Cryptocurrency, Imperfect Information, and Fraud

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Introduction

- Cryptocurrency is a new payment system that maintains a public transaction ledger in a distributed manner
- Everyone can have their own version of the ledger. Payers make payments by sending transaction messages to other participants
- In some cryptocurrency such as Bitcoin, record makers or traders are required to solve difficult hash problems to update the ledger, called **proof-of-work (PoW)**
- In some other cryptocurrencies, the updating of the ledger requires some deposits or stakes, called **proof-of-stake (PoS)**
- PoW and PoS generate a loss to the provider but does not directly benefit anyone

Imperfect Network: Consensus Algorithm

- If the message sending is perfect, then the message sending system itself can serve as a perfect settlement system (Yap island stone money)
- Why do we need PoW and PoS in cryptocurrency?
- In cryptocurrency, the messages are sent through the internet, which is an imperfect message sending system (missing, delay, and error)
- Through the imperfect system, participants may not receive messages as the order they were sent (disagreement)
- Consensus algorithms are applied to create agreements

Imperfect Network: Double Spending

- This imperfection also provide traders incentives to disrupt the consensus system and take advantage by sending inconsist messages
- The double spending fraud:
  - an attacker initially sends a message to make a payment to a merchant, receive goods, and then sends another message (double-spending message) to transfer her balance to another account owned by herself (or another merchant)
  - if the double spending message instead of the original message is recognized as the real one by the consensus system, the merchant will not receive the payment

Literature

- Computer science literature: Bitcoin protocol satisfies consistency if the computing power owned by adversary players is less than 50% (Pass, Seeman, shelaty 2016, Garay, Kiayias, Leonardosy 2017)
- The thing missing in computer science literature: rationality of players
- Literature in monetary search: Chiu and Koeppl 2017
- Counterfeiting/fraud: Wallace and Nosal 2007, Rocheteau, Li, Weill 2012

Main Results

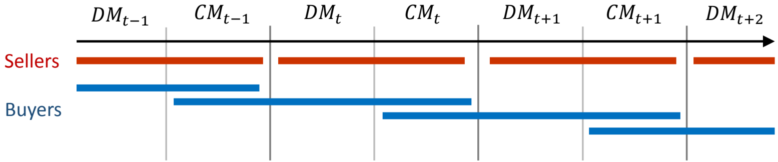
- We study the relationship between PoW, PoS, and the imperfectness of message sending
- PoW and PoS can deterring double spending and may improve the efficiency of cryptocurrency
- PoW and PoS are costly. Imposing a high PoW or PoS to deter double spending may not be optimal in some circumstance
- When the network imperfectness diminishes, cryptocurrency can serve as an efficient means of payment

The Model

- Lagos and Wright (2005), Rocheteau and Wright (2005)
- Two types of agents: buyers and sellers
- $t = 0, 1, 2, 3, \ldots$. Each period has two subperiods:
  - DM: a buyer and a seller meet bilaterally (trade stage)
  - CM: centralized market (settlement stage)

The Model

- Agents are short-lived in the economy
- Buyers enters at the CM, consumes at the next DM and leaves at the next CM
$$X_t + \beta \left[u_t(x_{t+1}) + X_{t+1} \right]$$
- Sellers enters at the DM, produce at the DM and leaves at the CM
$$-l_t + H_t$$
- Buyers and sellers can produce and consume at the CM

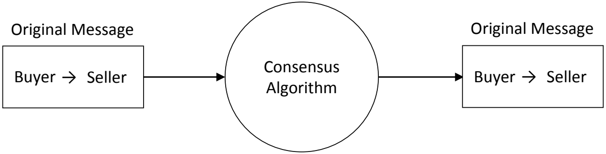


Cryptocurrency System

- No physical assets. No commitment. Traders are anonymous. No credit
- There is a cryptocurrency system
  - A set of digital addresses
  - A consensus algorithm
- Agents can create accounts on the addresses freely. They make payment between accounts by sending transaction messages
- We do not model details about consensus formations or blockchains and miners

Consensus Algorithm

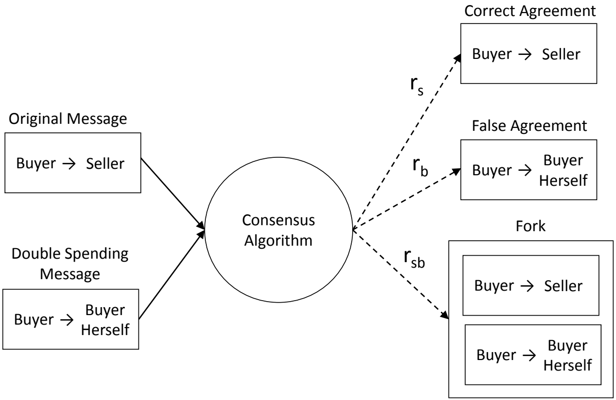
- We abstract the consensus algorithm as an imperfect message sending system
- People send messages to all others through the system. The outcome of the system is observable by all others (agreement)
- If the buyer sends only one transaction message (the original message), the message will be included in the consensus outcome (recognized by the system) for sure



▸ Appendix

Consensus Algorithm

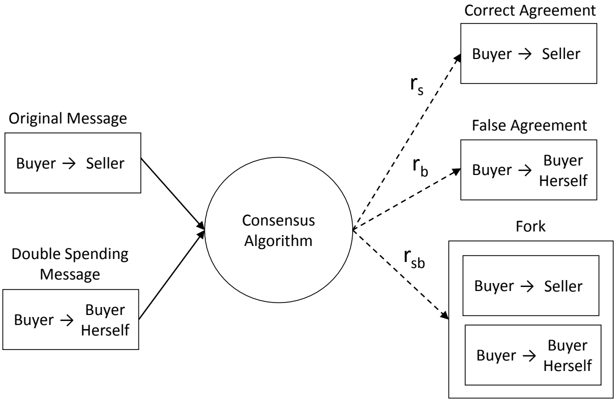
- The buyer can send a double spending message after the transaction to transfer the balance to another account owned by her
- Three mutually exclusive consensus outcomes may occur:



▸ Consensus

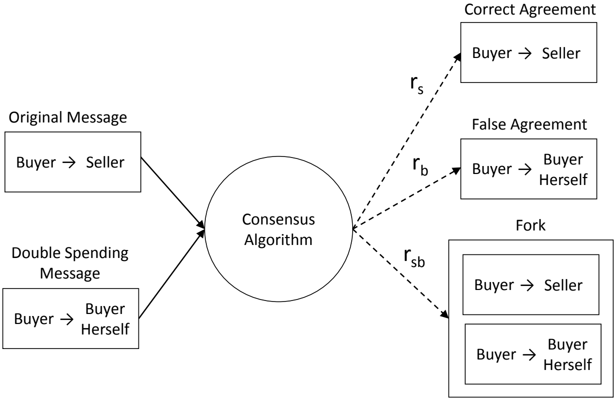
Consensus Algorithm

- (r_s, r_b, r_{sb}) is exogenously determined (by the development of the network)
- Assumption 1: $r_s > r_b$
- Assumption 2: $r_s + r_b + r_{sb} = 1$



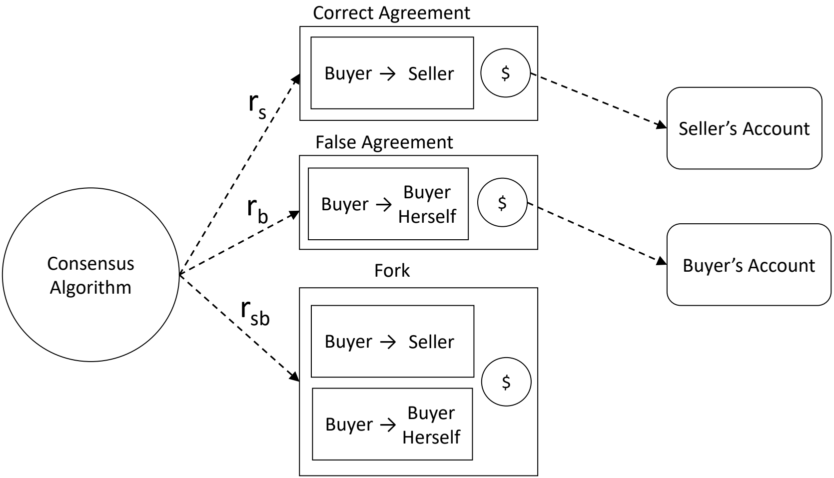
Consensus Algorithm

- Agents cannot distinguish an original message from a double spending messag, so they cannot tell whether the outcome is a correct agreement or a false agreement.
- They can only distinguish a single outcome from a fork
- Thus, forks can be applied as signals to detect double spending

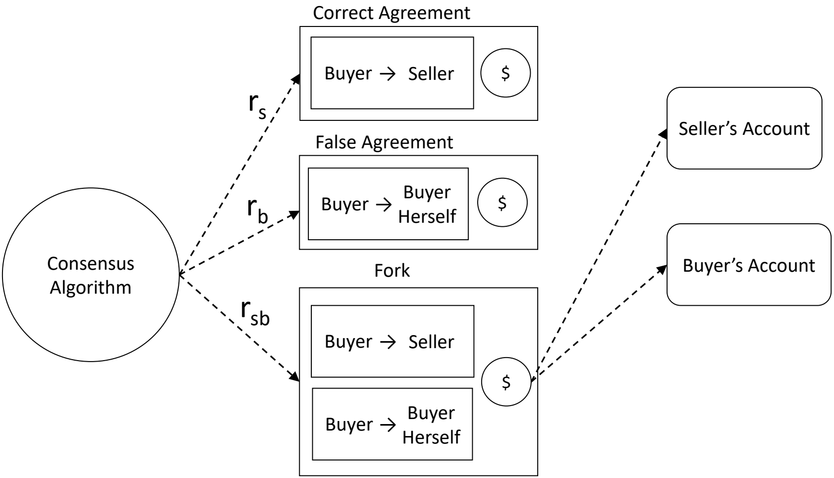


- Message sending is frictionless at the CM

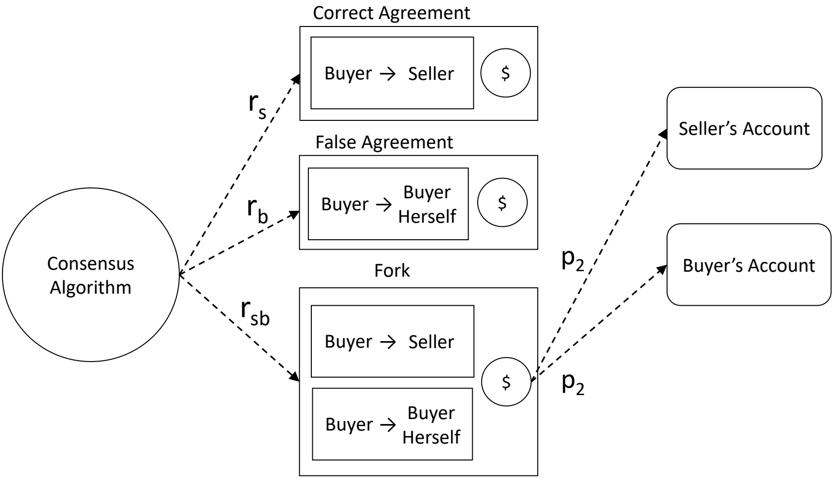
Consensus Algorithm



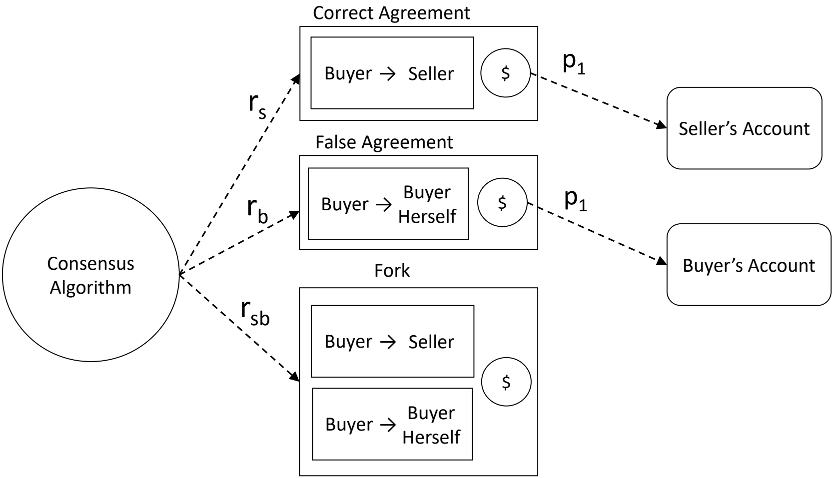
Consensus Algorithm



Consensus Algorithm



Consensus Algorithm



Preventing Double Spending

- If sending messages is costless, double spending will be a dominant strategy
- **PoW**: sending a message costs the payer k units of disutility
- **PoS**: the payer is required to put δ units of balance as deposits
- The return of the deposit can also be conditional on the consensus outcome, denoted by (q_1, q_2)

Preventing Double Spending

- $(k, \delta, p_i, q_i, \pi, \tau)$ is the mechanism
- The inflation rate: π
- Transaction subsidy: τ
- Let $\hat{z} \equiv z + \tau$ be the post-subsidy payment
- The transfer, $\hat{z}$, is observable in a message, so (p_i, q_i) can also depend on, $\hat{z}$ (Hu-Kennan-Wallace mechanism)
- Our goal is to solve for the optimal mechanism given the environment (r_s, r_b, r_{sb})

The Timeline

- We first analyze the trading game given the environment (r_s, r_b, r_{sb}) and the mechanism $(k, \delta, p_i, q_i, \tau, \pi)$

1. CM: Buyer purchases balance
2. DM: Three substages
   a. Offer stage: buyer provides a TIOLI offer $(\hat{z}, x)$ to the seller
   b. Response stage: Seller decides to accept or reject the offer
   c. Post-trade stage: buyer decides to double spend or not

Post-Trade Strategy

- Buyer's post-trade strategy (σ): the probability that the buyer is honest is determined by the cost and benefit of double spending

$$\sigma \in B(\hat{z}) = \begin{cases} 1 & \text{if } \theta_d^b \hat{z} < k + (\eta_h - \eta_d)\delta \\ [0,1] & \text{if } \theta_d^b \hat{z} = k + (\eta_h - \eta_d)\delta \\ 0 & \text{if } \theta_d^b \hat{z} > k + (\eta_h - \eta_d)\delta \end{cases}$$

where

θ_d^b : Pr (buyer receives the payment| buyer double spends)

η_d : Pr (buyer receives the deposit return| buyer double spends)

η_h : Pr (buyer receives the deposit return| buyer is honest)

$$\begin{aligned} \theta_d^b &= p_1 r_b + p_2 r_{sb} \\ \eta_d &= q_1 \\ \eta_d &= q_1 (r_s + r_b) + q_2 r_{sb} \end{aligned}$$

TIOLI Offer

- Under a TIOLI offer, the equilibrium DM production x^* must be equal to the seller's expected payoff

$$x^* = \tilde{x}(\hat{z}^*, \sigma^*) \equiv [\sigma^* \theta_h^s \hat{z}^* + (1 - \sigma^*)\theta_d^s \hat{z}^*]$$

where

θ_h^s : Pr (seller receives the payment| buyer is honest)

θ_d^s : Pr (seller receives the payment| buyer double spends)

$$\begin{aligned} \theta_h^s &= p_1 \\ \theta_d^s &= p_1 r_s + p_2 r_{sb} \end{aligned}$$

Pareto optimal SPE

- We consider the Pareto optimal SPE of the sequential game
- The equilibrium strategy $\left(\hat{z}^*, \sigma^*\right)$ maximizes the buyer's expected value at the CM

$$\left(\hat{z}^*, \sigma^*\right) = \arg\max_{\hat{z}, \sigma \in B(\hat{z})} \bar{V}(\hat{z}, \sigma) \qquad \text{(IC)}$$

where

$$\bar{V}(\hat{z}, \sigma) = \left\{ \begin{array}{c} -(1+\pi)\left(\hat{z} - \tau + \delta\right) \\ +\beta\left\{u\left[\tilde{x}(\hat{z}, \sigma)\right] - k + \varphi(\hat{z}, \sigma)\right\} \end{array} \right\},$$

  - $\varphi(\hat{z}, \sigma)$ is the post-trade gain

$$\varphi(\hat{z}, \sigma) = \sigma\left[\eta_h \delta\right] + (1-\sigma)\left[\theta_d^b \hat{z} + \eta_d \delta - k\right]$$

Money Market Clearing

- CM money market clearing condition

$$\underbrace{\left\{ \begin{array}{c} \left[\sigma\theta_h^s\hat{z} + (1-\sigma)\theta_d^s\hat{z}\right] \\ + \left[\sigma\eta_h\delta + (1-\sigma)\left(\theta_d^b\hat{z} + \eta_d\delta\right)\right] \end{array} \right\}}_{\text{CM money supply}} = \underbrace{(1+\pi)(\hat{z} - \tau + \delta)}_{\text{CM money demand}} \tag{MM}$$

- The CM money supply is equal to the aggregate balance holding at the end of DM (including buyers' and sellers' balance)

Money Market Clearing

$$\underbrace{\left\{\begin{array}{c}[\sigma\theta_h^s\hat{z}+(1-\sigma)\theta_d^s\hat{z}] \\ +\left[\sigma\eta_h\delta+(1-\sigma)\left(\theta_d^b\hat{z}+\eta_d\delta\right)\right]\end{array}\right\}}_{\text{CM money supply}} = \underbrace{(1+\pi)\left(\hat{z}-\tau+\delta\right)}_{\text{CM money demand}} \tag{MM}$$

- Double spending increases the buyer's balance
  1. crowds out the seller's balance holding
  2. increases the aggregate balance ⇒ increases inflation rate or increase the transaction fee ⇒ increases the cost of trade
- Only the balance received by seller can facilitate transactions, but balance received by buyer cannot, so double spending generates inefficiency to cryptocurrency

Stationary Equilibrium

- The participation constraint (IR) for the buyer in CM:

$$\bar{V}(\hat{z}^*, \sigma^*) \geq 0 \quad \text{(IR)}$$

Definition

Given (r_s, r_b, r_{sb}), a stationary equilibrium is a mechanism $(k, \delta, p_i, q_i, \tau, \pi)$, and a strategy $(\hat{z}^*, \sigma^*)$ such that $\frac{1+\pi}{\beta} \geq 1$ and

1. Buyers and sellers are rational: (IC)
2. CM money market clears: (MM)
3. The participation constraint holds: (IR)

Optimal Mechanism

- Given the environment (r_s, r_b, r_{sb}), we solve for the optimal mechanism $(k, \delta, p_i, q_i, \tau, \pi)$ that maximizes the social welfare
- We select two candidates for the optimal mechanism: a **simple honest mechanism** and a **simple double spending mechanism**
- We show that an equilibrium is either dominated by an equilibrium generated by a simple honest mechanism or a simple double spending mechanism
- It is sufficient to solve for the optimal mechanism from the two sets of mechanisms

Simple Mechanisms

1. In a **simple honest mechanism**, we apply PoW and PoS to deter double spending
   - We set $p_2 = q_2 = 0$: payments and deposits are forfeited as off-equilibrium punishment when forks occur $\Rightarrow$ diminishes the gain from double spending
2. In a **simple double spending mechanism**, neither PoW nor PoS is imposed, so buyers will double spend
   - We set $p_2 = 0$: receivers only receive payments in single outcomes but not forks, because sellers has an advantage over buyers in single outcomes ($r_s > r_b$)

▸ mechanisms

Optimal Simple Honest Equilibrium: Pure PoW

- We maximize the social welfare subject to the participation constraint (IR)

$$\max_{x,k} \quad u(x) - x - k$$

$$\text{subject to} \quad \begin{cases} -x + \beta\left[-k + \beta u(x)\right] \geq 0 & \text{(IR)} \\ k = r_b x \end{cases}$$

- Given the trade volume x, the required size of PoW to deter double spending, k, is determined by r_b
- The welfare of PoW equilibrium is determined by r_b
- When $r_b \to 0$, the welfare approaches to efficient level

Optimal Simple Honest Equilibrium: Pure PoS

$$\max_{x,\delta} \quad u(x) - x$$

$$\text{subject to} \quad \begin{cases} -[x+\delta] + \beta[u(x)+\delta] \geq 0 \quad \text{(IR)} \\ r_{sb}\delta = r_b x \end{cases}$$

- Difference between PoS and PoW
  1. PoS does not generate a direct loss to social welfare
  2. PoS applies forks to trigger punishments
- Given the trade volume, how much PoS is needed to deter double spending is determined by $\frac{r_b}{r_{sb}}$
- Given r_b, if r_{sb} is higher, PoS has more advantage over PoW and vice versa

Optimal Simple Honest Equilibrium: PoW and PoS

$$\max_{x,k,\delta} \quad u(x) - x - k \tag{1}$$

$$\text{subject to} \quad \begin{cases} -(x+\delta) + \beta\{u(x) + \delta - k\} \geq 0 & \text{(IR')} \\ k + r_{sb}\delta = r_b x \end{cases}.$$

- We can consider both PoW and PoS into the mechanism, then the trade volume x, can be supported by PoW and PoS all together.
- There is a region in which the optimal simple honest mechanism requires both PoS and PoW

Optimal Simple Double Spending Equilibrium

$$\max_{\hat{z}} \quad u(r_s\hat{z}) - r_s\hat{z}$$

$$\text{subject to} \quad -(r_s + r_b)\hat{z} + \beta\{u(r_s\hat{z}) + r_b\hat{z}\} \geq 0 \quad \text{(IR')}$$

- When the buyer makes $\hat{z}$ unit of payment, the seller only receives $r_s\hat{z}$ units, and the buyer receives $r_b\hat{z}$ units
- The efficiency of the payment system is determined by $\frac{r_b}{r_s}$
- We compare the simple double spending equilibrium and simple honest equilibrium
  - Fixed an r_b, if r_{sb} is high, double spending can be detected more easily, so simple honest mechanism will dominate simple double spending mechanism
  - If r_{sb} is lower, then r_s must be higher, so simple double spending eq will dominate simple honest mechanism

Conclusion

- We construct a model of cryptocurrency in which the main friction is the imperfect information transmission
- The model captures the following:
  - PoW and PoS emerges endogenously to improve efficiency
  - Tradeoff between safety and the cost of trade
  - The required PoW or PoS diminishes as message sending becomes perfect
- Literature: counterfeiting of fiat money (Wallace and Nosal 2007, Rocheteau, Li, Weill 2012)
- This paper: counterfeiting of transaction messages in cryptocurrency
- Coming soon: counterfeiting of transaction accounts in digital payment systems

Simple Honest Mechanism

- Simple honest mechanism M^h :

1. $p_2(\hat{z}) = q_2(\hat{z}) = 0$: **off-equilibrium punishment.** Minimize the gain from double spending and the required size of k and δ
2. $p_1(\hat{z})$ and $q_1(\hat{z})$ are set to be indicator functions, and that is,

$$\mathbb{1}_y(\hat{z}) = \begin{cases} 1 \text{ if } \hat{z} = y \\ 0 \text{ otherwise} \end{cases}, \text{ for some } y > 0,$$

   Punish deviations. If the payment deviates y, the receiver will not receive the payment
3. (k, δ) satisfies $\theta_d^b(y)y = k + [\eta_h(y) - \eta_d(y)]\,\delta$: PoW and PoS are sufficiently high and just enough to prevent double spending fraud

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Simple Double Spending Mechanism

- **Simple double spending mechanism**:

1. $k = 0, \delta = 0$: the buyer must double spend
2. $p_1(\hat{z})$ is set to be indicator functions

$$\mathbb{1}_y(\hat{z}) = \begin{cases} 1 \text{ if } \hat{z} = y \\ 0 \text{ otherwise} \end{cases}, \text{ for some } y > 0,$$

3. $p_2(\hat{z}) = 0$: Eliminate payments in forks
   - **Not for off-equilibrium punishment** because forks are not off-equilibrium outcomes
   - Because $r_s > r_b$, a single outcome can be a better signal to identify the seller than a fork

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Optimal Simple Honest Equilibrium: PoW

$$\max_{y} \quad u(y) - y - r_b y$$

$$\text{subject to} \quad -y + \beta\left[-r_b y + \beta u(y)\right] \geq 0 \quad \text{(IR)}$$

- The welfare of PoW equilibrium is determined by r_b

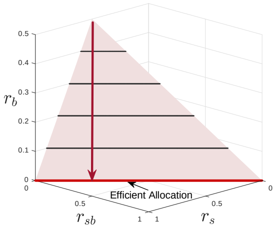


Figure:

Optimal Simple Honest Equilibrium: PoS

$$\max_{y} \quad u(y) - y$$

$$\text{subject to} \quad -\left[y + \frac{r_b}{r_{sb}} y\right] + \beta\left[u(y) + \frac{r_b}{r_{sb}} y\right] \geq 0 \quad \text{(IR)}$$

- The welfare of PoS equilibrium is determined by $\frac{r_b}{r_{sb}}$

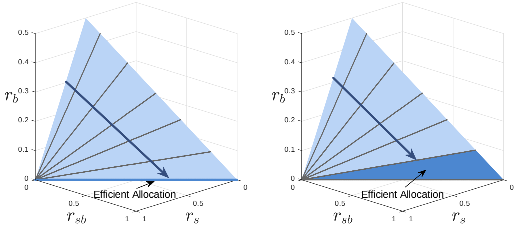


Figure:

Optimal Simple Honest Equilibrium: PoW and PoS

- Given r_b, if r_{sb} is higher, PoS has more advantage over PoW

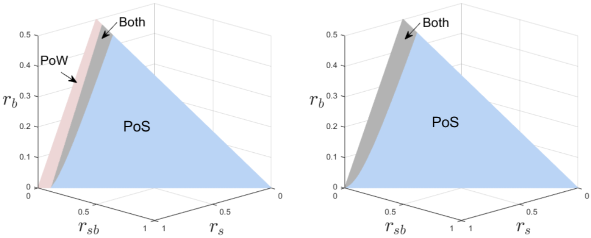


Figure:

Optimal Simple Double Spending Equilibrium

$$\max_{y} \quad u\left(r_s y\right) - r_s y$$
$$\text{subject to} \quad -(r_s + r_b)y + \beta\left\{u\left(r_s y\right) + r_b y\right\} \geq 0 \quad \text{(IR)} \quad .$$

- The ratio $\frac{r_b}{r_s}$ determines the efficiency of cryptocurrency in optimal simple double spending equilibrium

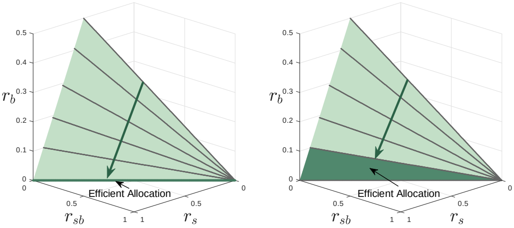


Figure:

Honest Equilibrium vs Double Spending Equilibrium

- Fixed an r_b, when r_{sb} is high, double spending can be detected easily, so simple honest mechanism of preventing the optimal simple honest equilibrium will dominate the optimal simple double spending equilibrium

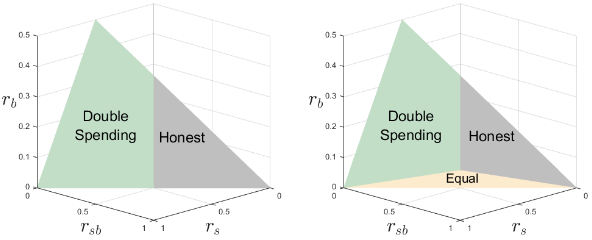
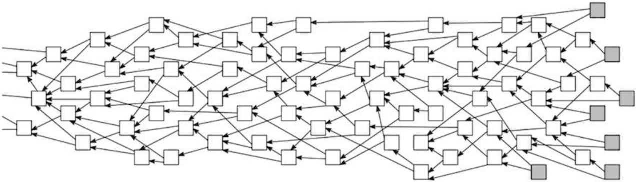


Figure:

Consensus Algorithm

- Alternative public ledger structures:
  - Iota (DAG public ledger, No miners, traders do PoW by themselves)



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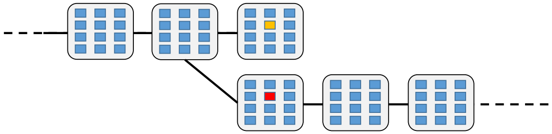
Double Spending

1. In Bitcoin, if the branch including the double spending becomes the longer branch, the payer takes the payment back



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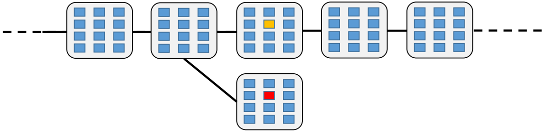
Double Spending

2. If the branch including the original message is the longer branch, then the payment is still received by the merchant



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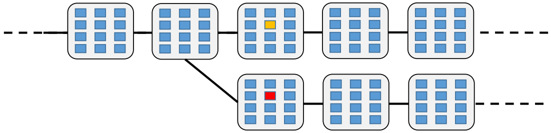
Double Spending

3. Two branches may coexist: a fork



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