More on Money Mining and Price Dynamics: Competing and Divisible Currencies

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Motivation

• Many digital monies are privately-produced through “mining”.
• Satoshi Nakamoto:

> The steady addition of a constant of amount of new (Bit)coins is analogous to gold miners expending resources to add gold to circulation. In our case, it is CPU time and electricity that is expended.

• How does mining affect the joint dynamics of the price and supply of privately-produced monies (e.g. gold and Bitcoin)?
Our approach

- We construct a continuous-time model where:
  1. Transactional role of money is endogenous
  2. Money is perfectly divisible and produced through mining with an explicit time dimension
  3. Different mining technologies for tangible or crypto-monies
  4. Different mining cost functions: exogenous flow cost or endogenous opportunity cost
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- Choi and Rocheteau (2020) study a related model with indivisible money. This model is useful because:
  1. Divisible money model is desirable for various applications.
  2. Easily incorporate competing private or government monies.
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Q: What is the dynamics of the value of money?
A: Depending on the mining technology and production cost, currency price can rise, fall, or be non-monotone over time.

Q: Can the government choose monetary policy to prevent the production of the private money?
A: Yes, but only when the private money is not widely acceptable.
Environment
Environment: Time, agents, goods

- Time is continuous: $t \in \mathbb{R}_+$
- A unit measure of buyers and a unit measure of sellers
- Trade numeraire $c$ in an ongoing competitive market
- Trade good $q$ in pairwise meetings
- All goods are non-storable
Environment: Meeting technology

- Meetings with single coincidence of want at Poisson rate $\alpha$
  - Utility of consumption in a pairwise meeting: $u(q)$ with $u(0) = 0$, $u' > 0$, $u'(0) = +\infty$, and $u'' < 0$.
  - Disutility of production in a pairwise meeting: $q$
- Anonymity: Individual trading histories are private
- Agents lack commitment, i.e., cannot commit to repay debt
Environment: Divisible assets

- Money is perfectly divisible and pays a dividend flow \( d \geq 0 \)
- In-between pairwise meetings agents can trade money with numeraire \( c \) in competitive exchanges
- Price of money in terms of numeraire: \( \phi_t \)
- Rate of return of money: \( r = \left( d + \dot{\phi} \right) / \phi \)
- Amount held by agents: \( A_t \leq \bar{A} \)
Timing

Pairwise meetings (DM)
Environment: Mining technology

- Aggregate mining intensity: \( m_t \equiv \int_0^1 e_i di \) where \( e_i \) is individual mining effort
- Individual discovery/mining rate: \( \Lambda(A, m)e \).

1. Example 1 (gold mining)

\[ \Lambda(A, m) = \lambda (\bar{A} - A) . \]
Environment: Mining technology

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1. **Example 1 (gold mining)**

   \[
   \Lambda(A, m) = \lambda (\bar{A} - A).
   \]

2. **Example 2 (crypto mining)**

   \[
   \Lambda(A, m) = \frac{\pi(A)}{m} A.
   \]

   In the case of Bitcoins,

   \[
   \pi(A) \approx \lambda \left( \frac{\bar{A} - A}{A} \right).
   \]
Environment: Mining cost

- Flow cost of mining: $C(e) \geq 0$

Example 1 (occupation choice): $e \in \{0, 1\}$ and $C(1)$ is an endogenous opportunity cost e.g., gold mining diverts resources that could be used for the production of goods and services.
Environment: Mining cost

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1. **Example 1** (occupation choice): \( e \in \{0, 1\} \) and \( C(1) \) is an endogenous opportunity cost
   e.g., gold mining diverts resources that could be used for the production of goods and services.

2. **Example 2** (linear cost): \( C(e) = ke \) where \( e \in \mathbb{R}_+ \) and \( k \geq 0 \)
   e.g., \( e \) is electricity and \( k \) is its unit price
Equilibrium
HJB equation of the buyers holding real balances $a$:

$$
\rho V^b_t(a) = \max_{a^*_t \geq 0}\{\rho (a - a^*_t) + r_t a^*_t \\
+ \alpha \chi_t \theta \{ u[q(a^*_t)] - q(a^*_t) \} \\
+ \dot{V}^b_t(a) \}.
$$

The quantity of trade $q(a^*_t)$ is determined by Kalai bargaining.
Mining decisions

HJB equation of the sellers facing an occupation choice:

\[ \rho V_t^s = \max_{e_t \in \{0,1\}} \left\{ \lambda (\bar{A} - A_t) e_t \phi_t + \alpha (1 - e_t) (1 - \theta) [u(q_t) - q_t] + \dot{V}_t^s \right\} . \]

Mining effort \( e_t = 1 \) iff

\[ \lambda (\bar{A} - A_t) \phi_t > \alpha (1 - \theta) [u(q_t) - q_t] . \]
Price dynamics

At any instance, the economy is characterized by a pair of \((A_t, \phi_t)\). The law of motion for the supply of money is:

\[
\dot{A} = \lambda (\bar{A} - A_t) m_t.
\]

Value of money \(\phi_t\) evolves according to

\[
\frac{\dot{\phi}_t + d}{\phi_t} = \rho - \alpha \chi_t \theta \left[ \frac{u' [q(\phi_t A_t)] - 1}{(1 - \theta) u' [q(\phi_t A_t)] + \theta} \right].
\]
Mining divisible assets

Scarcely liquidity

\[ \phi^s(A) \]

\[ m \in [0,1] \]

\[ m = 1 \]

\[ m = 0 \]

\[ \frac{d}{r} \]

\[ A^s \]

\[ \bar{A} \]

Abundant liquidity

\[ \phi^s = \frac{d}{r} \]

\[ m = 1 \]

\[ m = 0 \]
Mining fiat money

Fiat money

Increase in potential money supply
Crypto mining

Exogenous money growth rate $\pi(A)$:

$$\dot{A}_t = \pi(A_t)A_t.$$  

The HJB equation for a seller becomes

$$\rho V^s_t = \max_{e_t \in \mathbb{R}_+} \left\{ -e_t k + \frac{e_t}{m_t} \pi(A_t)Z_t + \alpha(1 - \theta) [u(q_t) - q_t] + \dot{V}^s_t \right\}$$

where $Z_t \equiv \phi_t A_t$. 
Figure: $Z_t$ rises while $\phi_t$ falls over time.
Competing Monies
Competing private monies

- Two private commodity monies: silver (Ag) and gold (Au).
- Generate the same flow dividend $d$.
- Perfect substitutes as a mean of payment.
- Buyers carry a portfolio of $a_t^* = (a_t^g, a_t^u)$.
- Miners can produce silver or gold, but not simultaneously.
Dual asset economy

\[ \phi \]

\[ m = 1 \]

\[ \phi^s \]

\[ d / \rho \]

\[ \hat{\bar{A}} \quad \bar{A}^g \quad \bar{A}^s \quad \bar{A} \]

\[ \Phi^s(A) \]
Government money

• Privately produced money \((b)\) vs government-produced \((g)\).
• Acceptabilities in different meetings:
  • A fraction \(\gamma_b\) of meetings where only \(b\) is acceptable.
  • A fraction \(\gamma_g\) of meetings where only \(g\) is acceptable
  • A fraction \(\gamma_2\) where both monies are acceptable.
• Pricing of money \(j = b, g\):

\[
\rho - r_j = \alpha \gamma_j (1 - m) \theta \left\{ \frac{u'(q_j) - 1}{(1 - \theta)u'(q_j) + \theta} \right\} \\
+ \alpha \gamma_2 (1 - m) \theta \left\{ \frac{u'(q_2) - 1}{(1 - \theta)u'(q_2) + \theta} \right\}
\]
Prevent the emergence of private monies

- Monetary policy aims at keeping $q_g$ constant
- There is no equilibrium with production of private money if
  \[
  \frac{\alpha \gamma_b \theta}{1 - \theta} + \alpha \gamma_2 \sigma \theta \left[ \frac{u'(q_g) - 1}{(1 - \theta)u'(q_g) + \theta} \right] < r.
  \]
- $q_g$ must be sufficiently high (government money is valuable)
- $\alpha \gamma_b$ is small (private money is not wildly accepted)
Conclusion

• Build a versatile model of privately-produced money
• New insights for private money production
  • boom and burst
  • velocity of money increases over time
• Fundamentals matter: mining technology, mining cost, intrinsic value of money ...
• Potential application: competition among currency designers (Fernandez-Villaverde and Sanches 2019).
Time varying acceptability

Constant acceptability: $\dot{\alpha} = 0$

Increasing acceptability: $\dot{\alpha} \geq 0$