Central Bank Digital Currency and Flight to Safety

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1 Introduction

Continuing advances in payments technologies have created a pressing need for central bankers to understand the implications for monetary policy, payments efficiency, and financial stability of the introduction of central bank digital currency (CBDC). Should a CBDC work as widely-accessible central bank reserve accounts, be traded using blockchain technologies, as with cryptocurrencies, or should CBDC function in some other fashion? Should CBDC replace old-fashioned physical currency, or should it coexist with physical currency, perhaps alongside physical currency in a modified form? Should CBDC be designed to avoid competition with private means of payment, or are there potential efficiency gains from moving electronic payments from private banks to central banks? These are only some of the interesting questions that need answers in this area of research.

This paper will be focused specifically on the implications of CBDC issue for financial stability. A model of banking and banking panics is developed which has some new features. These new features are designed to highlight how central bank action to replace physical currency with CBDC will affect the incidence of banking panics and their effects on economic welfare. We also want to show how monetary policy, in the form of conventional interest rate policy and central bank crisis intervention, matters in the context of physical currency and CBDC regimes.

What has evolved as the typical – though not universal – structure for central banks includes three key features: (i) a monopoly on the issue of physical currency; (ii) a constraint that the central bank's activities are restricted to swaps of its liabilities for other assets – principally the debt of the central government;

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(iii) a lender-of-last-resort function under which the central bank stands ready to lend to financial institutions. Thus, a typical central bank is in the retail payments business, transforming assets not used as means of payment into assets that are, and it may intervene in ways intended to mitigate dysfunction in the payments system, or in the financial system as a whole. Many central banks, since at least 2008, have been expanding the scope of their mandates, but we will not deal directly with issues related to unconventional monetary policy here.

Secular changes in the relative usage of means of payment issued by the central bank – physical currency – and privately supplied electronic means of payment have been well documented. For example, in Canada and the United States, surveys indicate that the use of currency has been declining relative to alternative means of payment, at least in legal, reported transactions. However, in most countries of the world, the ratio of currency to GDP has been increasing, as it has in Canada and the United States, potentially driven by an increase in the use of currency in the underground economy, to an increase in the size of the underground economy, or both (see, for example Williamson 2020).

The long-run changes in the demand for physical currency relative to other means of payment are of critical importance with regard to the potential introduction of CBDC. However, of greater importance for this paper is the short-run substitution from private means of payment to central bank means of payment that results from a change in perceptions about the stability of private financial institutions. Banking panics were, for example, an important feature of 19th century banking in the U.K. and in the United States in the late 19th century and the Great Depression. An idea enshrined in the Federal Reserve Act of 1913 in the United States is that the stock of physical currency supplied by the central bank should respond elastically to short run changes in demand for means of payment, which could be driven at times by banking panics. Following an approach invented by the Bank of England in the 19th century (see Bagehot 1873), a flight to safety – from the payments liabilities of other banks to central bank liabilities, could finance central bank crisis lending. This lending to illiquid but solvent banks – which presumably the central bank would be in a good position to distinguish from insolvent banks – could then replace the outflow of funding that otherwise sound banks were experiencing.

So, in terms of the traditional functions of central banking, the fact that physical currency supplied by the central bank could be a safe harbor need not be viewed as a bad thing. The supply of central bank currency expands during the panic, mitigating the disruption in retail payments, and this expansion in payments instruments finances central bank crisis lending that mitigates disruption in private financial intermediation. But, concern has been raised that central bank liabilities that are too attractive could engender financial instability. That is, in providing a safe harbor, the central bank could encourage financial instability by making it easy for the holders of private financial intermediary liabilities to flee at the first sign of trouble.

An example of concern over flight to attractive central bank liabilities relates to the Fed's reverse repurchase agreement (ON-RRP) facility, introduced in conjunction with the Fed's "normalization" policy beginning in 2015. The intention of the ON-RRP facility was to expand the reach of Fed liabilities, permitting better control of overnight interest rates. But, concern was raised about flight-to-safety, for example in Frost et al. (2015). That is, an ON-RRP facility, paying a high interest rate on overnight balances at the Fed, could potentially provide a too-attractive safe harbor for financial market participants, relative to private sector alternatives, or so it was argued. Alternatively, Greenwood et al. (2016) make an argument that a large central bank balance sheet, coupled with an active ON-RRP facility, could promote efficiency.

So, views about flight to safety apparently depend on the nature of the central bank liabilities that market participants might flee to. Currency might be seen as a limited vehicle for flight, as it is not useful in large transactions and in wholesale transactions. Currency can be withdrawn and used in small transactions, but otherwise can only be hoarded as a store of value. Further, currency is costly to store and move around, which limits its usefulness as a safe asset in a financial crisis. However, overnight reverse repos held with the Fed are different. Such assets are a safe and liquid alternative to private assets such as money market mutual funds, and are one step away from reserves, which are used in interbank transactions.

Given this logic, it is not surprising that, with rising interest among central bankers in CBDC, that there would be concern about how CBDC could lead to flight-to-safety and financial instability. At the extreme, Kumhof and Noone (2019) have argued that CBDC, if issued by central banks, should have features that make it inconvenient for its users. Private bank deposits and central bank reserves, as they argue, should not be freely convertible into CBDC. In this view, a concern about states of the world when CBDC could exacerbate banking panics leads to policies that throw sand in the gears of the financial system in all states of the world.

To explore these issues, we construct a model of payments supported by central bank liabilities and private bank deposits, in which banking panics can occur. Banks in the model perform two roles. First, banks issue liabilities that are acceptable as means of payment because banks have the incentive, at least in some states of the world, to ultimately redeem these liabilities, in spite of limited commitment. If a bank does not default on its liabilities this is because the bank has too much to lose from default. That is, the acceptability of its means of payment is supported by the bank's franchise value. Second, banks provide insurance against the event that a bank depositor needs central bank liabilities to execute a transaction, in a manner related to Diamond and Dybvig (1983), or to a particular adaptation of the Diamond-Dybvig insurance role for banking to Lagos and Wright (2005) environments as in Williamson (2012, 2016, 2018, 2019, 2020).

Banking panics, if they occur in the model, are driven by underlying beliefs. In particular, given how the acceptability of a bank's deposit liabilities in payment is supported, if all agents anticipate that a bank will not redeem liabilities issued in the future, then this implies the bank will not redeem liabilities issued today, as its future franchise value is zero. We suppose there is an aggregate sunspot variable that is observed prior to when default occurs, but after the bank has accepted deposit inflows during the current period. On observing a bad sunspot outcome, depositors know that a fraction of banks will fail – essentially, those banks are insolvent. But individual depositors do not know the fate of their own bank. Each depositor then makes a calculation as to whether to withdraw central bank liabilities, given what they know about the structure of the bank's assets. Depositors who need central bank liabilities to execute transactions will always withdraw, but those who can use private bank deposits in transactions may or may not withdraw. If all depositors request withdrawal, that is defined to be a banking panic. A panic can disrupt transactions, depending on the nature of central bank liabilities. For example, physical currency may not be accepted in the transactions that a depositor wants to make. So, a banking panic can disrupt retail transactions, because central bank liabilities can only be held temporarily as a store of value, for some depositors, and potentially because central bank liabilities are spread across a larger group of depositors making withdrawals during a panic. A feature this banking panic model has – which we view to be desirable – as that panics do not depend on sequential service, as in the Diamond-Dybvig (1983) model.

The approach taken here is to consider two alternative regimes, one in which the central bank issues physical currency, and one in which the central bank substitutes CBDC for physical currency. In the model, the differences between CBDC and physical currency are that the central bank can pay interest on CBDC, and not on physical currency, and CBDC is designed to be accepted in a wider array of transactions than is physical currency. Paying interest on CBDC potentially matters for the effects of conventional monetary policy on the incidence of banking panics and on economic welfare. The fact that CBDC is potentially more widely-used in transactions cuts two ways, if we take seriously the concerns voiced about flight to safety. That is, we might view CBDC as contributing to the efficiency of the payments system, but perhaps not if this leads to greater financial instability.

In a regime with physical currency, banking panic equilibria, in which panics occur on occasion (driven by sunspots) tend to exist when the nominal interest rate on government debt, supported by open market operations, is low. But conventional monetary policy can then eliminate banking panics, by maintaining a sufficiently high nominal interest rate. This need not be welfare maximizing, however. Banking panics are disruptive, in that retail transactions are foregone, and the full debt capacity of solvent banks is not used in support of transactions. But a higher nominal interest rate causes inefficiencies, reflected in a lower volume of transactions in all states of the world. It is shown that there are regions of the parameter space in which: (i) banking panic equilibria do not exist; (ii) banking panic equilibria exist at low nominal interest rates, but welfare is higher if monetary policy eliminates panics; (iii) banking panic equilibria exist at low nominal interest rates, and welfare would be lower if panics were eliminated. Banking panic equilibria may not exist because they violate feasibility. That is, the possibility of banking panics creates a high demand for safe assets, through a self-insurance effect – in anticipation that a panic can occur, banks hold a high quantity of liquid assets. If the supply of safe assets, determined by fiscal policy, is sufficiently low, then this may constrain monetary policy in a way that eliminates panics.

We then examine two alternative approaches to central bank crisis intervention. First, suppose that there is a central bank facility under which loans are advanced to solvent banks in a banking panic. That is, the central bank has superior information relative to depositors concerning which banks will fail, given that a bad sunspot is observed. The central bank lends only to banks that will repay loans, and these loans help finance withdrawals of currency in a banking panic. If monetary policy is conducted optimally, this crisis loan facility does nothing to expand the options of the central bank. But, by making solvent banks more liquid during a banking panic, the facility acts to encourage panics. There can be instances in which banking panic equilibria and equilibria with no panics coexist.

Second, we consider the possibility of emergency open market purchases during a banking panic. It has been argued, for example by Goodfriend and King (1988), that open market intervention during a banking crisis is superior to targeted central bank lending. In this case, we assume a central bank facility under which, during a banking panic, the central bank will swap currency for government debt at a specified rate. Private banks then accumulate government debt, anticipating that it will be useful in a panic. Perhaps surprisingly, this does not expand possibilities for the central bank, given that it conducts conventional policy optimally. But an open market crisis facility does even more, in a well defined sense, than central bank crisis lending to promote panics. Given the existence of the open market crisis facility, inferior banking panic equilibria can coexist with superior equilibria without panics, given conventional monetary policy.

Given how CBDC is modeled in this environment, it can be used in transactions in which physical currency would otherwise be used, and in transactions in which bank deposits are used. If CBDC is too attractive, that is if the interest rate on CBDC is sufficiently high, then it will drive deposit-taking banks out of the means-of-payment market altogether. Here we assume that, perhaps because of unspecified costs of CBDC issue, that CBDC is issued under conditions such that, in non-panic states of the world, bank deposits are used in some transactions.

With CBDC, in qualitative terms banking panic equilibria and equilibria with no panics have similar features to what happens in the physical currency regime. The key difference here is that existence of these equilibria depends on the difference between the interest rate on government debt and the interest rate on CBDC, rather than on the difference between the interest rate on government debt and zero-nominal-interest currency. Banking panic equilibria exist when the interest rate differential is low, and equilibria without panics exist when the interest rate differential is high. Further, CBDC tends to narrow the set of interest rate differentials where panics do not exist, and expand the set where they do.

But, the reason that CBDC encourages panics is that CBDC is less disruptive of retail payments when a panic occurs – those economic agents typically using bank deposits in exchange will withdraw CBDC and use it in transactions in a banking panic. So there is a greater tendency with CBDC for banking panic equilibria to dominate equilibria without panics, when the central bank is conducting conventional policy optimally.

As well, once a central bank crisis lending facility or an open market crisis facility are introduced, these do even more to encourage banking panics. In fact, with an open market crisis facility, if an equilibrium with no panics exists, then so does an equilibrium with banking panics. Further, the baking panic equilibrium typically dominates the equilibrium with no panics, in welfare terms. So, in contrast to what happens in typical Diamond-Dybvig (1983) models, in which banking panics are associated with inferior outcomes, here a banking panic in a CBDC regime can be associated with superior outcomes.

The nature of the banking panics that happen in this model is somewhat related to what occurs in models studied by Gertler and Kiyotaki (2015), Gertler, Kiyotaki, and Prestipino (2019), Gu et al. (2019), Andolfatto, Berentsen, and Martin (2019), and Robatto (2019). In contrast to Diamond and Dybvig (1983), as noted above, banking panics are unrelated to sequential service. Other work that explores the implications of CBDC issue includes Andolfatto (2018), Bech and Garratt (2017), Chiu et al. (2019), Davoodalhosseini (2018), Hendry and Zhu (2019), Keister and Sanches (2018), Brunnermeier and Niepelt (2019), and Williamson (2020). Fernadez-Villaverde et al. (2020) study banking panics in the context of CBDC issue, but their environment is more closely related to baseline Diamond-Dybvig (1983) constructs.

The remainder of the paper is organized as follows. In Section 2, the model is laid out. Then, in Sections 3 and 4, two regimes are considered, one where the central bank issues physical currency and no CBDC, and one where the central bank issues CBDC and no physical currency, respectively. The final section is a conclusion.

2 Model

This model builds on a Lagos and Wright (2005) or Rocheteau and Wright (2005) framework. Time is indexed by $t = 0, 1, 2, 3, ...$, and there are two sub-periods in each period, the CM (centralized market) followed by the DM (decentralized market). There are three types of infinite-lived agents, denoted buyers, sellers, and banks. There is a continuum of buyers with unit mass, and each buyer maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t \left[-H_t + u(x_t) \right], \tag{1}$$

where $0 < \beta < 1$, H_t denotes labor supply in the CM, and x_t is consumption in the DM. Assume that $u(\cdot)$ is strictly increasing, strictly concave, twice continuously differentiable, with $u'(0) = \infty$, $u(0) = 0$, and define x^* as the solution to $u'(x^*) = 1$. As well, assume that

$$-x\frac{u''(x)}{u'(x)} < 1, \tag{2}$$

for $x \geq 0$, which implies, roughly, that substitution effects dominate income effects in terms of the demands for assets – the demand for an asset increases with its own rate of return. There is a continuum of sellers with unit mass, and each seller maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - h_t], \tag{3}$$

where X_t denotes consumption in the CM, and h_t denotes labor supply in the DM. There exists an infinite mass of bankers, each of whom maximizes

$$E_0 \sum_{t=0}^{\infty} \beta^t [X_t - H_t], \tag{4}$$

where X_t and H_t are, respectively, the banker's consumption and labor supply in the CM.

One unit of labor supply produces one unit of the perishable consumption good for a producer in the CM or in the DM. In the CM, buyers and bankers can produce but sellers cannot, and in the DM sellers can produce but buyers cannot. In the CM, all agents are initially together in one location. At the beginning of the CM, debts from the previous period are settled, then production, consumption, asset trade, and contracting with banks occur. Finally, at the end of the CM, each buyer can contact his or her bank. In the DM, each buyer is randomly matched with a seller, and bankers are not present. In DM meetings, the seller does not know the buyer's history, but bankers' histories are public information. Each agent is subject to limited commitment, that is no one can be forced to work.

The consolidated government in this economy potentially issues three liabilities – physical currency, central bank digital currency (CBDC), and one-period government debt. There is government debt in all the regimes we consider, but the liabilities the central bank issues differ across regimes, for example we want to examine what happens if we replace physical currency with CBDC. Fraction ρ of sellers in the DM accept only physical currency or CBDC in exchange, while the remaining $1-\rho$ fraction of sellers accept only electronic means of payment – CBDC or claims on banks – and do not accept physical currency. That is, we assume that CBDC is designed so that it can be used in exchange in all circumstances, while other means of payment (both private and public) are limited in their acceptability.

The consolidated government budget constraints are

$$c_0 + m_0 + b_0 = \tau_0, \tag{5}$$

and

$$c_t + m_t + b_t = \frac{c_{t-1} + R_t^m m_{t-1} + R_t^b b_{t-1}}{\pi_t} + \tau_t, \tag{6}$$

for $t = 1, 2, 3, ...$, where equations (5) and (6) are written in units of the CM good in period 0 and period t, respectively. Here, c_t, m_t, and b_t denote, respectively, physical currency, CBDC, and government debt outstanding in period t, while τ_t is the lump-sum transfer per buyer in the CM. Finally, R_t^m and R_t^b denote the gross nominal interest rates on CBDC and government debt, respectively, and π_t is the gross inflation rate. Note that physical currency does not bear interest, while CBDC does. The fiscal authority targets the total quantity of consolidated government debt outstanding, that is w_t is exogenous, where

$$w_t = c_t + m_t + b_t, \tag{7}$$

so the fiscal authority manipulates lump-sum taxes, given monetary policy, to achieve an exogenous path for the total real quantity of consolidated government debt.

Each period there are active and inactive banks. An inactive bank cannot trade with anyone. To become active, a bank bears a fixed cost γ, in units of labor supply, in the CM, per unit mass of depositors served. Assume that γ is the cost per depositor of acquiring a technology that makes the bank's history part of the public record and that paying this cost implies that the bank has access to the technology forever. Bearing the cost γ is necessary for the bank's viability. But the bank's viability also depends on beliefs about the bank's future redemption of deposit liabilities, a feature which we will exploit in what follows.

3 Equilibrium with Physical Currency and No CBDC

First, suppose that the central bank's assets are government debt, and its liabilities are physical currency. We will examine equilibria without central bank crisis intervention, both without and with banking panics. Then we show what happens if the central bank intervenes by lending during bank panic episodes, or if it intervenes in panics using open market operations.

3.1 No Central Bank Crisis Intervention

We will assume, when banks write contracts with buyers in the CM, that buyers do not know what types of sellers they will meet in the subsequent DM. But, buyers learn this at the end of the CM, after production and consumption have taken place. A buyer's type is the means of payment accepted by the seller who meets the buyer in the DM. Type is private information, and known by the buyer at the end of the CM. Banks in this environment will have two roles. First, because banks' histories are known, their debt is held and traded in equilibrium, whereas the IOUs of buyers, for whom there is assumed to be no recordkeeping, will not be accepted in exchange. Second, banks will play an insurance role, in providing buyers with the appropriate means of payment contingent on who they will meet in the DM. The first role for banks is related to what is modeled in Cavalcanti and Wallace (1999), and the second bears some resemblance to a Diamond and Dybvig (1983) role for banks.

Let v_t denote the franchise value of an active bank at the beginning of the CM in period t. At the beginning of the CM in period t, there is an aggregate sunspot shock σ_t, where $\sigma_t \epsilon \{G, B\}$. If $\sigma_t = G$, then each bank that was active in period $t-1$ remains active in period t. However, if $\sigma_t = B$, then fraction α of banks active in period $t-1$ become inactive in period t, and will remain inactive forever, as $v_{t+s} = 0$ for those banks, $s = 0, 1, 2, ...$. No depositors will lend to these banks as their future franchise value is zero, which is then self-fulfilling. Assume that σ_t is i.i.d., and $\Pr[\sigma_t = G] = 1 - \delta$, $\Pr[\sigma_t = B] = \delta$, with $0 < \delta < 1$ and $0 < \alpha < 1$. Also assume that the realization of σ_t is publicly known at the end of the CM of period $t-1$, at the time buyers learn the type of the seller they will meet. But the central bank has more detailed information as, at the end of the CM in period $t-1$, the central bank knows the identities of the individual banks that will become inactive in period t. Before the aggregate shock is known, each bank has the same probability α of becoming inactive, conditional on $\sigma_t = B$. In the DM of period $t-1$, the status of each bank in period t becomes public information.

We will construct equilibria given alternative institutional setups, without banking panics (defined later) and with panics. These are credit equilibria, in which lending to banks by depositors is supported by beliefs about the future creditworthiness of banks. For simplicity, we confine attention to a particular class of equilibria that are straightforward to characterize. These equilibria have the property that there is a constant probability of an adverse event ($\sigma_t = B$) under which some constant fraction of banks will be effectively insolvent and will default on their deposit liabilities, because they have nothing to lose from doing so. But at the time when a bank depositor needs to make a decision as to whether to withdraw or not, he or she knows only whether or not there will be an adverse event, and not whether his or her bank will fail. The banking panics we will study are events where all depositors choose to withdraw at the end of the CM in period t when they observe $\sigma_{t+1} = B$.

We assume that the central bank observes the aggregate sunspot, and knows the identities of the failing banks. Bank failures may potentially lead to banking panics, defined to be events where it is in the interest of all bank depositors to make withdrawals in cash. For some depositors, this will imply that they forgo making an exchange with a bank deposit claim, even though they do not know for sure that their bank will fail, and in spite of the fact that the currency they withdraw can only be used as a store of value.

3.1.1 No Banking Panics

First, consider an equilibrium in which there are no banking panics. In this equilibrium, banks write deposit contracts with buyers at the beginning of the CM that give these depositors the option to either withdraw currency at the end of the CM, or trade claims on the bank in the DM. Then, at the end of the

CM, everyone observes the state $\sigma_{t+1}\epsilon\{G, B\}$, and depositors decide whether to request withdrawal of currency or not. A banking panic is defined to be a state in which all depositors choose to withdraw currency. Requests for withdrawal of currency occur simultaneously, so there is no sequential service, in contrast to Diamond-Dybvig (1983) type models. The bank's available currency is allocated equally among the depositors requesting withdrawal. In this equilibrium without bank runs, it must be optimal in all states of the world for only a fraction ρ of depositors to request withdrawal of currency.

A bank chooses a deposit contract (k_t, c_t, d_t), where k_t is the amount deposited by each depositor in the CM, c_t is the quantity of currency acquired by the bank, per depositor at the beginning of the CM, and d_t is the quantity of claims on CM goods in period $t+1$ that a depositor is permitted to trade in the DM if currency is not withdrawn at the end of the current CM. The deposit contract specifies that the depositor can either request withdrawal of currency at the end of the CM, with each depositor requesting withdrawal receiving an equal share of the total currency on hand, or the depositor can trade away d_t deposit claims that the bank will redeem in the CM of period $t+1$. The bank then solves

$$\max_{k_t,c_t,d_t}\left\{-k_t+\rho u\left(\frac{\beta c_t}{\rho\pi_{t+1}}\right)+(1-\rho)(1-\alpha\delta)u\left(\beta d_t\right)\right\} \tag{8}$$

subject to

$$v_t = k_t - c_t + \beta(1-\alpha\delta)\left[-(1-\rho)d_t + v_{t+1}\right] \geq \bar{v} \tag{9}$$

and

$$-(1-\rho)d_t + v_{t+1} \geq 0. \tag{10}$$

The objective function (8) is the expected utility of the depositor from the banking contract, as of the beginning of the CM of period t. That is, there is probability ρ that the depositor needs currency to trade in the DM, and requests withdrawal at the end of the CM. There is no banking panic in any state, so each of the fraction ρ of depositors requesting withdrawal receives $\frac{c_t}{\rho}$ units of currency. In the DM, each buyer with currency makes a take-it-or-leave-it offer to the seller, so the seller produces $\frac{\beta}{\pi_{t+1}}$ goods for each unit of currency (in units of CM goods in period t) received from the buyer. With probability $1-\rho$, the depositor learns at the end of the CM that he or she will trade with a seller in the DM who accepts only bank deposit claims. The buyer makes a take-it-or-leave-it offer, receiving β units of goods from the seller for each unit of deposit claims transferred to the seller. But the seller will accept the deposit claims only if the buyer's bank is viable in period $t+1$, as a non-viable bank defaults on its deposit claims, having nothing to lose from doing so. In the DM, each bank's viability in the next period is revealed publicly, so in the DM sellers know which banks will fail and which will not. Thus, at the beginning of the CM, a depositor knows that the bank will default with probability $\alpha\delta$, as $\sigma_{t+1} = B$ with probability δ, and each bank becomes nonviable with probability α, conditional on $\sigma_{t+1} = B$.

Each bank maximizes the expected utility of depositors (8) subject to the constraint (9) that the bank achieve a current franchise value of at least $\bar{v}$ (which will be determined endogenously), and subject to the incentive constraint (10). All agents, including banks, are subject to limited commitment, so even if the bank is viable in period $t+1$, it must prefer in equilibrium to pay off on its deposit liabilities rather than defaulting and being denied access to depositors in the future, as (10) states.

Assume that incentive constraint (10) does not bind, which implies, from (8) and (9) that

$$d_t = \frac{x^*}{\beta}, \tag{11}$$

Free entry into banking implies that the franchise value of a bank is equal to the entry cost, i.e. $v_t = \gamma$ for all t, so given (11), the incentive constraint (10) does not bind if and only if

$$\gamma \geq \frac{(1-\rho)x^*}{\beta} \tag{12}$$

which we will assume henceforth. That is, the entry cost into banking must be sufficiently large, so as to support a high enough franchise value to make default by the bank on deposit claims suboptimal.

Then, confine attention to stationary policies (R^b, w), where R^b is the gross nominal interest rate on government debt, chosen by the monetary authority, and w is the real quantity of consolidated government debt, chosen by the fiscal authority. The choice of a nominal interest rate target by the central bank is then supported by open market operations. Given $R_t^b = R^b$ and $w_t = w$ for all t, look only for stationary equilibria. Government debt is held only as a store of value, so in equilibrium

$$R^b = \frac{\pi}{\beta}, \tag{13}$$

Then, from (8), (9), and (13),

$$-R^b + u'\left(\frac{c}{\rho R^b}\right) = 0, \tag{14}$$

which solves for the quantity of currency acquired per depositor, c. As well, since $v_t = \gamma$ for all t, and (9) holds with equality,

$$k = c + [1 - \beta(1-\alpha\delta)]\,\gamma + (1-\rho)(1-\alpha\delta)x^*, \tag{15}$$

which solves for k given c.

In a stationary equilibrium, a bank depositor who uses bank deposits in exchange in the DM will choose not to panic and request withdrawal in currency when $\sigma_{t+1} = B$ if and only if

$$\frac{c}{\rho R^b} \leq (1-\alpha)u(x^*), \tag{16}$$

i.e. if and only if the depositor receives lower utility from withdrawing currency and holding it as a store of value until the next CM, rather than trading deposits in the DM and taking the chance that the depositor's bank fails and the deposits are worthless.

We need to also consider the possibility that the equilibrium does not exist because depositors prefer holding currency rather than depositing in the bank at the beginning of the CM. If a buyer were to deviate and hold currency at the beginning of the CM, then he or she solves

$$\max_{c^d}[-c^d + \rho u\left(\frac{c^d}{R^b}\right) + (1-\rho)\frac{c^d}{R^b}] \tag{17}$$

where c^d is the quantity of currency acquired, in units of the CM good. The first-order condition for an optimum, given deviation, is

$$R^b = \rho u'\left(\frac{c^d}{R^b}\right) + 1 - \rho. \tag{18}$$

For convenience assume that

$$u(x) = \frac{x^{1-\omega}}{1-\omega}, \tag{19}$$

with $0 < \omega < 1$, to be consistent with assumption (2), and with $u(0) = 0$. Assuming the utility function satisfies constant relative risk aversion does little or no harm. Then, let x denote the consumption of currency users in the DM each period, so we can solve for x given (18) and (19) to obtain

$$x = (R^b)^{-\frac{1}{\omega}}. \tag{20}$$

Then, (16) implies that a banking panic will not occur if

$$R^b \geq \left(\frac{1-\omega}{1-\alpha}\right)^{\omega}, \tag{21}$$

So, an equilibrium with no banking panics exists if the nominal interest rate is sufficiently high. In that case, each bank has a small enough stock of currency on hand that no depositor who uses deposits in transactions chooses to withdraw in a state of the world when the failure of some banks is imminent. Further, the higher the conditional probability an individual bank fails given the imminent bank failure state, the higher the nominal interest rate threshold that prevents a banking panic from occurring. If $\alpha \leq \omega$ then an equilibrium with no bank failures exists for any positive nominal interest rate ($R^b \geq 1$), but if $\alpha > \omega$, then if the nominal interest rate is low enough this equilibrium will not exist.

To support this equilibrium, the entry cost for a bank must be sufficiently large that (12) is satisfied, so that each bank has a sufficient franchise value that its incentive constraint does not bind. Also, the entry cost must be sufficiently small that depositors do not want to deviate to using only currency in transactions. Given our assumed constant relative risk aversion utility function, we can determine the expected period utility of a depositor which is, using (8), (15), and (20),

$$U^{Nc} = -\left[1-\beta(1-\alpha\delta)\right]\gamma + \left(\frac{\omega}{1-\omega}\right)\left[(1-\rho)(1-\alpha\delta) + \rho(R^b)^{-\frac{(1-\omega)}{\omega}}\right] \quad (22)$$

Then, if we determine the utility from deviating from the bank deposit contract, from (17) and (18), it can be shown that if

$$\omega \geq 1-\beta(1-\alpha\delta) \quad (23)$$

then there is a range for γ that supports this equilibrium, i.e. the unconditional probability of a bank failure cannot be too large.

We also need to check that the fiscal authority issues enough government debt that the central bank can buy to support currency demanded in equilibrium, as determined by monetary policy R^b. That is, from (7),

$$w \geq c, \quad (24)$$

or, from (20), (24), and (14),

$$R \geq \left(\frac{\rho}{w}\right)^{\frac{\omega}{1-\omega}} \quad (25)$$

which puts a lower bound on the nominal interest rate, i.e. the nominal interest rate needs to be sufficiently high, otherwise the demand for currency exceeds the real value of the consolidated government debt, determined by fiscal policy.

In comparing performance in this equilibrium with alternatives note that, if we sum expected utility across agents, then all that will matter for comparing alternative arrangements is expected surplus in the DM. As in other Lagos-Wright (2005) structures, if we sum utility across agents, then most production in the CM nets out with consumption in the CM. A difference here is that we have to account for the entry costs incurred by banks in the CM. Entry costs are borne in period 0, for all banks, and in any period when $\sigma_t = B$, when new banks enter to replace failed banks. But these entry costs are invariant to the alternative arrangements we consider. So, for the welfare comparisons we make in what follows, total expected surplus in the DM in this equilibrium, from (8) and (20) is

$$S^{Nc} = \frac{\rho(R^b)^{-\frac{1}{\omega}}\left(R^b - 1 + \omega\right)}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)(1-\rho)(1-\alpha\delta) \quad (26)$$

3.1.2 Banking Panic Equilibrium

In this equilibrium, if $\sigma_{t+1} = B$ is observed at the end of the CM of period t, then all depositors choose to withdraw currency from the bank, but if $\sigma_{t+1} = G$ is observed, only those who will need currency for transactions in the DM

withdraw it. So, when $\sigma_{t+1} = B$ is observed, this triggers insolvency for some banks. Depositors do not know whether their own bank will be insolvent, but the probability of insolvency is high enough that all depositors choose to withdraw currency given $\sigma_{t+1} = B$. In a banking panic equilibrium, the bank solves

$$\max_{c_t,k_t,d_t} \left\{ \begin{array}{c} -k_t + \delta\left[\rho u(\frac{\beta c_t}{\pi_{t+1}}) + (1-\rho)\frac{\beta c_t}{\pi_{t+1}}\right] \\ +(1-\delta)\left[\rho \min\left[u(x^*), u(\frac{\beta c_t}{\rho\pi_{t+1}})\right] + \rho\max\left(0, -x^* + \frac{\beta c_t}{\rho\pi_{t+1}}\right) + (1-\rho)u\left(\beta d_t\right)\right] \end{array} \right\} \tag{27}$$

subject to

$$v_t = k_t - c_t + \beta\delta(1-\alpha)v_{t+1} + \beta(1-\delta)\left[-(1-\rho)d_t + v_{t+1}\right] \geq \bar{v} \tag{28}$$

and

$$-(1-\rho)d_t + v_{t+1} \geq 0. \tag{29}$$

Assuming (12), and again confining attention to stationary policies and stationary equilibria, (11) and (13) hold, and then (27) and (28) give

$$R^b = \delta\left[\rho u'\left(\frac{c}{R^b}\right) + 1 - \rho\right] + (1-\delta)\max\left[1, u'\left(\frac{c}{\rho R^b}\right)\right] \tag{30}$$

and

$$k = c + \left[1 - \beta\left(1 - \delta\alpha\right)\right]\gamma + (1-\delta)(1-\rho)x^* \tag{31}$$

In this equilibrium, it must be optimal for all buyers to withdraw currency in the event that $\sigma_{t+1} = B$ is observed. That is,

$$\frac{c}{R^b} \geq (1-\alpha)u\left(x^*\right). \tag{32}$$

So, from (32), a necessary condition for existence of an equilibrium with panics is

$$x^* \geq (1-\alpha)u(x^*). \tag{33}$$

As in the equilibrium with no banking panics, assume (19) with $0 < \omega < 1$, and let x denote consumption for a depositor using currency, in the state of the world in which banks fail. Then, from equation (30), first consider the case in which $x \geq \rho$, so there is efficient trade in the DM for depositors using currency in the state of the world in which banks do not fail. Then, from (30), we obtain

$$x = (R^b - 1 + \rho\delta)^{-\frac{1}{\omega}}\delta^{\frac{1}{\omega}}\rho^{\frac{1}{\omega}}. \tag{34}$$

So, in this case we require $x \geq \rho$, or from (34),

$$R^b \leq 1 - \rho\delta + \delta\rho^{1-\omega}. \tag{35}$$

As well, a banking panic must be rational, i.e. (32) must hold, so given (34),

$$R^b \leq 1 - \rho\delta + \rho\delta\left(\frac{1-\omega}{1-\alpha}\right)^{\omega}. \tag{36}$$

Next, consider the case in which $x \leq \rho$. Then, from from (30),

$$x = \left[R^b - \delta(1-\rho)\right]^{-\frac{1}{\omega}} \left[\delta\rho + (1-\delta)\rho^\omega\right]^{\frac{1}{\omega}}. \tag{37}$$

Then, to satisfy $x \leq \rho$, from (37) we need

$$R^b \geq \delta\rho^{1-\omega} + 1 - \delta\rho. \tag{38}$$

As well, from (32) and (37), bank panics are rational if

$$R^b \leq \delta(1-\rho) + [\delta\rho + (1-\delta)\rho^\omega]\left(\frac{1-\omega}{1-\alpha}\right)^\omega \tag{39}$$

So, from (34)-(39), if

$$\rho \leq \frac{1-\alpha}{1-\omega}, \tag{40}$$

then the equilibrium with bank panics exists when

$$1 \leq R^b \leq 1 - \rho\delta + \rho\delta\left(\frac{1-\omega}{1-\alpha}\right)^\omega \tag{41}$$

holds, x is determined by (34), and the equilibrium does not otherwise exist. However, if

$$\rho \geq \frac{1-\alpha}{1-\omega}, \tag{42}$$

then an equilibrium with banking panics exists with x determined by (34) if

$$1 \leq R^b \leq 1 - \rho\delta + \delta\rho^{1-\omega}, \tag{43}$$

and an equilibrium with banking panics exists with x determined by (37) if

$$1 - \rho\delta + \delta\rho^{1-\omega} \leq R^b \leq \delta(1-\rho) + [\delta\rho + (1-\delta)\rho^\omega]\left(\frac{1-\omega}{1-\alpha}\right)^\omega. \tag{44}$$

Otherwise, this equilibrium does not exist.

As in the equilibrium with no panics, we can calculate total expected surplus in the DM, for the purposes of making welfare comparisons. For the case in which $x \geq \rho$, from (27) and (34), the surplus measure is

$$S^{Pc1} = \frac{(R^b - 1 + \rho\delta)^{-\frac{1}{\omega}}\delta^{\frac{1}{\omega}}\rho^{\frac{1}{\omega}}(R^b - 1 + \rho\delta\omega)}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)(1-\delta), \tag{45}$$

and for the case $x \leq \rho$, from (27) and (37), we get

$$S^{Pc2} = \frac{\left[R^b - \delta(1-\rho)\right]^{-\frac{1}{\omega}}\left[\delta\rho + (1-\delta)\rho^\omega\right]^{\frac{1}{\omega}}\left[R^b - \delta(1-\rho) - \rho(1-\omega)\right]}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)(1-\rho)(1-\delta) \tag{46}$$

In general, we can say that, if a banking panic equilibrium exists for some nominal interest rate i.e if, from (33), if

$$\alpha > \omega, \tag{47}$$

then banking panic equilibria exist for low nominal interest rates, and equilibria with no panics exist for high nominal interest rates. In particular, if $\alpha > \omega$, then the lower bound on the nominal interest rate in an equilibrium with no panics, from (21), exceeds the upper bound on the nominal interest rate in an equilibrium with panics, from (41) and (44). To generate banking panic equilibria requires that a sufficiently large number of banks fail when $\sigma_t = B$, and panic equilibria occur at low interest rates because a low interest rate on government debt implies that banks have more currency on hand at the end of the CM, which encourages a banking panic when $\sigma_{t+1} = B$ is observed.

3.1.3 Optimal Interest Rate Policy If There is No Central Bank Crisis Intervention

In this subsection, we want to determine, given our setup thus far, how the central bank should conduct interest rate policy so as to maximize welfare. First, if (47) holds, then an equilibrium with panics does not exist for any $R^b \geq 1$. So, in this case, an optimal policy is a setting for R^b that maximizes welfare in an equilibrium with no panics. As discussed previously, our welfare measure is total expected surplus in the DM. Then, from (26), welfare is maximized when $R^b = 1$,which is a Friedman rule. Further, this result is not particular to our assumption of a constant relative risk aversion utility function. So, if (47) holds, so that banking panics are not a possibility, then we get a standard prescription for interest rate policy, that the central bank should satiate the private sector with currency.

We are in a different realm, however, if (47) does not hold. Then, a banking panic equilibrium exists for low nominal interest rates, and an equilibrium with no panics exists for high nominal interest rates. So, in choosing its interest rate policy, the central bank can choose between a world with panics and a world without panics. But, doing away with panics also implies living with the distortion caused by a high nominal interest rate, so it is not immediately clear what constitutes optimal policy.

If a central banker is to choose between a world with banking panics and a world without, then this involves first choosing an optimal policy given panics, choosing an optimal policy if there were no panics, and then comparing the outcomes in the two cases. In an equilibrium with no panics, welfare is maximized when R^b is at the lower bound given by (26), which implies that total expected surplus in the DM is

$$S^{Nc*} = \frac{\rho\left[\left(\frac{1-\alpha}{1-\omega}\right)^{(1-\omega)} - (1-\alpha)\right]}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)(1-\rho)(1-\alpha\delta). \tag{48}$$

As well, in a panic equilibrium, welfare is maximized for $R^b = 1$, so expected surplus, from (45) is

$$S^{Pc*} = \left(\frac{\omega}{1-\omega}\right)(\delta\rho + 1 - \delta) \tag{49}$$

So, we have

$$S^{Nc*} - S^{Pc*} = \frac{\rho\left[\left(\frac{1-\alpha}{1-\omega}\right)^{(1-\omega)} - (1-\alpha)\right]}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)\left[-\rho + \delta(1-\alpha)(1-\rho)\right]. \tag{50}$$

Then, from (50), we could have $S^{Nc*} - S^{Pc*} > 0$ or $S^{Nc*} - S^{Pc*} < 0$, depending on parameters. For example, if

$$\rho \leq \frac{\delta(1-\alpha)}{1+\delta(1-\alpha)}, \tag{51}$$

then $S^{Nc*} - S^{Pc*} > 0$, and if $\alpha = 1$, then $S^{Nc*} - S^{Pc*} < 0$. Therefore, depending on parameters, an optimal monetary policy could be to set $R^b = 1$ and tolerate the welfare losses from banking panics, or to set $R^b = \left(\frac{1-\omega}{1-\alpha}\right)^{\omega}$, forego banking panics altogether, and bear the distortion from high nominal interest rates. Note that the expression on the right-hand side of (50) is strictly increasing in δ, and strictly decreasing in α. That is, the relative welfare loss from banking panics increases with the frequency of panics in the banking panic equilibrium, as banking panics disrupt retail payments. But there is less disruption of payments, conditional on a panic occuring, the greater is the bank failure rate, as a failing bank will disrupt payments even when a panic does not occur.

Even in cases in which $S^{Nc*} - S^{Pc*} < 0$, the optimal banking panic equilibrium may not be feasible, due to fiscal policy. That is, (24) must hold in equilibrium, that is the demand for currency cannot exceed the quantity of consolidated government debt. At the optimum, in an equliibrium with no panics, in our example, the demand for currency, from (8), (20), and (21), is

$$c^N = \rho\left(\frac{1-\alpha}{1-\omega}\right)^{1-\omega}. \tag{52}$$

Similarly, the demand for currency at the optimum in the banking panic equilibrium, from (27) and (34), is

$$c^P = 1. \tag{53}$$

So, in the relevant case in which $\alpha > \omega$, $c^N < c^P$, so the fiscal constraint (24) could be violated at the optimum for the panic equilibrium, but not for the equilibrium with no panics, at the optimum. That is, in a banking panic equilibrium, the demand for safe assets is higher, as banks insure depositors by holding large quantities of currency. So, an insufficient supply of safe assets may constrain monetary policy so that it is optimal to eliminate banking panics.

3.2 Equilibrium with Central Bank Crisis Lending

There is of course a long history of central bank intervention in banking panics, in the form of lending from the central bank. In this section we use the model to illustrate how central bank crisis lending affects banking panic equilibria, and how the central bank uses its superior information in a crisis.

At the end of the CM in period t, the central bank observes the aggregate shock σ_{t+1}, and it also knows which banks will become inactive and default on their liabilities. Suppose, in addition, that the central bank can observe withdrawal requests from each bank. For simplicity, suppose that the central bank sets up a facility for crisis lending under which, if the central bank observes $\sigma_{t+1} = B$, it will lend to any banks that will remain active and which experience requests for withdrawal from all the bank's depositors at the end of the CM in period t. The central bank will only lend an amount to a bank that it knows will be repaid in the next CM. Central bank crisis loans are financed by issuing currency, and the central bank sets a nominal interest rate of zero on central bank loans, which assures that the currency injection in the CM in period t, when $\sigma_{t+1} = B$, will be withdrawn when the loans are repaid in the next CM. Also, assume that the bank's depositors must decide whether to request withdrawal of currency before knowing whether or not their bank will receive a loan from the central bank. That is, when a depositor observes $\sigma_{t+1} = B$, he or she only knows that there is probability α that his or her bank fails.

This regime is intended to capture the nature of actual central bank crisis lending. Typically central banks lend in a crisis to banks, but not to those which are insolvent, which in the model consists of the fraction α of non-viable banks. For the banks that receive central bank loans in a crisis, central bank funding replaces funding from depositors, and all depositors withdraw cash.

The bank then solves

$$\max_{c_t,k_t,d_t,c_t^B} \left\{ \begin{array}{c} -k_t + \delta(1-\alpha)\rho u(\frac{\beta c_t + \beta c_t^B}{\pi_{t+1}}) + \delta(1-\alpha)(1-\rho)(\frac{\beta c_t + \beta c_t^B}{\pi_{t+1}}) \\ \delta\alpha \left[\rho u(\frac{\beta c_t}{\pi_{t+1}}) + (1-\rho)\frac{\beta c_t}{\pi_{t+1}}\right] + \\ (1-\delta)\left\{\rho \min\left[u(x^*), u(\frac{\beta c_t}{\rho \pi_{t+1}})\right] + \rho\max(0, \frac{\beta c_t}{\rho\pi_{t+1}} - x^*) + (1-\rho)u(\beta d_t)\right\} \end{array} \right\} \tag{54}$$

subject to

$$v_t = k_t - c_t + \beta\delta(1-\alpha)\left(-\frac{c_t^B}{\pi_{t+1}} + v_{t+1}\right) + \beta(1-\delta)\left[-(1-\rho)d_t + v_{t+1}\right] \geq \bar{v}, \tag{55}$$

$$-(1-\rho)d_t + v_{t+1} \geq 0, \tag{56}$$

and

$$-\frac{c_t^B}{\pi_{t+1}} + v_{t+1} \geq 0. \tag{57}$$

As in the simpler problem in which banks do not have access to central bank lending in a panic, the buyer deposits k_t with the bank at the beginning of the CM. Each bank acquires c_t units of currency. At the end of the CM, in (54), with probability δ the buyer observes $\sigma_{t+1} = B$, and withdraws currency from the bank, whether or not the buyer needs currency for transactions in the DM. If $\sigma_{t+1} = B$ and the bank will remain active in period $t+1$, the bank takes out a central bank loan, borrowing c_t^B in currency, per depositor, and each depositor withdraws $c_t + c_t^B$. But, if $\sigma_{t+1} = B$ and the bank will not be active in period $t+1$, then the bank does not receive a central bank loan, and each depositor withdraws c_t from the bank. If, with probability $1-\delta$, $\sigma_{t+1} = G$ at the end of the CM, then each buyer who needs currency withdraws $\frac{c_t}{\rho}$ from the bank, and those buyers who need bank deposits to trade in the DM each trade claims to d_t units of consumption in the next CM. Inequalities (56) and (57) are incentive constraints which state, respectively, that the bank has the incentive to pay off on its deposit liabilities when $\sigma_{t+1} = G$, and has the incentive to pay off its central bank loan if $\sigma_{t+1} = B$ and the bank will remain active in period $t+1$.

We will continue to assume stationary policies and to focus on stationary equilibria. As in the previous subsections, free entry into banking gives $v_t = \gamma$. Continue to assume that γ is large enough that the incentive constraints (56) and (57) do not bind, so c and c^b solve

$$c + c^B = R^b x^*. \tag{58}$$

and

$$\begin{aligned} &-R^b + \delta(1-\alpha) \\ &\qquad + \delta\alpha\left[\rho u'\left(\frac{c}{R^b}\right) + 1 - \rho\right] + (1-\delta)\max\left[1, u'\left(\frac{c}{\rho R^b}\right)\right] = 0. \end{aligned} \tag{59}$$

Then, from (55), in equilibrium

$$k = c\left(1 - \frac{\delta(1-\alpha)}{R^b}\right) + \left[1 - \rho(1-\delta) - \alpha\delta\right]x^* + \left[1 - \beta(1-\alpha\delta)\right]\gamma \tag{60}$$

In this regime we are assuming that, at the end of the CM, depositors have to first request withdrawal on observing the aggregate state, without knowing whether their banks are viable. So, for this equilibrium to exist,

$$(1-\alpha)x^* + \alpha\frac{c}{R^b} \geq (1-\alpha)u(x^*) \tag{61}$$

Then, following our example with a constant relative risk aversion utility function, similar to the banking panic equilibrium, a necessary condition for existence of this equilibrium is

$$\alpha > \omega. \tag{62}$$

If (62) holds, and if

$$\rho \leq \frac{\omega(1-\alpha)}{\alpha(1-\omega)}, \tag{63}$$

then an equilibrium with central bank lending exists if

$$1 \leq R^b \leq 1 - \delta\alpha\rho + \alpha\delta\rho\left[\frac{(1-\omega)\alpha}{\omega(1-\alpha)}\right]^{\omega}. \tag{64}$$

In this case, from (59),

$$x = (R^b - 1 + \delta\alpha\rho)^{-\frac{1}{\omega}}\delta^{\frac{1}{\omega}}\alpha^{\frac{1}{\omega}}\rho^{\frac{1}{\omega}}, \tag{65}$$

and from (54) and (65), expected surplus in the DM is

$$S^{Lc1} = \frac{\left(R^b - 1 + \delta\alpha\rho\right)^{-\frac{1}{\omega}}\delta^{\frac{1}{\omega}}\alpha^{\frac{1}{\omega}}\rho^{\frac{1}{\omega}}\left(R^b - 1 + \omega\delta\alpha\rho\right)}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)[\delta(1-\alpha)\rho + 1 - \delta] \tag{66}$$

However, if (62) holds and

$$\rho \geq \frac{\omega(1-\alpha)}{\alpha(1-\omega)}, \tag{67}$$

then if

$$1 \leq R^b \leq 1 - \delta\alpha\rho + \alpha\delta\rho^{1-\omega}, \tag{68}$$

then x is determined by (65), and expected surplus is given by (66). But, if

$$1 - \delta\alpha\rho + \alpha\delta\rho^{1-\omega} \leq R^b \leq \delta(1-\alpha\rho) + [\delta\alpha\rho + (1-\delta)\rho^{\omega}]\left[\frac{(1-\omega)\alpha}{\omega(1-\alpha)}\right]^{\omega} \tag{69}$$

then from (59),

$$x = [R^b - \delta(1-\alpha\rho)]^{-\frac{1}{\omega}}[\delta\alpha\rho + (1-\delta)\rho^{\omega}]^{\frac{1}{\omega}}, \tag{70}$$

and from (54) and (70), expected surplus is

$$S^{Lc2} = \frac{[R^b - \delta(1-\rho)]^{-\frac{1}{\omega}}[\delta\alpha\rho + (1-\delta)\rho^{\omega}]^{\frac{1}{\omega}}\left\{R^b - \delta(1-\rho) - \rho[1-\delta(1-\alpha)]\right\}}{1-\omega} \tag{71}$$

$$+\left(\frac{\omega}{1-\omega}\right)[\delta(1-\alpha)\rho + (1-\delta)(1-\rho)] \tag{72}$$

So, if (62) holds, then an optimal policy in a panic equilibrium with central bank lending is $R^b = 1$, and so expected surplus at the optimum, from (66), is

$$S^{Lc*} = \left(\frac{\omega}{1-\omega}\right)(\delta\rho + 1 - \delta). \tag{73}$$

So, from (49) and (73), given an optimal policy, welfare is the same with central bank crisis lending, as in a banking panic equilibrium with no crisis intervention. That is, at the Friedman rule, $R^b = 1$, banks are satiated with currency, and there is no central bank lending in a banking panic, as banks choose not to borrow, given the central bank lending facility. Further, from (54) and (65) the demand for safe assets is equal to 1 in this equilibrium when $R^b = 1$, so the demand for safe assets is identical to what it is at the optimum in a banking panic equilibrium without central bank lending. So, central bank crisis lending is not advantageous, given optimal central bank interest rate policy.

However, a central bank crisis lending facility can affect existence of equilibrium, in a manner that encourages panics. For example, suppose that $\alpha \geq \omega$, so that a banking panic equilibrium with central bank lending exists for some $R^b \geq 1$. Then, let $\omega \to 0$, which implies that, from (50), $S^{Nc*} - S^{Pc*} > 0$, so that a policy of $R^b = \left(\frac{1-\omega}{1-\alpha}\right)^{\omega}$ would be optimal in the absence of central bank crisis lending, and this optimal policy would eliminate banking panics. But, then (67) holds, so given $R^b = \left(\frac{1-\omega}{1-\alpha}\right)^{\omega}$, from (21) and (69), a banking panic equilibrium with central bank crisis lending and an equilibrium with no panics coexist, so a high interest rate need not eliminate the panic equilibrium with the central bank crisis lending facility in place. So, crisis lending can be destabilizing and reduce welfare.

3.3 Central Bank Crisis Intervention Through Open Market Operations

There are differing views about how a central bank should intervene in a crisis, and how much it should intervene. For example, a dimension of this policy debate concerns whether liquidity injections should occur in a crisis through directed central bank lending, or more broad-based open market operations. One view is that of Goodfriend and King (1988), who argue that issues regarding the insolvency of specific banks should be separated from crisis intervention, the latter executed entirely, as they argue, through open market operations. Thus, it seems useful to use our model to address how crisis intervention via open market operations differs in its effects from lending to individual institutions based on solvency.

In the model, conventional open market operations consist of asset purchases in the CM, before the central bank has information on future private bank solvency. To capture the effects of crisis intervention through open market operations, we will suppose that there is another market on which government debt can be exchanged for money, with this market opening at the end of the CM, after σ_{t+1} is observed. If the central bank intervenes in this market, we assume it will not use information on individual bank solvency, in line with the ideas of Goodfriend and King (1988), for example. The central bank will intervene at the end of the CM by swapping money for government debt, but will do this only when the realization is $\sigma_{t+1} = B$. Assume that banks acquire government debt in the CM at the time they write deposit contracts with buyers and take deposits. Thus, banks acquire government debt so it can be exchanged for currency if $\sigma_{t+1} = B$ is observed, but banks will just carry the government debt into the next CM if $\sigma_{t+1} = G$ is observed. Also assume that, when the central bank intervenes at the end of the CM, that it purchases government debt at face value. This assures that the open market operation conducted in a crisis has no implications for the consolidated government budget constraint, and therefore will not matter for inflation, given the future path for taxes. Then, a bank's problem in this regime is

$$\max_{c_t,k_t,d_t,}\left\{\begin{array}{c} -k_t+\delta\left[\rho u(\frac{\beta c_t+\beta b_t R_t^b}{\pi_{t+1}})+(1-\rho)(\frac{\beta c_t+\beta b_t R_t^b}{\pi_{t+1}})\right] \\ +(1-\delta)\left[\rho u(\frac{\beta c_t}{\rho\pi_{t+1}})+(1-\rho)u\left(\beta d_t\right)\right] \end{array}\right\} \tag{74}$$

subject to

$$v_t=k_t-c_t-b_t+\beta\delta(1-\alpha)v_{t+1}+\beta(1-\delta)\left[-(1-\rho)d_t+v_{t+1}+\frac{b_t R_t^b}{\pi_{t+1}}\right]\geq\bar{v}, \tag{75}$$

and

$$-(1-\rho)d_t+v_{t+1}\geq 0. \tag{76}$$

In the bank's problem, b_t denotes the quantity of government bonds acquired in the CM by the bank. If, at the end of the CM in period t, everyone observes that $\sigma_{t+1}=B$, then the central bank conducts an open market operation at the end of the CM, in which any bank can exchange government debt for currency, at face value. In this state, all depositors panic and withdraw currency from their bank, and depositors get the same quantity of currency (in contrast to central bank crisis lending) whether the bank is solvent or insolvent. As before, assume that the incentive constraint (76) does not bind. In a stationary equilibrium, from (74) and (75), optimal choice of c_t and b_t, respectively, by the bank give

$$-1+\delta\frac{\beta}{\pi}\left[\rho u'\left(\frac{\beta c+\beta b R^b}{\pi}\right)+1-\rho\right]+(1-\delta)\frac{\beta}{\pi}u'\left(\frac{\beta c}{\rho\pi}\right)=0, \tag{77}$$

and

$$-1+\delta\frac{\beta R^b}{\pi}\left[\rho u'\left(\frac{\beta c+\beta b R^b}{\pi}\right)+1-\rho\right]+(1-\delta)\frac{\beta R^b}{\pi}=0. \tag{78}$$

In equilibrium, (13) holds and we can write (77) as

$$R^b=\delta+(1-\delta)u'\left(\frac{c}{\rho R^b}\right), \tag{79}$$

which determines c given the gross nominal interest rate R^b. For this equilibrium to exist, it must be in the interest of bank depositors who use deposits in exchange to withdraw currency at the end of the CM when they observe $\sigma_{t+1}=B$, that is

$$x^*\geq(1-\alpha)u(x^*). \tag{80}$$

So, in our running example with a constant relative risk aversion utility function, solve for x, the quantity of consumption for currency users in non-panic states, from (79), obtaining

$$x=(R-\delta)^{-\frac{1}{\omega}}(1-\delta)^{\frac{1}{\omega}}, \tag{81}$$

and note that, from (80), that the equilibrium exists if

$$\alpha\geq\omega. \tag{82}$$

Then, in equilibrium, expected surplus in the DM, from (74) and (81), is

$$S^{Oc} = \frac{\rho(R-\delta)^{-\frac{1}{\omega}}(1-\delta)^{\frac{1}{\omega}}\left[R-1+\omega(1-\delta)\right]}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)\left[\delta\rho + (1-\delta)(1-\rho)\right] \tag{83}$$

So, if (82) holds, then an optimal monetary policy, given crisis intervention via open market operations, is $R^b = 1$, and from (83) optimal expected surplus in the DM is

$$S^{Oc*} = \left(\frac{\omega}{1-\omega}\right)(\delta\rho + 1 - \delta), \tag{84}$$

which is identical to what we obtained in a banking panic equilibrium with no intervention (from (49)) and with central bank crisis lending (from (73)). Further, the demand for safe assets at the optimum is the same as in the other two types of panic equilibria. So, panic intervention via open market operations achieves nothing relative to central bank lending, or no intervention, given optimal central bank interest rate policy.

But, in terms of promoting instability, an open market crisis facility of this type is even worse than crisis lending. If (82) holds, and if an equilibrium with no panics exists, then so does the equilibrium with panics and open market crisis intervention. So, there are instances in which, from (50), $S^{Nc*} > S^{Pc*}$, but there are multiple equilibria, so that an inferior panic equilibrium coexists with the equilibrium with no panics. That is, the open market crisis facility can act to encourage welfare-reducing panics.

4 Equilibrium with CBDC and No Physical Currency

There are many possible ways to model the introduction of CBDC. Here, we will assume that CBDC is introduced by the central bank as a replacement for physical currency. In the model, the difference between CBDC and physical currency is that CBDC can bear interest and can potentially be used in transactions in which bank deposits are used. Throughout, we will assume that CBDC is used in transactions as a substitute for bank deposits only in bank panic states. That is, the gross nominal interest rate on CBDC, denoted R_t^m will be set low enough that depositors in banks will prefer to use bank deposits in transactions rather than CBDC, in the transactions in which sellers in the DM accept only deposits or CBDC. As a result, CBDC will be used in non-panic states of the world only when sellers accept only CBDC, but in panic states all bank depositors use CBDC in transactions.

4.1 No Central Bank Crisis Intervention, Equilibrium Without Banking Panics

In this case, the bank chooses a deposit contract (k_t, m_t, d_t) under which each depositor deposits k_t units of goods with the bank at the beginning of the CM.

Then, in equilibrium, depositors who cannot use bank deposits in transactions collectively withdraw m_t units of CBDC from the bank at the end of the CM, and the remaining depositors trade bank deposits if $\sigma_{t+1} = G$. Bank deposits are not traded only if $\sigma_{t+1} = B$, and the bank will become inactive, in which case the bank's deposit claims are not accepted in payment in the DM. The bank's problem is then

$$\max_{k_t, m_t, d_t} \left[-k_t + \rho u \left(\frac{\beta R^m_{t+1} m_t}{\rho \pi_{t+1}} \right) + (1-\rho)(1-\delta\alpha) u(\beta d_t) \right] \tag{85}$$

subject to

$$v_t = k_t - m_t + \beta(1-\alpha\delta) \left[-(1-\rho) d_t + v_{t+1} \right] \geq \bar{v}, \tag{86}$$

and

$$-(1-\rho) d_t + v_{t+1} \geq 0. \tag{87}$$

This is similar to the banking problems in the other regimes we have examined, with the constraint (86) stating the the bank must attain an expected value of $\bar{v}$, while (87) is the incentive constraint.

In a stationary equilibrium, $v_t = \gamma$ (13) holds and, assuming that (87) does not bind, d, m, and k solve

$$d = \frac{x^*}{\beta}, \tag{88}$$

$$R = u' \left(\frac{m}{\rho R} \right), \tag{89}$$

$$k = \left[1 - \beta(1-\alpha\delta) \right] \gamma + (1-\rho)(1-\alpha\delta) x^* + m. \tag{90}$$

In equations (88) and (89), R is defined to be the ratio of the gross nominal interest rate on government debt to the gross nominal interest rate on CBDC,

$$R \equiv \frac{R^b}{R^m}. \tag{91}$$

and in this regime, policy is given by (R^b, R^m, w), where the gross nominal interest rate on government debt R^b is supported by open market operations, and the gross nominal interest rate on CBDC, R^m, is set administratively.

In equilibrium, bank depositors must not want to panic when $\sigma_{t+1} = B$, so

$$u \left(\frac{m}{\rho R} \right) \leq (1-\alpha) u(x^*). \tag{92}$$

Also, in this equilibrium, depositors must prefer the contract chosen in (85) subject to (86) and (87) rather than foregoing the deposit contract, acquiring CBDC in the CM, and trading it in the DM, that is

$$-k + \rho u \left(\frac{m}{\rho R} \right) + (1-\rho)(1-\delta\alpha) u(x^*) \geq -m^d + u \left(\frac{m^d}{R} \right), \tag{93}$$

where m^d is chosen optimally, so

$$R = u'\left(\frac{m^d}{R}\right), \tag{94}$$

and k and m solve (89) and (90). Therefore, to support this equilibrium requires $R > 1$, as this is necessary to make CBDC sufficiently unattractive relative to bank deposits, both in the ex ante sense, and as a vehicle to flee to in the event that $\sigma_{t+1} = B$.

To characterize the equilibrium, continue with our running example of a utility function with constant relative risk aversion, i.e. (19), with $\omega < 1$. Let $x = \frac{m}{\rho R}$, where x is consumption by CBDC users in the DM. Then, from (19), (89), and (92),

$$x = R^{-\frac{1}{\omega}}, \tag{95}$$

and from (92) a panic will not occur so long as

$$R \geq (1-\alpha)^{-\frac{\omega}{1-\omega}} \tag{96}$$

So, this works qualitatively like the regime with physical currency, in that an absence of banking panics is supported by high interest rates, but here what matters is the interest rate on government debt relative to the interest rate on CBDC, not the interest rate on government debt relative to zero-nominal-interest currency.

Given our assumed utility function, we can write the inequality (93) as

$$\left[1-\beta(1-\alpha\delta)\right]\gamma \leq (1-\rho)\left(\frac{\omega}{1-\omega}\right)\left[(1-\delta\alpha) - R^{-\frac{(1-\omega)}{\omega}}\right] \tag{97}$$

So note, in (97), that banking is more costly the larger is γ, the entry cost, and the larger is $\delta\alpha$, the unconditional probability an individual bank fails. So (97) states that, to support this equilibrium, banking cannot be too costly, and R must be sufficiently large, so that CBDC is not too attractive.

As well, the entry cost of banking must be sufficiently large that banks have something to lose, in equilibrium, from defaulting on their deposit liabilities. That is, for the incentive constraint (87) not to bind, given the assumed utility function,

$$\gamma \geq \frac{1-\rho}{\beta} \tag{98}$$

So the entry cost can be neither too large nor too small. For there to exist some entry cost, and some sufficiently high R, such that this equilibrium exists requires that

$$\omega \geq 1-\beta(1-\alpha\delta), \tag{99}$$

That is, there must be sufficient curvature in the utility function, a sufficiently low unconditional probability of bank failure, and a sufficiently high discount factor.

Finally, expected surplus in this equilibrium is given by

$$S^{Nm} = \frac{\rho R^{-\frac{1}{\omega}}\left(R-1+\omega\right)}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)(1-\rho)(1-\alpha\delta) \tag{100}$$

4.2 No Central Bank Crisis Intervention, Banking Panic Equilibrium

Similar to the banking panic equilibrium in a regime with physical currency, in the event that $\sigma_{t+1} = B$ is observed at the end of the CM, each depositor withdraws CBDC. A difference here is that all of these depositors can spend CBDC in the DM.The bank chooses a deposit contract (k_t, m_t, d_t), where k_t denotes the amount deposited in the CM, m_t is the quantity of CBDC the bank acquires, and d_t is the quantity of deposit claims that can be traded by the depositor, provided the bank will remain active in the following period. The bank's problem is

$$\max_{k_t, m_t, d_t} \left\{ \begin{array}{c} -k_t + \delta u(\frac{\beta R^m_{t+1} m_t}{\pi_{t+1}}) + \rho(1-\delta) \left[u\left(\min\left(x^*, \frac{\beta R^m_{t+1} m_t}{\rho \pi_{t+}} \right) \right) + \max\left(-x^* + \frac{\beta R^m_{t+1} m_t}{\rho \pi_{t+1}}, 0 \right) \right] \\ +(1-\rho)(1-\delta) u\left(\beta d_t\right) \end{array} \right\} \tag{101}$$

subject to

$$v_t = k_t - m_t + \beta\delta(1-\alpha)v_{t+1} + \beta(1-\delta)\left[-(1-\rho)d_t + v_{t+1}\right] \geq \bar{v}, \tag{102}$$

and

$$-(1-\rho)d_t + v_{t+1} \geq 0. \tag{103}$$

In the objective function (101) note that, in the event that $\sigma_{t+1} = B$, all depositors withdraw CBDC at the end of the CM, and each obtains an equal share of the bank's CBDC, which is then traded in the DM subject to take-it-or-leave-it offers. If $\sigma_{t+1} = G$, then only those depositors who need CBDC to make a transaction withdraw, and the remaining depositors trade bank deposits. Constraint (102) states that the bank earns an expected lifetime payoff of at least $\bar{v}$.Inequality (103) is the bank's incentive constraint.

In equilibrium, as with our approach in the previous section, suppose that the incentive constraint (103) does not bind. Then, in a stationary equilibrium, from (101) and (102),

$$d = \frac{x^*}{\beta} \tag{104}$$

The gross nominal interest rate on government debt is given by (13) in equilibrium, so m is determined, from (101) and (102), by

$$R = \delta u'\left(\frac{m}{R}\right) + (1-\delta)\max\left[1, u'\left(\frac{m}{\rho R}\right)\right], \tag{105}$$

and then, given free entry into banking, implying $v_t = \gamma$ for all t, we get

$$k = \left[1 - \beta(1-\alpha\delta)\right]\gamma + m + (1-\rho)(1-\delta)x^* \tag{106}$$

For this equilibrium with bank runs to exist it is necessary that it be in the interest of depositors using bank deposits in transactions to withdraw when observing $\sigma_{t+1} = B$, so

$$u\left(\frac{m}{R}\right) \geq (1-\alpha)u(x^*) \tag{107}$$

As well, it must be suboptimal for bank depositors to defect to an arrangement in which they acquire CBDC in the CM, and then trade it in the DM, so

$$-m^d+u\left(\frac{m^d}{R}\right) \leq -k+\delta u\left(\frac{m}{R}\right)+\rho(1-\delta)\left[u\left(\frac{m}{\rho R}\right)+\max\left(-x^*+\frac{m}{\rho R},0\right)\right]+(1-\rho)(1-\delta)u\left(x^*\right), \tag{108}$$

where m^d solves (94), and m and k solve (106) and (107).

As in the previous subsection assume that the utility function has constant relative risk aversion, i.e. (19) holds. Let x denote consumption by currency users in the DM when a bank failure episode is imminent. Then, from (105) and (107), x is determined by

$$R = \delta x^{-\omega} + (1-\delta)\max\left[1, \left(\frac{x}{\rho}\right)^{-\omega}\right], \tag{109}$$

and from (107), x must satisfy

$$x^{1-\omega} \geq 1-\alpha. \tag{110}$$

So, if

$$1 \leq R \leq \delta\rho^{-\omega} + 1 - \delta, \tag{111}$$

then from (109),

$$x = \delta^{\frac{1}{\omega}}(R-1+\delta)^{-\frac{1}{\omega}}, \tag{112}$$

and from (110),

$$R \leq 1-\delta+\delta(1-\alpha)^{-\frac{\omega}{1-\omega}} \tag{113}$$

But, if

$$R \geq \delta\rho^{-\omega} + 1 - \delta, \tag{114}$$

then from (109),

$$x = \left[\delta + (1-\delta)\rho^{\omega}\right]^{\frac{1}{\omega}} R^{-\frac{1}{\omega}}, \tag{115}$$

and, from (110) and (115),

$$R \leq [\delta + (1-\delta)\rho^{\omega}](1-\alpha)^{-\frac{\omega}{1-\omega}} \tag{116}$$

So, if

$$\rho \leq (1-\alpha)^{\frac{1}{1-\omega}}, \tag{117}$$

and

$$1 \leq R \leq 1-\delta+\delta(1-\alpha)^{-\frac{\omega}{1-\omega}}, \tag{118}$$

then a banking panic equilibrium exists in which x is determined by (112). But if

$$\rho \geq (1-\alpha)^{\frac{1}{1-\omega}}, \tag{119}$$

and (111) holds, then a banking panic equilibrium exists in which x is determined by (112), and if (119) and

$$\delta\rho^{-\omega} + 1 - \delta \leq R \leq [\delta + (1-\delta)\rho^{\omega}](1-\alpha)^{-\frac{\omega}{1-\omega}}, \tag{120}$$

hold, then a banking panic equilibrium exists, and x is determined by (115).

In equilibrium, the entry cost into banking, γ, can be neither too small nor too large. In particular, as in the equilibrium with no panics, (98) must hold. As well, if (112) holds, then from (108),

$$-[1-\beta(1-\alpha\delta)]\gamma+\left(\frac{\omega}{1-\omega}\right)\left[\delta^{\frac{1}{\omega}}(R-1+\delta)^{-\frac{(1-\omega)}{\omega}}+1-\delta-R^{-\frac{(1-\omega)}{\omega}}\right]\geq 0, \tag{121}$$

and if (115) holds, then

$$-[1-\beta(1-\alpha\delta)]\gamma+\left(\frac{\omega}{1-\omega}\right)\left\{R^{-\frac{(1-\omega)}{\omega}}\left[\delta+(1-\delta)\rho^{\omega}\right]^{\frac{1}{\omega}}+(1-\rho)(1-\delta)-R^{-\frac{(1-\omega)}{\omega}}\right\}\geq 0. \tag{122}$$

Finally, expected surplus in the DM if (112) holds, is

$$S^{Pm1}=\frac{\delta^{\frac{1}{\omega}}\left(R-1+\delta\right)^{-\frac{1}{\omega}}\left(R-1+\delta\omega\right)}{1-\omega}+\left(\frac{\omega}{1-\omega}\right)(1-\delta), \tag{123}$$

and if (115) holds, then expected surplus in the DM is

$$S^{Pm2}=\frac{R^{-\frac{1}{\omega}}\left[\delta+(1-\delta)\rho^{\omega}\right]^{\frac{1}{\omega}}\left(R-1+\omega\right)}{1-\omega}+\left(\frac{\omega}{1-\omega}\right)(1-\delta)(1-\rho) \tag{124}$$

4.3 Comparing Equilibria With and Without Panics

A first question we could ask is how the replacement of currency by CBDC affects the existence of equilibria without and with panics. Is there some sense in which CBDC encourages panics? First, in general the critical interest rate quantity is the margin between the interest rate on government debt and the interest rate on central bank liabilities, that is R^b in the currency regime and R in the CBDC regime. Then, from (21) and (96), we can say that the introduction of CBDC tends to encourage panics, in the sense that, with CBDC, the interest rate margin needs to be higher to induce an equilibrium in which banking panics do not occur. We get this result because CBDC can be used in all transactions, whereas currency cannot be used in transactions by some buyers. So, if a panic occurs, this is more disruptive of retail payments in a currency regime than with CBDC. Therefore, CBDC tends to encourage banking panics.

As well, recall that in the currency regime, an equilibrium with banking panics does not exist if $\alpha<\omega$, i.e. if the bank failure rate, conditional on $\sigma_t=B$, is sufficiently low. However, with CBDC, as long as R is sufficiently low, a banking panic equilibrium exists. Thus, in this second sense, CBDC tends to encourage banking panics, again because panics are less disruptive of retail payments with CBDC than with currency.

CBDC may encourage banking panics, but this happens because CBDC is more useful in retail payments than is currency, so it is possible that banking panics are not so bad with CBDC, or that banking panic equilibria may dominate equilibria without panics in welfare terms. That is, just as with physical currency, a high interest rate margin is required to eliminate banking panics, when banking panic equilibria exist, and high R produces welfare losses. But, we assume here that the central bank chooses R so that CBDC does not compete with private bank deposits. This can be viewed either as a constraint on policy, or as a stand-in for unspecified costs of operating the CBDC system. So, with CBDC, it is possible that R must be high anyway, so that CBDC does not eliminate banks from the retail payments system.

To see what is going on, if it is feasible to support an equilibrium without banking panics with CBDC at the lower bound on the nominal interest rate given by (96), which will maximize welfare in the equilibrium with no runs, then from (97),

$$-[1-\beta(1-\alpha\delta)]\gamma + \left(\frac{\omega}{1-\omega}\right)(1-\rho)\alpha(1-\delta) \geq 0, \tag{125}$$

which puts a lower bound on the bank entry cost, γ, required to support this equilibrium. Similarly, if (117) holds, suppose we consider the banking panic equilibrium with the highest R, and thus the lowest welfare, where R reaches the upper bound given by (118). Then, given this interest rate, from (121),

$$-[1-\beta(1-\alpha\delta)]\gamma + \left(\frac{\omega}{1-\omega}\right)\left\{1-\alpha\delta-\left[1-\delta+\delta(1-\alpha)^{-\frac{\omega}{(1-\omega)}}\right]^{-\frac{(1-\omega)}{\omega}}\right\} \geq 0 \tag{126}$$

Then, if we evaluate expected surplus in the DM in the equilibrium with no panics, at the lower bound given by (96), from (100) we get

$$S^{Nml} = \frac{\rho(1-\alpha)^{\frac{1}{1-\omega}}\left[(1-\alpha)^{-\frac{\omega}{1-\omega}}-(1-\omega)\right]}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)(1-\rho)(1-\alpha\delta), \tag{127}$$

and similarly, if (117) holds, then in the banking panic equilibrium given the upper bound on R given by (118), expected surplus is

$$S^{Pmu} = \frac{\delta(1-\alpha)^{\frac{1}{1-\omega}}\left[(1-\alpha)^{-\frac{\omega}{1-\omega}}-(1-\omega)\right]}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)(1-\delta) \tag{128}$$

So, if we subtract the left-hand side of (126) from the left-hand side of (125), we obtain

$$\chi_1 = \left(\frac{\omega}{1-\omega}\right)\left\{-1+\alpha\left[1-\rho(1-\delta)\right]+\left[1-\delta+\delta(1-\alpha)^{-\frac{\omega}{(1-\omega)}}\right]^{-\frac{(1-\omega)}{\omega}}\right\} \tag{129}$$

And, from (127) and (128), we get

$$S^{Nml}-S^{Pmu} = \frac{(\rho-\delta)(1-\alpha)^{\frac{1}{1-\omega}}\left[(1-\alpha)^{-\frac{\omega}{1-\omega}}-(1-\omega)\right]}{1-\omega} + \left(\frac{\omega}{1-\omega}\right)\left[\delta(1-\alpha)-\rho(1-\alpha\delta)\right] \tag{130}$$

So, suppose that $\rho = 0$ and $\delta = 1$, which implies, from (129) and (130), respectively, that

$$\chi_1 = 0, \tag{131}$$

and

$$S^{Nml} - S^{Pmu} = \frac{(1-\alpha)[(1-\alpha)^{-\frac{\omega}{1-\omega}} + \omega]\left[(1-\alpha)^{-\frac{\omega}{1-\omega}} - 1\right]}{1-\omega} < 0 \tag{132}$$

So, by continuity, there are circumstances under which, if the equilibrium without panics is feasible, then the equilibrium with bank panics is feasible, and dominates the equilibrium without panics in welfare terms. Thus, the fact that CBDC encourages panics need not be a bad thing. Similarly, suppose that $\delta = 0$. Then, from (129) and (130), respectively, we get $\chi_1 > 0$, and $S^{Nml} - S^{Pmu} > 0$, so there exist parameter values such that, if the equilibrium with panics is feasible, then so is the equilibrium with no panics, and the equilibrium with no panics dominates in welfare terms. So, it is possible that it is advantageous to tolerate high R because this eliminates panics, or because the panic equilibrium is not feasible.

We could compare equilibria with and without panics in the case in which (115) holds, but we would not learn anything that was not illustrated in the case where (117) holds.

4.4 Equilibrium With Central Bank Intervention: Central Bank Lending

As in the regime with physical currency, suppose that if the central bank observes $\sigma_{t+1} = B$ it extends loans to viable banks, financed by issuing CBDC. The central bank sets the nominal interest rate on these loans equal to the interest rate on CBDC, so that the injection of CBDC in the crisis state in the CM is temporary, with the CBDC issue withdrawn in the next CM. The bank then chooses a deposit contract (k_t, m_t, d_t, m_t^b), where k_t is the amount a buyer deposits, m_t is CBDC acquired by the bank at the beginning of the CM, d_t are the deposit claims that a depositor can trade if he or she does not withdraw CBDC, and m_t^b is CBDC borrowed by a viable bank at the end of the CM if $\sigma_{t+1} = B$. The bank solves

$$\max_{k_t, m_t, d_t, m_t^b} \left[\begin{array}{c} -k_t + \delta(1-\alpha)u\left(\frac{\beta R_{t+1}^m(m_t+m_t^b)}{\pi_{t+1}}\right) \\ +\delta\alpha u\left(\frac{\beta R_{t+1}^m m_t}{\pi_{t+1}}\right) + \rho(1-\delta)\left\{\min\left[u(x^*), u\left(\frac{\beta R_{t+1}^m m_t}{\rho\pi_{t+1}}\right)\right] + \max\left[0, -x^* + \frac{\beta R_{t+1}^m m_t}{\rho\pi_{t+1}}\right]\right\} \\ +(1-\rho)(1-\delta)u\left(\beta d_t\right) \end{array} \right] \tag{133}$$

subject to

$$v_t = k_t - m_t + \beta\delta(1-\alpha)\left(-\frac{R_{t+1}^m m_t^B}{\pi_{t+1}} + v_{t+1}\right) + \beta(1-\delta)\left[-(1-\rho)d_t + v_{t+1}\right] \geq \bar{v}, \tag{134}$$

$$-(1-\rho)d_t + v_{t+1} \geq 0, \tag{135}$$

and

$$-\frac{R^m_{t+1} m^B_t}{\pi_{t+1}} + v_{t+1} \geq 0. \tag{136}$$

Assume, as in the other regimes, that the incentive constraints (135) and (136) do not bind. As before, in a stationary equilibrium, (13) holds. As well, free entry into banking assures that $v_t = \gamma$. So, in a stationary equilibrium, from (133) subject to (134)-(136), we get

$$R = \delta(1-\alpha) + \delta\alpha u'\left(\frac{m}{R}\right) + (1-\delta)\max\left[1, u'\left(\frac{m}{\rho R}\right)\right], \tag{137}$$

$$k = [1-\beta(1-\alpha\delta)]\,\gamma + m + \delta(1-\alpha)\frac{m^b}{R} + (1-\rho)(1-\delta)x^*. \tag{138}$$

For this equilibrium to exist, depositors must prefer to panic when $\sigma_{t+1} = B$, that is

$$\alpha u\left(\frac{m}{R}\right) + (1-\alpha)u(x^*) \geq (1-\alpha)u(x^*), \tag{139}$$

which always holds, so given this central bank lending arrangement, the banking panic equilibrium exists for any $R \geq 1$, provided depositors will not defect to conducting all transactions in CBDC. That is, depositors in viable banks do as well in a bank run as when the run does not occur, and depositors in nonviable banks do strictly better, as they can trade CBDC in the DM rather than receiving nothing.

$$R \leq 1 - \delta\alpha + \rho^{-\omega}\delta\alpha, \tag{140}$$

then from (137), we can solve for x, the quantity of consumption in for all depositors in a panic state, with

$$x = (R-1+\delta\alpha)^{-\frac{1}{\omega}}\delta^{\frac{1}{\omega}}\alpha^{\frac{1}{\omega}}. \tag{141}$$

As in the equilibria without central bank crisis intervention, it cannot be advantageous in equilibrium for bank depositors to defect from the deposit contract and conduct transactions only with CBDC, so from (133), (138), and (141),

$$-[1-\beta(1-\alpha\delta)]\gamma + \left(\frac{\omega}{1-\omega}\right)\left[1-\delta\alpha + (R-1+\delta\alpha)^{-\frac{(1-\omega)}{\omega}}\delta^{\frac{1}{\omega}}\alpha^{\frac{1}{\omega}} - R^{-\frac{(1-\omega)}{\omega}}\right] \geq 0. \tag{142}$$

As well, given (133) and (141), expected surplus in the DM in equilibrium in this case is given by

$$S^{Lm1} = \left(\frac{\omega}{1-\omega}\right)(1-\alpha\delta) + \left(\frac{1}{1-\omega}\right)(R-1+\delta\alpha)^{-\frac{1}{\omega}}\delta^{\frac{1}{\omega}}\alpha^{\frac{1}{\omega}}(R-1+\delta\alpha\omega) \tag{143}$$

Similarly, if

$$R \geq 1 - \delta\alpha + \rho^{-\omega}\delta\alpha, \tag{144}$$

then

$$x = [R - \delta(1-\alpha)]^{-\frac{1}{\omega}} \left[\delta\alpha + (1-\delta)\rho^{\omega}\right]^{\frac{1}{\omega}}. \tag{145}$$

So, if deviation to the use of only CBDC in transactions cannot occur in equilibrium, we require

$$\begin{aligned} &-[1-\beta(1-\alpha\delta)]\gamma \\ &+\left(\frac{\omega}{1-\omega}\right)\left\{1-\rho(1-\delta)-\delta\alpha+[R-\delta(1-\alpha)]^{-\frac{1-\omega}{\omega}}\left[\delta\alpha+(1-\delta)\rho^{\omega}\right]^{\frac{1}{\omega}}-R^{-\frac{(1-\omega)}{\omega}}\right\} \geq 0, \end{aligned} \tag{146}$$

and expected surplus in the DM in equilibrium is given by

$$\begin{aligned} S^{Lm2} = &\left(\frac{\omega}{1-\omega}\right)[1-\alpha\delta-\rho(1-\delta)] \\ &+\left(\frac{1}{1-\omega}\right)[R-\delta(1-\alpha)]^{-\frac{1}{\omega}}\left[\delta+(1-\delta)\rho^{\omega}\right]^{\frac{1}{\omega}}[R-(1-\omega)-\delta(1-\alpha)\omega] \end{aligned} \tag{147}$$

In contrast to the banking panic equilibrium with no central bank crisis intervention, with central bank lending in a crisis the banking panic equilibrium exists for any $R \geq 1$, so long as the entry cost into banking is not too small or too large. In this sense, central bank lending does more to encourage banking panics than in a regime with physical currency, again because CBDC, when withdrawn in a panic, can always be used in DM transactions.

But, as in the banking panic equilibrium with no intervention, the fact that introducing CBDC encourages banking panics need not mean that a banking panic equilibrium has inferior welfare properties. Compared to the equilibrium with no runs, with central bank lending in a banking panic, buyers who use either CBDC or bank deposits in transactions do strictly better in the lending equilibrium than in the equlilibrium with no panics. But buyers who use only CBDC do better in a panic with lending than with no panics, if their bank does not fail. But if the buyer's bank fails, they do worse when $\sigma_{t+1} = B$ than in the non-panic equilibrium. This is because, in the equilibrium with no panics, banks have a greater incentive to self-insure by holding CBDC, which works to the advantage of the depositor who uses CBDC in transactions. When $\sigma_{t+1} = G$, buyers who use CBDC in transactions may do better or worse in a banking panic equilibrium with central bank lending than in an equilibrium no banking panics. If we compare welfare in a banking panic equilibrium with lending, and in the equilibrium with no panics, it is hard to make definitive general statements, but we can say the following. Using expected surplus in the DM as our welfare measure, from (100) and (143), expected surplus is strictly higher in the banking panic equilibrium than in the equilibrium with no panics, when $R = 1$, if $R = 1$ were feasible. But $R = 1$ is not feasible, as in either equilibrium buyers would defect to CBDC from banking arrangements. Similarly, if we let $R \to \infty$, then from (100) and (147), then expected surplus is higher in the banking panic equilibrium. So, given R, we know that expected surplus is higher in the banking panic equilibrium with lending for high R and for low R. In either equilibrium, expected surplus is declining in R, which works against the equilibrium with no panics, since it exists only for high R, while the panic equilibrium exists for any $R \geq 1$, so long as it is feasible. Further, from (97), (142), and (146), we can say that, given R, there is a stronger incentive to defect under an equilibrium with no panics than in an equilibrium with banking panics and central bank lending.

We can conclude that there are some parameter values such that banking panic equilibria with central bank lending are superior to equilibria with no panics. But policy cannot necessarily eliminate panics, as the cases in which banking panic equilibria are superior may also be cases in which the equilibrium with no panics exists. This works contrary to the conventional wisdom that comes from Diamond-Dybvig (1983) type models of banking panics. In those setups there can be multiple equilibria in which banking panic equilibria are inferior. Here there can be multiple equilbria, with inferior equilibria having no panics.

4.5 Central Bank Intervention Through Open Market Operations

Similar to the case with physical currency, suppose the central bank intervenes when $\sigma_{t+1} = B$ is observed, by trading CBDC for government debt. Then, a bank solves

$$\max_{c_t, k_t, d_t,} \left\{ \begin{array}{c} -k_t + \delta u(\frac{\beta m_t R^m_{t+1} + \beta b_t R^b_{t+1}}{\pi_{t+1}}) \\ +(1-\delta)\left[\rho u(\frac{\beta R^m_{t+1} m_t}{\rho \pi_{t+1}}) + (1-\rho) u\left(\beta d_t\right)\right] \end{array} \right\} \tag{148}$$

subject to

$$v_t = k_t - m_t - b_t + \beta\delta(1-\alpha)v_{t+1} + \beta(1-\delta)\left[-(1-\rho)d_t + v_{t+1}\right] + \beta b_t (1-\delta)\frac{R^b_{t+1}}{\pi_{t+1}} \geq \bar{v}, \tag{149}$$

and

$$-(1-\rho)d_t + v_{t+1} \geq 0. \tag{150}$$

In a stationary equilibrium in which government bonds are plentiful, that is

$$\frac{w}{R} - b\left(1 - \frac{1}{R}\right) \geq x^*, \tag{151}$$

then exchange in the DM is efficient when $\sigma_{t+1} = b$, and (13) holds.

If government debt is plentiful, then for it to be optimal for all depositors to run to the bank when $\sigma_{t+1} = B$ requires that

$$u(x^*) \geq (1-\alpha)u(x^*), \tag{152}$$

which always holds.

In a stationary equilibrium in which government bonds are plentiful, we can write (149) as

$$k = [1 - \beta(1 - \alpha\delta)]\,\gamma + m + \delta b + (1 - \delta)(1 - \rho)x^*, \tag{153}$$

Then, in our running example, assuming (19) with $\omega < 1$, let x denote the consumption of depositors who use CBDC in transactions in the DM, in states in which banks do not fail, from (148) and (153) we obtain

$$x = (R - \delta)^{-\frac{1}{\omega}}(1 - \delta)^{\frac{1}{\omega}}, \tag{154}$$

and, so that depositors do not defect to using only CBDC in transactions, we require

$$-\,[1 - \beta(1 - \alpha\delta)]\,\gamma + \left(\frac{\omega}{1 - \omega}\right)\left[1 - \rho(1 - \delta) + \rho(R - \delta)^{-\frac{(1-\omega)}{\omega}}(1 - \delta)^{\frac{1}{\omega}} - R^{-\frac{(1-\omega)}{\omega}}\right] \geq 0. \tag{155}$$

As well, from (148) and (154), expected surplus in the DM in equilibrium is

$$S^{Om} = \frac{\rho(R - \delta)^{-\frac{1}{\omega}}(1 - \delta)^{\frac{1}{\omega}}\,[R - 1 + (1 - \delta)\omega]}{1 - \omega} + \left(\frac{\omega}{1 - \omega}\right)[1 - \rho(1 - \delta)] \tag{156}$$

As with the banking panic equilibrium with central bank lending, introducing CBDC along with an open market crisis facility acts to encourage panics. That is, this equilibrium exists for all $R \geq 1$, so long as (155) and (98) hold, while the equilibrium with no panics requires that (96) holds. But, as with central bank crisis lending, we can show that welfare must be higher in the banking panic equilibrium with an open market crisis facility than in the equilibrium with no panics. As well, if (97) holds, then so does (155), given R. Therefore the banking panic equilibrium with an open market crisis facility has greater immunity from defection to trading in CBDC only than is the case in an equilibrium with no panics.

So, this equilibrium also has the property that there can be multiple equilibria, but the equilibrium with banking panics is actually the superior equilibrium. Thus, it may not be possible for conventional interest rate policy to select the equilibrium.

5 Conclusion

Our main conclusion from the model constructed here is that, were the central bank to replace physical currency with CBDC, this would indeed encourage banking panics. But the reason for this is that CBDC is potentially more useful in transactions than is physical currency. This not only mitigates the damage done by a banking panic, but could imply that, in an ex ante sense, economic welfare is higher in an economy that experiences banking panics than in an economy that does not.

The model of banking panics constructed here has some novel features, and it can be argued that it improves in important ways on traditional models of financial crises and panics. The model does not rely on sequential service, which is not a feature associated with modern electronic banking arrangements. Also, while bank failures and panics are driven by beliefs, it is essentially the potential for bank insolvency that leads to a banking panic, and this seems to correspond with what we know about historical banking panic episodes, for example in the US before the Federal Reserve Act of 1913.

What does the model leave out that might be important to the problem at hand? Potentially, deposit insurance could be important. We could defend leaving deposit insurance out by arguing that banks typically have a significant quantity of uninsured liabilities which, while not traditional retail transactions deposits, serve a transactions role that is close to that of such deposits. But, including deposit insurance would be an interesting avenue for further research.

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