More on Money Mining and Price Dynamics: Competing and Divisible Currencies

Michael Choi
University of California, Irvine

Guillaume Rocheteau
University of California, Irvine

The Economics of Digital Currencies, Oct 7, 2020

Motivation

- Many digital monies are privately-produced through "mining".
- Satoshi Nakamoto:

  The steady addition of a constant of amount of new (Bit)coins is analogous to gold miners expending resources to add gold to circulation. In our case, it is CPU time and electricity that is expended.

- How does mining affect the joint dynamics of the price and supply of privately-produced monies (e.g. gold and Bitcoin)?

Our approach

- We construct a continuous-time model where:
  1. Transactional role of money is endogenous
  2. Money is perfectly divisible and produced through mining with an explicit time dimension
  3. Different mining technologies for tangible or crypto-monies
  4. Different mining cost functions: exogenous flow cost or endogenous opportunity cost

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- Choi and Rocheteau (2020) study a related model with indivisible money. This model is useful because:
  1. Divisible money model is desirable for various applications.
  2. Easily incorporate competing private or government monies.

Result highlights

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1. Q: Can privately-produced money have a positive price?
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2. Q: What is the dynamics of the value of money?
   A: Depending on the mining technology and production cost, currency price can rise, fall, or be non-monotone over time.
3. Q: Can the government choose monetary policy to prevent the production of the private money?
   A: Yes, but only when the private money is not widely acceptable.

Environment

Environment: Time, agents, goods

- Time is continuous: $t \in \mathbb{R}_+$
- A unit measure of buyers and a unit measure of sellers
- Trade numeraire c in an ongoing competitive market
- Trade good q in pairwise meetings
- All goods are non-storable

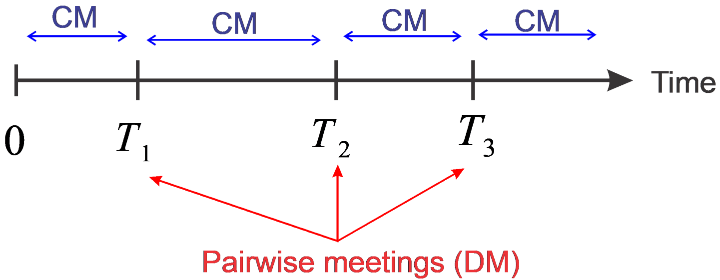
Environment: Meeting technology

- Meetings with single coincidence of want at Poisson rate α
  - Utility of consumption in a pairwise meeting: $u(q)$ with $u(0) = 0$, $u' > 0$, $u'(0) = +\infty$, and $u'' < 0$.
  - Disutility of production in a pairwise meeting: q
- Anonymity: Individual trading histories are private
- Agents lack commitment, i.e., cannot commit to repay debt

Environment: Divisible assets

- Money is perfectly divisible and pays a dividend flow $d \geq 0$
- In-between pairwise meetings agents can trade money with numeraire c in competitive exchanges
- Price of money in terms of numeraire: ϕ_t
- Rate of return of money: $r = \left(d + \dot{\phi}\right) / \phi$
- Amount held by agents: $A_t \leq \bar{A}$

Timing



Environment: Mining technology

- Aggregate mining intensity: $m_t \equiv \int_0^1 e_i di$ where e_i is individual mining effort
- Individual discovery/mining rate: $\Lambda(A, m)e$.

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1. Example 1 **(gold mining)**

$$\Lambda(A, m) = \lambda \left(\bar{A} - A\right).$$

2. Example 2 **(crypto mining)**

$$\Lambda(A, m) = \frac{\overbrace{\pi(A)}^{\text{money growth rate}} \quad A}{m}.$$

In the case of Bitcoins,

$$\pi(A) \approx \lambda \left(\frac{\bar{A} - A}{A}\right).$$

Environment: Mining cost

- Flow cost of mining: $C(e) \geq 0$

1. Example 1 (occupation choice): $e \in \{0, 1\}$ and $C(1)$ is an endogenous opportunity cost
   e.g., gold mining diverts resources that could be used for the production of goods and services.

Environment: Mining cost

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1. Example 1 (occupation choice): $e \in \{0, 1\}$ and $C(1)$ is an endogenous opportunity cost
   e.g., gold mining diverts resources that could be used for the production of goods and services.
2. Example 2 (linear cost): $C(e) = ke$ where $e \in \mathbb{R}_+$ and $k \geq 0$
   e.g., e is electricity and k is its unit price

Equilibrium

Money holding decisions

HJB equation of the buyers holding real balances a:

$$\rho V_t^b(a) = \max_{\mathbf{a}_t^* \geq 0} \{\rho\,(a - a_t^*) + r_t a_t^* \\ + \alpha \chi_t \theta \left\{ u\left[q(a_t^*)\right] - q(a_t^*) \right\} \\ + \dot{V}_t^b(a)\}.$$

The quantity of trade $q(a_t^*)$ is determined by Kalai bargaining.

Mining decisions

HJB equation of the sellers facing an occupation choice:

$$\rho V_t^s = \max_{e_t \in \{0,1\}} \left\{ \lambda(\bar{A} - A_t) e_t \phi_t + \alpha(1 - e_t)(1 - \theta)\left[u\left(q_t\right) - q_t\right] + \dot{V}_t^s \right\}.$$

Mining effort $e_t = 1$ iff

$$\lambda(\bar{A} - A_t)\phi_t > \alpha(1 - \theta)\left[u\left(q_t\right) - q_t\right].$$

Price dynamics

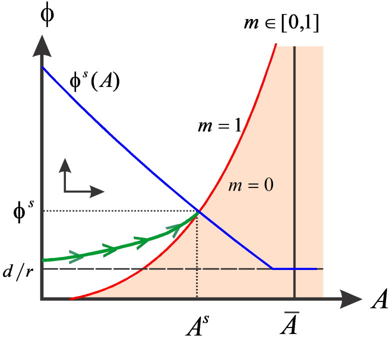
At any instance, the economy is characterized by a pair of (A_t, ϕ_t). The law of motion for the supply of money is:

$$\dot{A} = \lambda(\bar{A} - A_t)m_t.$$

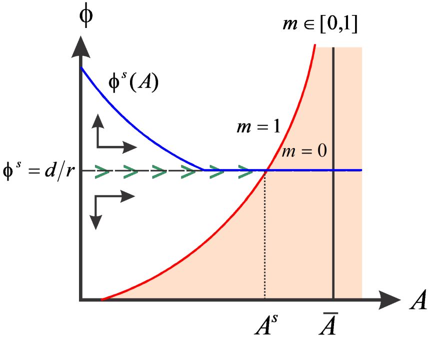
Value of money ϕ_t evolves according to

$$\frac{\dot{\phi}_t + d}{\phi_t} = \rho - \alpha\chi_t\theta\left[\frac{u'\left[q(\phi_t A_t)\right] - 1}{(1-\theta)u'\left[q(\phi_t A_t)\right] + \theta}\right].$$

Mining divisible assets

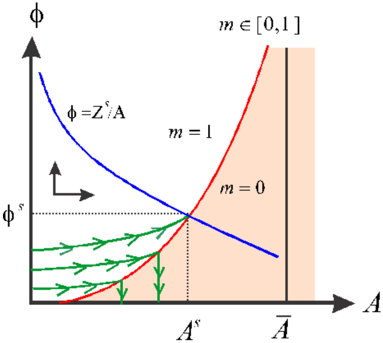


Scarce liquidity

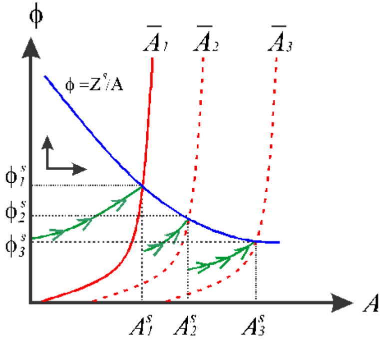


Abundant liquidity

Mining fiat money



Fiat money



Increase in potential money supply

Crypto mining

Exogenous money growth rate $\pi(A)$:

$$\dot{A}_t = \pi(A_t)A_t.$$

The HJB equation for a seller becomes

$$\rho V_t^s = \max_{e_t \in \mathbb{R}_+} \left\{ -e_t k + \frac{e_t}{m_t} \pi(A_t) Z_t + \alpha(1-\theta)\left[u\left(q_t\right) - q_t\right] + \dot{V}_t^s \right\}$$

where $Z_t \equiv \phi_t A_t$.

Crypto mining

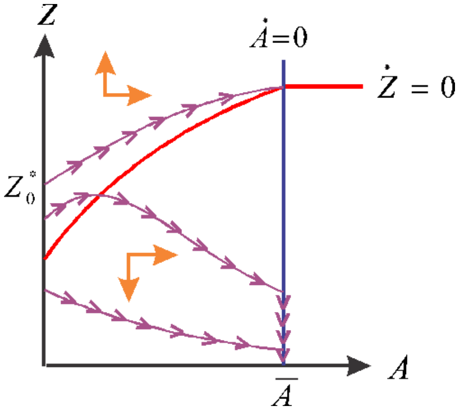


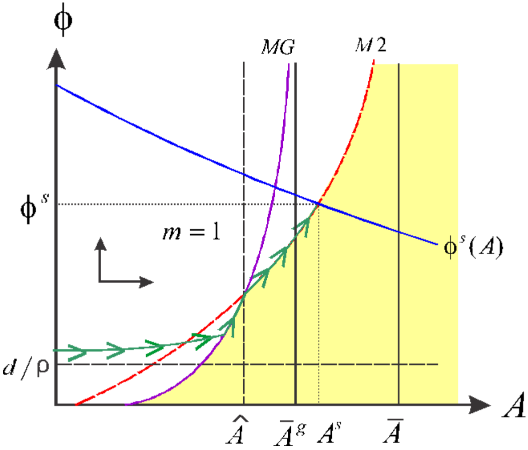
Figure: Z_t rises while ϕ_t falls over time.

Competing Monies

Competing private monies

- Two private commodity monies: silver (Ag) and gold (Au).
- Generate the same flow dividend d.
- Perfect substitutes as a mean of payment.
- Buyers carry a portfolio of $\mathbf{a}_t^* = \left(a_t^{g*}, a_t^{u*}\right)$.
- Miners can produce silver or gold, but not simultaneously.

Dual asset economy



Government money

- Privately produced money (b) vs government-produced (g).
- Acceptabilities in different meetings:
  - A fraction γ_b of meetings where only b is acceptable.
  - A fraction γ_g of meetings where only g is acceptable
  - A fraction γ_2 where both monies are acceptable.
- Pricing of money $j = b, g$:

$$\rho - r_j = \alpha\gamma_j(1-m)\theta\left\{\frac{u'(q_j)-1}{(1-\theta)u'(q_j)+\theta}\right\} + \alpha\gamma_2(1-m)\theta\left\{\frac{u'(q_2)-1}{(1-\theta)u'(q_2)+\theta}\right\}$$

Prevent the emergence of private monies

- Monetary policy aims at keeping q_g constant
- There is no equilibrium with production of private money if

$$\frac{\alpha\gamma_b\theta}{1-\theta} + \alpha\gamma_2\sigma\theta\left[\frac{u'(q_g)-1}{(1-\theta)u'(q_g)+\theta}\right] < r.$$

- q_g must be sufficiently high (government money is valuable)
- $\alpha\gamma_b$ is small (private money is not wildly accepted)

Conclusion

- Build a versatile model of privately-produced money
- New insights for private money production
  - boom and burst
  - velocity of money increases over time
- Fundamentals matter: mining technology, mining cost, intrinsic value of money ...
- Potential application: competition among currency designers (Fernandez-Villaverde and Sanches 2019).

Time varying acceptability

Constant acceptability: $\dot{\alpha}_t = 0$ Increasing acceptability: $\dot{\alpha}_t \geq 0$