{r+\gamma}G_1(\eta)$ and $\tau = \eta$. This corresponds to case i of Lemma 1, and hence $q_t = [1 - \tau_C(i)]^{1/\eta - 1}$ and $y_t = (1-\eta)^{1/\eta}$. Rearranging terms, we have $\tau_C(i) < \eta$ is equivalent to $i < i^*$, corresponding to part (a) of Proposition 1.

Consider case (b). Since $G_1(\tau)$ is single peaked at $\tau = \eta$ but $\eta \leq \tau_C(i)$, we must have $\max_{\tau_t \geq \tau_C(i)} G_1(\tau_t) = G_1[\tau_C(i)]$. Notice that

$$\Pi_2^C = \max_{\tau_t \leq \tau_C(i)} G_2(\tau_t; i) = G_2[\tau_C(i); i] = G_1[\tau_C(i)] = \max_{\tau_t \geq \tau_C(i)} G_1(\tau_t) = \Pi_1^C.$$

Thus we have $\tau = \tau_C(i)$ and $\Pi^C = \frac{\alpha\sigma}{r+\gamma} G_1[\tau_C(i)]$. This corresponds to case i of Lemma 1, and hence $q_t = [1 - \tau_C(i)]^{1/\eta - 1}$ and $y_t = [1 - \tau_C(i)]^{1/\eta}$. Rearranging terms, we have $\tau_C(i) \in [\eta, \bar{\tau}]$ is equivalent to $i \in [i^*, \hat{\imath}]$, corresponding to part (b) of Proposition 1.

Consider case (c). The premise of case (c) implies that

$$\Pi_1^C = \max_{\tau_t \geq \tau_C(i)} G_1(\tau_t) = G_1[\tau_C(i); i] = G_2[\tau_C(i); i] < \max_{\tau_t \leq \tau_C(i)} G_2(\tau_t; i) = \Pi_2^C.$$

Thus we have $\tau = \bar{\tau}$ and $\Pi^C = \frac{\alpha\sigma}{r+\gamma} G_2(\bar{\tau}; i)$. This corresponds to case i of Lemma 1, and hence $q_t = m[\tau_C(i)]$ and $y_t = (1 - \bar{\tau}) q_t$. Rearranging terms, we have $\tau_C(i) > \bar{\tau}$ is equivalent to $i > \hat{\imath}$, corresponding to part (c) of Proposition 1. ■

C Proof of Proposition 2

Proof. We omit the proof of the comparative statics of γ as they are straightforward. In the case of $i < i^*$, y_t is constant so its comparative statics is always zero. Given the definition of $\tau_C(i)$, we have

$$\frac{\partial \tau_C(i)}{\partial i} > 0, \frac{\partial \tau_C(i)}{\partial \alpha} > 0. \tag{C.1}$$

Given $q_t = [1 - \tau_C(i)]^{1/\eta - 1}$ in the case of $i < i^*$, we have $\frac{\partial q_t}{\partial i} < 0$ and $\frac{\partial q_t}{\partial \alpha} < 0$. Since $G_1(\eta)$ is constant, given $\Pi^C = \frac{\alpha\sigma}{r+\gamma} G_1(\eta)$ we have $\frac{\partial \Pi^C}{\partial i} = 0$ and $\frac{\partial \Pi^C}{\partial \alpha} > 0$.

In the case of $i \in [i^*, \hat{i}]$, notice that the comparative statics of q_t is the same as in the case of $i < i^*$, and y_t is a monotone transformation of q_t, so its comparative static is also negative. Since we have $\eta \leq \tau_C(i)$ in the case of $i \in [i^*, \hat{i}]$ (see the proof of Proposition 1), we have $G_1'[\tau_C(i)] < 0$ as $\tau = \tau_C(i)$ is on the RHS of the single peak $\tau = \eta$ of $G_1(\tau)$. Applying (C.1) on $\Pi^C = \frac{\alpha\sigma}{r+\gamma} G_1[\tau_C(i)]$, we have $\frac{\partial \Pi^C}{\partial i} < 0$ but $\frac{\partial \Pi^C}{\partial \alpha}$ ambiguous.

In the case of $i > \hat{i}$, notice that $\bar{\tau}$ is independent to i. Given

$$q_t = \left(1 + \frac{i}{\sigma}\right)^{-1/\eta} \left[1 - \alpha + \alpha (1 - \bar{\tau})^{1-\eta}\right]^{1/\eta} \Rightarrow \frac{\partial q_t}{\partial i} < 0,$$

$$y_t = (1 - \bar{\tau}) q_t \Rightarrow \frac{\partial y_t}{\partial i} < 0$$

$$\Pi^C = \frac{\alpha\sigma}{r+\gamma} \left(1 + \frac{i}{\sigma}\right)^{-1/\eta} \max_{\tau_t} \left\{\tau_t \left[1 - \alpha + \alpha (1 - \tau_t)^{1-\eta}\right]^{1/\eta}\right\} \Rightarrow \frac{\partial \Pi^C}{\partial i} < 0.$$

Notice that

$$\begin{aligned}
\left.\frac{\partial^2 G_2(\tau; i)}{\partial \alpha \partial \tau}\right|_{\tau=\bar{\tau}} &= G_2(\bar{\tau}; i) \left.\frac{\partial^2}{\partial \alpha \partial \tau} \log G_2(\tau; i)\right|_{\tau=\bar{\tau}} = -\frac{1}{\alpha} \frac{(1-\alpha)(1-\eta)}{\bar{\tau} - \eta} G_2(\bar{\tau}; i), \\
\left.\frac{\partial^2 G_2(\tau; i)}{\partial^2 \tau}\right|_{\tau=\bar{\tau}} &= G_2(\bar{\tau}; i) \left.\frac{\partial^2}{\partial \tau^2} \log G_2(\tau; i)\right|_{\tau=\bar{\tau}} = -\frac{(1-\alpha)(1-\tau) + \eta[\tau - \eta]}{\tau^2 (1-\eta)(1-\alpha)(1-\tau)} G_2(\bar{\tau}; i).
\end{aligned}$$

Thus we have

$$\frac{\partial \bar{\tau}}{\partial \alpha} = -\left.\frac{\partial^2 G_2(\tau; i)}{\partial \alpha \partial \tau}\right|_{\tau=\bar{\tau}} \Big/ \left.\frac{\partial^2 G_2(\tau; i)}{\partial^2 \tau}\right|_{\tau=\bar{\tau}} < 0.$$

Since $\bar{\tau}$ solves $G_2'(\bar{\tau}; i) = 0$, we have

$$\alpha = \left[1 + (1 - \bar{\tau})^{1-\eta} \left(\frac{\bar{\tau}}{\eta} \frac{1-\eta}{1-\bar{\tau}} - 1\right)\right]^{-1}.$$

Substituting the above to q_t, we have

$$\begin{aligned} q_t &= \left(1+\frac{i}{\sigma}\right)^{-1/\eta}\left[\frac{\bar{\tau}-\eta+\eta\left(1-\bar{\tau}\right)^{\eta}}{\bar{\tau}\left(1-\eta\right)}\right]^{-1/\eta}, \\ y_t &= \left(1+\frac{i}{\sigma}\right)^{-1/\eta}\left(1-\bar{\tau}\right)\left[\frac{\bar{\tau}-\eta+\eta\left(1-\bar{\tau}\right)^{\eta}}{\bar{\tau}\left(1-\eta\right)}\right]^{-1/\eta}, \end{aligned}$$

where are non-monotone in $\bar{\tau}$, thus we have $\frac{\partial q_t}{\partial \alpha}$, $\frac{\partial y_t}{\partial \alpha}$ and $\frac{\partial \Pi^C}{\partial \alpha}$ ambiguous. ■

D Proof of Proposition 3

Proof. After substituting ρ_t, the platform problem is a repetition of the following static problem:

$$\Pi^T = -\kappa\alpha + \frac{\sigma\alpha}{r+\gamma} \max_{b_t, k_t, \tau_t \geq 0} \left\{ \begin{array}{l} \left[(1-\tau_t)\left[1-\gamma\left(1-b_t\right)\right]k_t\right]^{1-\eta} \\ -\left[\left(\gamma + \frac{i}{\sigma\alpha}\right)b_t + (1-\gamma)(1-\tau_t) + \gamma\tau_t b_t\right]k_t \end{array} \right\}.$$

The nonnegativity constraint of k_t is never binding, since the profit is zero when $k_t = 0$, which is dominated by any positive k_t. Similarly, it is never optimal to have $k_t = \infty$. Thus, the first order condition with respect to k_t must hold with equality, and hence $y_t = (1-\tau_t)\left[1-\gamma\left(1-b_t\right)\right]k_t$ must be in the interior of $\mathbb{R}_+$. The repeated static problem is reduced to

$$\Pi^T = -\kappa\alpha + \frac{\sigma\alpha}{r+\gamma} \max_{b_t, y_t, \tau_t \geq 0} \left\{ y_t^{1-\eta} - \left[1 + \frac{i}{\sigma\alpha}\frac{1}{1-\tau_t}\frac{b_t}{1-\gamma\left(1-b_t\right)}\right] y_t \right\}.$$

Thus b_t and τ_t solve

$$, \min_{b_t, \tau_t \geq 0} \frac{1}{1-\tau_t}\frac{b_t}{1-\gamma\left(1-b_t\right)},$$

.where the solution is simply $b_t = \tau_t = 0$. Given $b_t = \tau_t = 0$, the first order condition with respect to k implies,

$$y_t = (1-\eta)^{1/\eta}.$$

Substituting the above solution, we have proved Proposition 3. ■

E Proof of Proposition 4

Proof. We omit the proof as it is straightforward from Proposition 3. ■

F Proof of Lemma 2

Proof. Consider the two cases: $y_t > 0$ and $y_t = 0$. In the first case, we have

$$\rho_t' = \sigma\alpha \left\{ (1-\tau_t)\, y_t'^{-\eta} - 1 \right\}_+ .$$

Thus, we have

$$\begin{aligned} y_t' &= (1-\tau_t)^{1/\eta} \left(1 + \frac{\rho_t'}{\sigma\alpha}\right)^{-1/\eta}, \\ m_t &= \left[1 + \frac{i - \rho_t'}{\sigma\,(1-\alpha)}\right]^{-1/\eta}. \end{aligned}$$

The premise $y_t > 0$, i.e., $y_t' > (1-\tau_t)\, m_t$, is satisfied if

$$\rho_t' < \sigma\alpha \frac{(1-\alpha)\left[1 - (1-\tau_t)^{\eta-1}\right] + \frac{i}{\sigma}}{(1-\alpha)\,(1-\tau_t)^{\eta-1} + \alpha}.$$

In the second case, the results are the same as Proposition 1. ■

G Proof of Proposition 5

Proof. Consider two cases, $y_t > 0$ and $y_t = 0$. In the first case, obviously $b_t = 0$ solves (22). Substituting y_t of (24) into (22), the platform problem becomes

$$\Pi^D = -\kappa\alpha + \frac{\sigma\alpha}{r+\gamma} \max_{y_t', \tau_t} \left\{ \begin{array}{l} \left(y_t'^{-\eta} - 1\right) y_t' \\ -\left[(1-\tau_t)\, y_t'^{-\eta} - 1\right] \left[1 + \frac{i - \sigma\alpha\left[(1-\tau_t) y_t'^{-\eta} - 1\right]}{\sigma(1-\alpha)}\right]^{-1/\eta} \end{array} \right\} \text{ s.t. (23).}$$

It is clear that the above profit is always lower than Π^T, thus when the platform allows both cash and token, the profit is always dominated by issuing tokens only. In this second case $y_t = 0$, the profit is exactly equal to Π^C. ■

H Proof of Lemma 3

Proof. Denote $\Delta\Pi(i) \equiv \left(\Pi^C - \Pi^T\right)/\alpha$. Using Proposition 1 and Proposition 3, we have

$$\Delta\Pi(i) = \begin{cases} \kappa \text{ if } i < i^* \\ \kappa + \frac{\sigma}{r+\gamma}\left\{\underbrace{\left[1 - \left[1 + \frac{i}{\sigma(1-\alpha)}\right]^{-1/(1-\eta)}\right]\left[1 + \frac{i}{\sigma(1-\alpha)}\right]^{-1/\eta}}_{G_1[\tau_C(i)]} - \underbrace{\eta(1-\eta)^{1/\eta}}_{G_1(\eta)}\right\} \text{ if } i \in [i^*, \hat{\imath}] \\ \kappa + \frac{\sigma}{r+\gamma}\left[\underbrace{\left(1 + \frac{i}{\sigma}\right)^{-1/\eta}\bar{\tau}\left[1 - \alpha + \alpha(1-\bar{\tau})^{1-\eta}\right]^{1/\eta}}_{G_2(\bar{\tau};i)} - \underbrace{\eta(1-\eta)^{1/\eta}}_{G_1(\eta)}\right] \text{ if } i > \hat{\imath} \end{cases}$$

Notice that $\Delta\Pi(0) > 0$ and $\Delta\Pi(\infty) < 0$. Using Proposition 2, we have $\partial\Delta\Pi(i)/\partial i \leq 0$ (strict inequality when $i > i^*$), so there exist a unique ι^Π such that $\Delta\Pi\left(\iota^\Pi\right) = 0$ and $\Pi^C < \Pi^T$ if and only if $i > \iota^\Pi$. Notice that

$$\frac{\partial\Delta\Pi(i)}{\partial\alpha} = \begin{cases} 0 \text{ if } i < i^* \\ \frac{\partial\Pi^C}{\partial i}\frac{\alpha}{1-\alpha} < 0 \text{ if } i \in [i^*, \hat{\imath}] \\ -\frac{\sigma\bar{\tau}\left(1+\frac{i}{\sigma}\right)^{-1/\eta}}{\eta(r+\gamma)}\left[1 - \alpha + \alpha(1-\bar{\tau})^{1-\eta}\right]^{1/\eta-1}\left[1 - (1-\bar{\tau})^{1-\eta}\right] < 0 \text{ if } i > \hat{\imath} \end{cases}$$

Using $\frac{\partial\iota^\Pi}{\partial\alpha} = -\frac{\partial\Delta\Pi(\iota^\Pi)}{\partial\alpha}\Big/\frac{\partial\Delta\Pi(\iota^\Pi)}{\partial i} \leq 0$ (strict inequality when $i > i^*$), we have proved that ι is decreasing in α. Finally, notice that $G_1(\eta) > G_1[\tau_C(i)] > G_2(\bar{\tau}; i)$, we have $\partial\Delta\Pi(i)/\partial\gamma \geq 0$ (strict inequality when $i > i^*$) and hence $\frac{\partial\iota^\Pi}{\partial\gamma} = -\frac{\partial\Delta\Pi(\iota^\Pi)}{\partial\gamma}\Big/\frac{\partial\Delta\Pi(\iota^\Pi)}{\partial i} \geq 0$. ■

I Proof of Proposition 6

Proof. We consider the three cases in proposition 1. In case (a), a token platform is not adopted. In case (b), $y^C < y^T$, according to propositions 1 to 3. Also, $q^C = q^T = [1 - \tau_C(i)]^{1/\eta - 1}$. In case (c), $y^C < y^T$, according to lemma 1 to 3. Also, $q^C = m(\bar{\tau})$ and $q^T = [1 - \tau_C(i)]^{1/\eta - 1}$ in this case. At $i = \hat{\imath}$, we have $m(\bar{\tau}) = [1 - \tau_C(\hat{\imath})]^{1/\eta - 1}$. For $i > \hat{\imath}$, we have $m(\bar{\tau})$ fixed and $[1 - \tau_C(i)]^{1/\eta - 1}$ declining in i. Therefore, we have $q^C \geq q^T$ in this case. The third statement is obvious given the defination of $\mathcal{S}$ and the result that reserve is $b = 0$. ■

J Proof of Lemma 4

Proof. Define $\Delta\mathcal{W}(i) \equiv \left(\mathcal{W}^C - \mathcal{W}^T\right)/\alpha$. Recall the allocations under the cash and token platforms are given by

	τ	q_t^C	q_t^T	y_t^C
(a). $i < i^*$	η	$[1-\tau_C(i)]^{1/\eta-1}$	$[1-\tau_C(i)]^{1/\eta-1}$	$(1-\eta)^{1/\eta}$
(b). $i \in [i^*, \hat{\imath}]$	$\tau_C(i)$	$[1-\tau_C(i)]^{1/\eta-1}$	$[1-\tau_C(i)]^{1/\eta-1}$	$[1-\tau_C(i)]^{1/\eta}$
(c). $i > \hat{\imath}$	$\bar{\tau}$	$m(\bar{\tau})$	$[1-\tau_C(i)]^{1/\eta-1}$	$(1-\bar{\tau})\, q_t$

	y_t^T	Π^C	Π^T
(a). $i < i^*$	$(1-\eta)^{1/\eta}$	$\frac{\alpha\sigma}{r+\gamma} G_1(\eta)$	$-\frac{\kappa\alpha}{r+\gamma} + \frac{\sigma\alpha\eta}{r+\gamma}(1-\eta)^{1/\eta-1}$
(b). $i \in [i^*, \hat{\imath}]$	$(1-\eta)^{1/\eta}$	$\frac{\alpha\sigma}{r+\gamma} G_1[\tau_C(i)]$	$-\frac{\kappa\alpha}{r+\gamma} + \frac{\sigma\alpha\eta}{r+\gamma}(1-\eta)^{1/\eta-1}$
(c). $i > \hat{\imath}$	$(1-\eta)^{1/\eta}$	$\frac{\alpha\sigma}{r+\gamma} G_2(\bar{\tau}; i)$	$-\frac{\kappa\alpha}{r+\gamma} + \frac{\sigma\alpha\eta}{r+\gamma}(1-\eta)^{1/\eta-1}$

Using Proposition 1 and Proposition 3, we have

$$\Delta\mathcal{W}(i) = \begin{cases} \kappa \text{ if } i < i^*; \\ \kappa + \frac{\sigma}{r+\gamma}\left\{u\left(y^C\right) - y^C - u\left(y^T\right) + y^T\right\} \text{ if } i \in [i^*, \hat{\imath}]; \\ \kappa + \frac{\sigma}{r+\gamma}\left\{u\left(y^C\right) - y^C - u\left(y^T\right) + y^T\right\} \\ \quad + \frac{\sigma}{r+\gamma}(1/\alpha - 1)\left\{u\left(q^C\right) - q_t^C - u\left(q^T\right) + q^T\right\} \text{ if } i > \hat{\imath}. \end{cases}$$

In the case of $i \in [i^*, \hat{\imath}]$, it is straightforward to see that $\frac{\partial\Delta\mathcal{W}(i)}{\partial i} < 0$. In the case of $i > \hat{\imath}$, differentiating the above and using the fact that $y_t^C = (1-\bar{\tau})\, q_t^C$ from Proposition 1, we have

$$\frac{\partial\Delta\mathcal{W}(i)}{\partial i} \begin{cases} \leq 0 \text{ if } i \leq \hat{\imath} \\ = \frac{i}{\alpha(r+\gamma)}\left[\left(1 + \frac{\sigma\alpha\bar{\tau}}{i}\right)\frac{\partial q^C}{\partial i} - \frac{\partial q^T}{\partial i}\right] \text{ if } i > \hat{\imath} \end{cases}$$

For the derivative of i, we have

$$\left(1 + \frac{\sigma\alpha\bar{\tau}}{i}\right)\frac{\partial q^C}{\partial i} - \frac{\partial q^T}{\partial i} = \frac{1}{\eta\sigma(1-\alpha)}\left(1 + \frac{i}{\sigma(1-\alpha)}\right)^{-1/\eta-1} H(i),$$

where

$$H(i) = 1 + \frac{\sigma\alpha\bar{\tau}}{i} - (1-\alpha)^{-1/\eta}\left(1 - \frac{\alpha}{1+\frac{i}{\sigma}}\right)^{1/\eta+1}\left[1 - \alpha + \alpha(1-\bar{\tau})^{1-\eta}\right]^{1/\eta}.$$

The sign of $\frac{\partial\Delta\mathcal{W}_t(i)}{\partial i}$ in the case of $i > \hat{\imath}$ depends on $H(i)$, which is decreasing in i. Notice that

$$H(\hat{\imath}) \equiv \alpha + \frac{\sigma\alpha\bar{\tau}}{\hat{\imath}} > 0, H(\infty) = 1 - \left[1 + \frac{\alpha(1-\bar{\tau})^{1-\eta}}{1-\alpha}\right]^{1/\eta} < 0.$$

Also, $\frac{\partial\Delta\mathcal{W}(i)}{\partial i} = \beta^{-1}\kappa\alpha > 0$ and $\frac{\partial\Delta\mathcal{W}(i)}{\partial i} \propto -\Pi^T < 0$, so there exists a unique $\iota^{\mathcal{W}}$ such that $\left.\frac{\Delta\mathcal{W}(i)}{\partial i}\right|_{i=\iota^{\mathcal{W}}} = 0$ and $\mathcal{W}^T < \mathcal{W}^C$ if and only if $i < \iota^{\mathcal{W}}$. For the derivative of α, using Proposition 1

and Proposition 3 we have

$$\frac{\partial \Delta \mathcal{W}(i)}{\partial \alpha} \leq 0 \text{ if } i \leq \hat{\imath},$$

Thus we have $\frac{\partial \iota^{\mathcal{W}}}{\partial \alpha} \equiv - \frac{\partial \Delta \mathcal{W}(\iota^{\mathcal{W}})}{\partial \alpha} \Big/ \frac{\partial \Delta \mathcal{W}(\iota^{\mathcal{W}})}{\partial i} < 0$ if $i \leq \hat{\imath}$. For the derivative of γ, notice that

$$\begin{aligned} \frac{\partial \Delta \mathcal{W}\left(\iota^{\mathcal{W}}\right)}{\partial \gamma} &= - \left[\begin{array}{c} \frac{\sigma}{(r+\gamma)^2} \left\{ u\left(y^C\right) - y^C - u\left(y^T\right) + y^T \right\} \\ + \frac{\sigma(1/\alpha - 1)}{(r+\gamma)^2} \left\{ u\left(y^C\right) - q^C - u\left(q^T\right) + q^T \right\} \end{array} \Bigg|_{i=\iota^{\mathcal{W}}} \right] \\ &= \frac{\kappa}{r+\gamma} > 0 \end{aligned}$$

Thus we have $\frac{\partial \iota^{\mathcal{W}}}{\partial \gamma} \equiv - \frac{\partial \Delta \mathcal{W}(\iota^{\mathcal{W}})}{\partial \gamma} \Big/ \frac{\partial \Delta \mathcal{W}(\iota^{\mathcal{W}})}{\partial i} > 0$. ■

K Proof of Proposition 9

Proof. By substituting $y_t = (1 - \tau_t)\, k_t$, the repeated static problem is reduced to

$$\max_{y_t, \tau_t \geq 0} \left\{ y_t^{1-\eta} - y_t \right\},$$

which is independent to τ_t. Thus, τ_t is indeterminate, as long as $\rho_t \geq 0$, which implies $\tau_t \in [0, \eta]$. The first order condition with respect to y_t implies

$$y_t = (1 - \eta)^{1/\eta}.$$

Substituting the above solution, we have proved Proposition 9. ■

L Proof of Proposition 7

Proof. The repeated static problem is reduced to

$$\Pi^T = -\kappa\alpha + \frac{\sigma\alpha}{r+\gamma} \max_{b_t \geq \bar{b}, y_t, \tau_t \geq 0} \left\{ y_t^{1-\eta} - \left[1 + \frac{\gamma\tau_t + \frac{i}{\sigma\alpha}}{1-\tau_t} \frac{b_t}{1-\gamma(1-b_t)} \right] y_t \right\},$$

where the solution is simply $b_t = \bar{b}$ and $\tau_t = 0$. Given $\bar{b} = 1$ and $\tau_t = 0$, the first order condition with respect to y_t implies

$$y_t = \left[\frac{1-\eta}{1 + \frac{i}{\sigma\alpha} \frac{\bar{b}}{1-\gamma\left(1-\bar{b}\right)}} \right]^{1/\eta}.$$

Substituting the above solution, we have proved Proposition 7. ■

M Proof of Proposition 8

Proof. Consider the relaxed problem where the incentive compatibility constraint (26) is not binding. Using Proposition 3 the platform solution is given by $b_t = 0$, $k_t = (1-\eta)^{1/\eta} / (1-\gamma)$ and $\Pi^T = -\kappa\alpha + \frac{\sigma\alpha\eta}{r+\gamma}(1-\eta)^{1/\eta-1}$. The incentive compatibility constraint (26) becomes

$$(1+r)\,\kappa\alpha \leq \left[(1-\gamma)\frac{\sigma\alpha\eta}{r+\gamma} - \frac{1}{1-\gamma}\right](1-\eta)^{1/\eta}$$

Thus condition (26) is violated if the first term on the RHS of above inequality is negative, which happens if and only if condition (27) is satisfied. ■

N Proof of Lemma 5

Proof. Consider two different cases: in the first case ($m_t > 0$), we have $i = \sigma(1-\alpha)\left\{(m_t + k_t')^{-\eta} - 1\right\}_+$, and in the second case ($m_t = 0$) we have $i > \sigma(1-\alpha)\left\{(m_t + k_t')^{-\eta} - 1\right\}_+$. In the first case, we have $m_t + k_t' = \{1 + i/[\sigma(1-\alpha)]\}^{-1/\eta}$ and $k_t'^{-\eta} \geq 1 + i/[\sigma(1-\alpha)] = (m_t + k_t')^{-\eta} > 1$. Thus, we have $\rho_t = -\gamma(1-b) + \sigma[1-\gamma(1-b)]\left[\alpha\left(k_t'^{-\eta} - 1\right) + \frac{i}{\sigma}\right]$, and hence

$$k_t' = \left[1 + \frac{1}{\sigma\alpha}\left[\frac{\rho_t + \gamma(1-b)}{1-\gamma(1-b)} - i\right]\right]^{-1/\eta}.$$

The premise $m_t > 0$ is satisfied if

$$m_t + k_t' = \left[1 + \frac{i}{\sigma(1-\alpha)}\right]^{-1/\eta} > k_t' = \left[1 + \frac{1}{\sigma\alpha}\left[\frac{\rho_t + \gamma(1-b)}{1-\gamma(1-b)} - i\right]\right]^{-1/\eta} \Leftrightarrow i \leq (1-\alpha)\frac{\rho_t + \gamma(1-b)}{1-\gamma(1-b)}.$$

In the second case, rearranging terms we have $k_t'^{-\eta} < 1 + i/[\sigma(1-\alpha)]$. Thus, we have $\rho_t = -\gamma(1-b) + \sigma[1-\gamma(1-b)]\left(k_t'^{-\eta} - 1\right)$, and hence

$$k_t' = \left[1 + \frac{1}{\sigma}\frac{\rho_t + \gamma(1-b)}{1-\gamma(1-b)}\right]^{-1/\eta}.$$

■

N.1 Deposit insurance

To deal with the collapse risk, a commonly used regulation is deposit insurance. One can treat the platform as a bank and tokens as deposits. Suppose the central bank requires a token platform to pay a premium at the rate φ if the cyber shock does not hit. When the shock realizes, the insurance will compensate token holders for the loss of the token value. Since the token holders are fully insured against the cyber risk, the nominal price of retail goods in terms of tokens is equal to that in terms of cash, i.e., $P_t^T = P_t = 1/\phi_t$. In this case, it is obvious that the platform never holds reserves ($b_t = 0$), because the loss is fully covered by deposit insurance. The shadow interest rate of tokens becomes

$$\rho_t = \sigma\alpha \left\{ (1-\tau_t)^{1-\eta} k_t^{-\eta} - 1 \right\}_+ .$$

The initial profit flow of the platform in stage 2 is

$$c_0 = p_0 K_1 - \kappa\alpha.$$

The expected profit flow in the latter period is

$$c_t = (1-\gamma) p_t \left[K_{t+1} - K_t - \varphi K_t \right] + \sigma\alpha \frac{\tau_t \phi_t P_t^T y_t}{1-\tau_t}.$$

The platform's profit is given by $\Pi^D = \max \left\{ c_0 + \sum_{t=1}^{\infty} (1-\gamma)^{t-1} \beta^t c_t \right\}$. By collecting terms of the same t across periods, the platform's profit can be expressed as

$$\begin{aligned} \Pi^D &= -\kappa\alpha + \max_{k_t, \tau_t \geq 0} \left\{ \begin{array}{l} p_0 K_1 \\ + \sum_{t=1}^{\infty} (1-\gamma)^{t-1} \beta^t (1-\gamma) \frac{p_t}{p_{t+1}} p_{t+1} K_{t+1} \\ + \sum_{t=1}^{\infty} (1-\gamma)^{t-1} \beta^t \left[-(1-\gamma)(1+\varphi) p_t K_t + \sigma\alpha \frac{\tau_t \phi_t P_t^T y_t}{1-\tau_t} \right] \end{array} \right\} \\ &= -\kappa\alpha + \max_{k_t, \tau_t \geq 0} \left\{ \sum_{t=1}^{\infty} (1-\gamma)^{t-1} \beta^t \left[1 + \rho_t - (1-\gamma)(1+\varphi) + \sigma\alpha\tau_t \right] k_t \right\}, \end{aligned} \tag{N.1}$$

where the platform takes as given the premium rate φ. In equilibrium, the premium rate of the deposit insurance is actuarially fair, such that

$$(1-\gamma)\varphi = \gamma. \tag{N.2}$$

We assume that the insurance premium (and payout after the cyber shock) are rebated to consumers as lump sum transfers.

Proposition 9 *The platform is indifferent to any $\tau_t \in [0, \eta]$. The equilibrium allocation of a token platform is the same, i.e., the deposit insurance does not affect the equilibrium allocation and profits*

of the token platform, i.e.,

$$\begin{aligned}
\rho_t &= \sigma\alpha\frac{\eta-\tau_t}{1-\eta}, \\
q_t &= \left[1-\tau_C\left(i\right)\right]^{1/\eta-1}, \\
y_t &= \left(1-\eta\right)^{1/\eta}, \\
\Pi^D &= -\kappa\alpha+\frac{\sigma\alpha\eta}{r+\gamma}\left(1-\eta\right)^{1/\eta-1}.
\end{aligned}$$

Corollary 4 *The deposit insurance does not affect welfare but slows down the growth rate of the nominal token price g.*

Here is why deposit insurance is neutral in this economy. Recall that the platform can always self-insure against the cyber risk by holding reserves b. Deposit insurance is cheaper than holding reserves because buying deposit insurance does not incur the opportunity cost of nominal interest i. However, this cheaper insurance option is not useful for the platform because the platform chooses to hold zero reserves in the absence of deposit insurance. Instead, the platform compensates the consumers' expected loss from the cyber risk by engineering a bigger token "bubble" (higher token price growth g_t). Thus, the rise of insurance premium is offset one-to-one by the corresponding decrease in g_t, while the profits and allocation stay unchanged.