Currency Stability Using Blockchain Technology*

Bryan Routledge[†] Ariel Zetlin-Jones[‡]

12.26.2019

Abstract

To date, the volatility of most cryptocurrency prices has impeded their adoption as a money. Actively stabilizing cryptocurrency prices with a policy of a peg is difficult. When a currency is not 100% backed, arbitrary speculative attacks on currencies can arise from self-fulfilling expectations (as in Obstfeld (1996)). Building on Green and Lin (2003), we derive the optimal conditional policy that considers traders in sequence and adjusts the conversion rate based on demand-to-date. We show that such a policy can eliminate the speculative attack since traders late in the sequence have a dominant strategy not to speculate. We discuss how the conditional peg policy can be implemented using smart contract blockchain environment such as Ethereum Network.

*Thanks to Fahad Saleh for helpful comments. We are grateful for the financial support of the PNC Center for Financial Services Innovation at Carnegie Mellon University and The Ripple Foundation.

[†]Tepper School of Business, Carnegie Mellon University; routledge@cmu.edu.

[‡]Tepper School of Business, Carnegie Mellon University; azj@andrew.cmu.edu.

1 Introduction

A central component of most blockchain technologies is a token that can be used as a method of payment. For some blockchains like Bitcoin, the token is a crypto-currency and its use as a means of payment is its primary purpose. For other blockchains, the token serves as a "utility token" to perform transactions on the blockchain. An Ether token, for example, is a means of payment for computer processing on Ethereum Virtual Machine, the runtime environment for smart contracts in Ethereum blockchain technology. In either case, the conversion between crypto-currencies and between crypto-currencies and government issued currencies is important. To date, existing crypto-currencies are simply too volatile to be an effective medium of exchange or a store of value. Figure 1 plots the standard deviation of daily USD price changes of Bitcoin from January 2015 to December 2019. Relative to the Euro-Dollar exchange rate, the price of gold, or even the US stock market, the volatility is an order of magnitude larger.

More generally, currency prices are volatile. Exchange rates fluctuate far more than country differences in economic activity or aggregate price levels (Rogoff (2001)). There is a long history of currency issuers—historically, governments—following policies to stabilize the price of their currency. Typically these policies involve using a "peg" to a more stable currency like the U.S. dollar or commodity price like gold. To maintain the peg, the issuer maintains a stock of reserves (dollars, gold) to redeem their currency at a fixed rate. Currently, countries such as Qatar, Cuba, and Panama have pegged their exchange rate to the U.S. dollar. Previously, Mexico and Argentina had pegged to the dollar but abandoned that policy. For crypto-currencies, Tether has pegged their exchange rate to the U.S. dollar.

Currency stability, even with a policy of a peg is difficult. When a currency is not 100% backed, it is vulnerable to a speculative attack. If enough traders sell (short) the currency, these traders can force a full depletion of the government's foreign reserves leading to a devaluation of the currency. This means that if each trader believes that enough other traders will sell (short) the currency, then it is also optimal for each trader to short the currency in the expectation of a devaluation. As a result, a change in traders' beliefs alone about the likelihood that others traders will speculate against the currency is sufficient to induce a run on the currency. Obstfeld (1996) characterizes this mechanism for how currency pegs can be subject to arbitrary speculative attacks.

The canonical "peg" policy uses a fixed amount of foreign currency reserves, R, and redeems domestic currency into foreign reserves at a fixed exchange rate e. Should reserves be exhausted, the currency floats at rate $e^f < e$. The policy "works" as long as demand for the foreign currency is not large (relative to R). The policy, however, permits equilibria with speculative attacks. A single trader, who does not have a fundamental need for a foreign currency, might choose to trade at e with the anticipation of unwinding the trade at the floating rate, e^f. The speculative profit, net of a transaction cost t, is $e/e^f - t$. If this trader believes that all traders will demand redemption, then the trader rationally anticipates that the currency will depreciate and demands redemption. This canonical "peg" policy is "unconditional" in the sense that the conversion rate e does not depend on demand (except of course, after reserves are exhausted).

In contrast, in this paper, we develop a new theory of optimal exchange rate pegs that are less than 100% backed by foreign (U.S. dollar) reserves and are also immune to speculative attacks. We do this based on the observation that the classical problem of speculative attacks arises from the hoc restriction that exchange rate policy be unconditional. We derive the optimal conditional policy that considers traders in sequence and adjusts the conversion rate based on demand-to-date. We show that such a policy can eliminate the speculative attack since traders late in the sequence have a dominant strategy not to speculate. Either remaining reserves are sufficiently large to cover possible remaining demand or reserves are low but the offered conversion rate is sufficiently low that any further depreciation is modest. In either case, speculating is not profitable. Iterating on this logic, by backward induction, speculating is a dominated strategy and there is no equilibrium that supports a speculative attack. This optimal dynamic exchange rate policy captures much of the stability of a pegged exchange rate while also being immune to speculative attacks. Our framework builds on Green and Lin (2003) who use a dynamic liquidity redemption policy to prevent the classic bank run in a banking model.

The model we develop is agnostic about the currencies involved. It applies equally well to a government issued fiat currency or a blockchain crypto-currency. The particular importance of blockchain technology is two-fold. First, the new technology has spawned a large number of coins. CoinMarketCap tracks the market capitalization (price times outstanding currency) of 200 different cryptocurrencies with a total market cap of approximately $200 billion. The ten largest each have a market capitalization over one billion dollars.[1] Currency stability is directly relevant for these blockchain-driven businesses. For many of the coins like Bitcoin Cash, Ethereum's Ether, Litecoin, price volatility is as large as Bitcoin (Figure 1); the exchange rate with US dollars is extremely volatile. Several crypto-currencies have been designed with price stability as the objective. Tether, one of the largest coins (by

[1]As of December 29, 2019. `https://coinmarketcap.com/`.

market capitalization) with an exchange rate "peg" claims a one-for-one reserves in U.S. dollars. But the exact mechanism Tether is using to maintain price stability is not transparent and has lead to claims of market manipulation[2]. It is often hard to verify the reserve holdings. Tether, for example, severed its relationship with its auditor in January 2018 and then used $850 million (US dollar) of reserves to cover losses in a related company.[3] Other crypto-currencies that aim for price stability include Nubits, Dia, Basecoin. All of these protocols feature collateral or reserves at 100% (or more). Others, at least on the surface, appear to have no workable mechanism for stability other than the name.

Second, and perhaps more importantly, blockchain technology has the potential to credibly implement complicated peg policies. Specifying and communicating a policy that depends on real-time currency demand may not be easy. Moreover, conditional policies may appear less credible since they are more complicated to monitor (think: "rules versus discretion" as in Kydland and Prescott (1977)). "Smart Contracts," such as those on the Ethereum Network are rich state-contingent contracts that are credible since they are immutable and enforced by an irreversible distributed-ledger blockchain technology. That is, they work as unchangeable code that run on decentralized computer network outside the control of the policy setting body. For tokens and currencies inside the Ethereum – so called ERC20 tokens – our conditional peg policy can be implemented with an Ethereum smart contract. Implementing our policy "across ledgers," such as a policy linking a cryptocurrency price to US dollars is more challenging since the US dollars are not transferable inside the blockchain. Interestingly, many central banks are considering digital or cryptocurrencies. The Marshall Islands and Venezuela have issued an official crypto currency (with limited success). To date, these and none of the central bank seem to be on a common blockchain network making that would facilitate an automated "smart contract" currency peg implementation.

What follows is a model without aggregate risk to outline the basic structure. We then introduce aggregate risk where the population of traders is finite so fundamental demand for foreign currency is stochastic. Here, we adapt the sequential-trader mechanism of Green and Lin (2003) and solve explicitly in a three-person setting. For larger economies, Section 4 characterizes some numerical examples. Finally in Section 5, we explore issues of implementation using the Ethereum smart-contracting blockchain as a concrete example.

[2] Griffin and Shams (2018). https://www.bloomberg.com/graphics/2018-tether-kraken-trades/

[3] See New York State Attorney General Letitia James press release of April 25, 2019. https://on.ny.gov/356dsbf.

2 A Model of Currency Crises without Aggregate Risk

In this section, we describe a theoretical model of currency crises in the spirit of Obstfeld (1996) and Morris and Shin (1998). We demonstrate that the existence of an equilibrium resembling a speculative attack is an artifact of an ad hoc restriction on the policy space. When a government (or currency issuer) is permitted to use state-contingent policies to defend an exchange rate peg, then the speculative attack equilibrium does not exist.

2.1 Model Environment

The model economy lasts for two periods and features a continuum of measure one of traders. Each trader owns one unit of *crypto-pesos*. While we describe their fiat endowments at crypto-pesos, for the purpose of our theory one may equivalently think of these as units of paper currency. The economy features two goods: crypto-goods and foreign goods. We normalize the price of the crypto-good to be 1 (in crypto-pesos) while the price of the foreign good in period $t = 0, 1$ is e_t crypto-pesos.

Before the beginning of period 0, each trader is identical and has a privately observed, uninsurable risk of being of type C or type F. Each trader learns their own type at the beginning of period 0. With probability $\mu_F > 0$, traders are of type F, whom we refer to as *foreign* (consumption) traders, and care only about period 0 consumption of foreign goods. With probability $\mu_C > 0$, traders are of type C, whom we refer to as *crypto* traders, and care only about period 1 consumption of a bundle of foreign and crypto-goods. Given x units of crypto-pesos in period 1, we assume that a crypto trader's preferences are such that she will spend a portion $\lambda x > 0$ of her crypto-pesos on crypto goods and a portion $(1 - \lambda)x$ of her crypto pesos on foreign goods netting her a total consumption bundle of $[(1 - \lambda)e_1 + \lambda]x$.

In addition, we allow traders to convert crypto-pesos into foreign currency in period 0 (at price e_0), store the foreign currency until period 1, and convert back into crypto-pesos (at price $1/e_1$) in period 1 as a form of speculation. We assume that this speculation bears a fixed cost $t > 0$ denominated in crypto-pesos. Formally, let d_t^j denote the units of crypto-pesos trader j requests to convert into foreign currency in period t, let $d_t = (d_t^j)_{j\in[0,1]}$, and let $(\phi_0(d_0), \phi_1(d_0, d_1))$ denote the fraction of crypto-pesos the trader is able to convert into foreign currency in period 0 and period 1. (We define ϕ_t for given exchange rate policies below.)

Since foreign traders only care about period 0 consumption, each such trader will always submit $d_0^j = 1$. Crypto traders may wish to speculate by submitting a conversion request of $d_0^j > 0$ depending on their perceptions of the path of exchange rates. Note that given (e_t, ϕ_t), a crypto trader who submits conversion demand d_0^j in period 0 will enter period 1 with

$$x_1^j = 1 - \phi_0(d_0)d_0^j + \frac{e_0}{e_1}\phi_0(d_0)d_0^j - t\mathbb{1}_{[d_0^j > 0]} \tag{1}$$

crypto-pesos (with $\mathbb{1}$ as an indicator function). The amount d_1^j reflects the un-coverted crypto-pesos, $1 - \phi_0(d_0)d_0^j$, the period 1 value of the crypto-pesos converted in period 0 (denominated in crypto-pesos), $\frac{e_0}{e_1}\phi_0(d_0)d_0^j$, less any transaction cost. Our assumption on preferences implies that such a trader will submit a conversion demand $d_1^j = (1-\lambda)x_1^j$ in period 1. Finally, we assume that foreign investors stand willing to convert crypto-pesos into foreign currency at a floating price e^f.

Given a path of exchange rates $e = (e_0, e_1)$, conversion rates $\phi = (\phi_0, \phi_1)$, and foreign currency demands $d = (d_0, d_1)$, each trader $j \in [0,1]$ has utility function of the form

$$U(d_0^j; e, \phi, d) = \begin{cases} u(e_0\phi(d_0)d_0^j) & \text{if } j \text{ is of type } F \\ u\left(\left[(1-\lambda)[\phi_1(d_0,d_1)e_1 + (1-\phi_1(d_0,d_1))e^f] + \lambda\right] x_1^j\right) & \text{if } j \text{ is of type } C \end{cases} \tag{2}$$

where x_1^j is given by (1).

2.2 Optimal Policy

We envision a policy maker that chooses an optimal exchange rate policy. We refer to the policy maker as a government but this could be a currency board or a crypto-currency issuer. The government has an initial endowment of R_0 foreign reserves. The policy choice is an *exchange rate policy* $(e_0(d_0), e_1(d_0, d_1), \phi_0(d_0), \phi_1(d_0, d_1))$ that specifyies exchange and conversion rates in each period as a function of the relevant history. The exchange rate policies must be feasible.

Definition 1: An exchange rate policy is *feasible*, if and only if for all d_t,

$$e_0(d_0)\phi_0(d_0)\int d_0^j dj \leq R_0 \tag{3}$$

$$e_1(d_1)\phi_1(d_0,d_1)\int d_1^j dj \leq R_0 - e_0(d_0)\phi_0(d_0)\int d_0^j dj. \tag{4}$$

Optimal Policy with Limited Contingency Policies. Motivated by Obstfeld (1996), we now place a restriction on the class of policies the government may consider and demonstrate that under this restriction, the optimal exchange rate policy admits a speculative attack equilibrium. Consider first a government that defends an exchange rate peg—a fixed e_0 in our model—until it runs out of reserves in which case it converts demand uniformly in period 0 and allows the exchange rate to float to e^f in period 1. We refer to such a policy as a *limited contingency policy.*

Definition 2: An exchange rate policy is a *limited contingency policy* if it satisfies

$$e_0(d_0) = \bar{e}_0 \quad \forall d_0 \tag{5}$$

$$\phi_0(d_0) = 1 \quad \forall d_0 \textit{ such that } \bar{e}_0 \int d_0^j dj \leq R_0 \tag{6}$$

$$\phi_0(d_0) = R_0/[\bar{e}_0 \int d_0^j dj] \quad \forall d_0 \textit{ such that } \bar{e}_0 \int d_0^j dj > R_0. \tag{7}$$

We note two important observations about limited contingency policies. First, if the government is not able to defend the peg against a given level of foreign currency demand, then it necessarily exhausts its supply of foreign reserves and allows the exchange rate to float in period 1 (or, for all such d_0, it follows that $\phi_1(d_0, d_1) = 0$). Second, when the government is not able to defend the peg, it treats all depositors who demand conversion equally. These two features of limited contingency policies play a key role in allowing for the possibility of speculative equilibria.

Consider next the problem of the government choosing among limited contingency exchange rate policies to maximize the expected utility of the traders subject to feasibility constraints and a no speculation constraint which ensures cypto traders prefer to submit a bid $d_0^j = 0$ rather than $d_0^j > 0$. In such a case, it is immediate that the government will choose $\phi_0(d_0) = \phi_1(d_0, d_1) = 1$ when $\int d_0^j dj = \mu_F$ and $\int d_1^j dj = (1-\lambda)\mu_C$ and will set $\bar{e}_0$ and $\bar{e}_1$ to solve

$$\max \mu_F u(\bar{e}_0) + \mu_C u((1-\lambda)\bar{e}_1 + \lambda) \tag{8}$$

subject to the feasibility constraints (3)-(4) and the speculation constraint,

$$u((1-\lambda)\bar{e}_1 + \lambda) \geq u\left([(1-\lambda)\bar{e}_1 + \lambda]\left[\frac{\bar{e}_0}{\bar{e}_1} - t\right]\right). \tag{9}$$

Since the feasibility constraint (4) necessarily binds, if the no speculation constraint is slack, an optimum exchange rate policy is characterized by the optimality condition,

$$u'(\bar{e}_0) = u'\left(\frac{R_0 - \mu_F \bar{e}_0}{\mu_C} + \lambda\right) \tag{10}$$

which implies $\bar{e}_0 = R_0 + \mu_C \lambda$. To verify this value of $\bar{e}_0$ is optimal, one need only verify the feasibility and no speculation constraint, which yields the following proposition.

Proposition 1: Suppose $\mu_F < R_0$ and λ sufficiently close to 1.[4] *Then the optimal limited contingency policy satisfies*

$$\bar{e}_0 = R_0 + \mu_C \lambda, \bar{e}_1 = \frac{1}{1-\lambda}\left(R_0 - \mu_F \lambda\right)$$

where $\bar{e}_1$ is defined for d_1 such that $\int d_1^j dj = (1-\lambda)\mu_C$.

Notice that as $\lambda \to 1$, if traders believe that crypto traders will not speculate, then traders should rationally anticipate an appreciation of the currency. As a result, a single crypto trader will find speculation to be an unprofitable strategy. Since each crypto trader's best response to the belief that other crypto traders will not speculate is to also not speculate, no speculation is an equilibrium of the optimal limited contingency policy.

Moreover, note that when $\mu_F < R_0$, the government's policy satisfies $\mu_F \bar{e}_0 < R_0 < \bar{e}_0$. This inequality implies that while the government chooses a policy that will not exhaust its reserves if no crypto traders demand currency in period 0, this limited contingency policy will necessarily exhaust all of the government's reserves if all crypto traders demand conversion.

Consider then a crypto trader's incentives to speculate when she believes all other crypto traders will also speculate (choose $d_0^j = 1$). Under these beliefs, the crypto trader rationally anticipates $\phi_0(d_0) = R_0/\bar{e}_0$ and $\phi_1(d_0, d_1) = 0$. As a result, her payoffs from not speculating are given by $u((1-\lambda)e^f + \lambda)$ and from speculating (with $d_0^j = 1$) are given by

$$u\left([(1-\lambda)e^f + \lambda]\left[1 - \frac{R_0}{\bar{e}_0} + \frac{\bar{e}_0}{e^f}\frac{R_0}{\bar{e}_0} - t\right]\right). \tag{11}$$

It follows that whenever $(R_0/e^f) - (R_0/\bar{e}_0) \geq t$, the crypto trader will find it optimal to speculate. We have proved the following lemma.

Lemma 2: If $\frac{R_o}{e^f} - \frac{R_0}{R_0 + \mu_C \lambda} \geq t$ (or e^f is sufficiently small), then the optimal limited contingency policy admits an equilibrium where all crypto traders speculate.

We have shown that limited contingency policies suffice to deliver efficient insurance arrangements against traders uncertain needs for foreign currency. However, as in Obstfeld

[4] Formally, we require $(R_0 + \mu_C \lambda)(1-\lambda)/[R_0 - \mu_F \lambda] \leq t$.

(1996), such policies allow for too much volatility in exchange rates in the sense that they also admit other equilibria. Note that if the floating exchange rate e^f is sufficiently small, then small changes in e^f would induce no change in the optimal exchange rate policy—its solution under Proposition 1 is independent of the floating rate for such values. However, under any equilibrium selection that admits the speculative equilibrium as an outcome (e.g. under a sunspot selection criteria), the model would feature variation in the floating rate price of crypto-pesos.

We now examine optimal policies without the ad hoc restriction on policy contingencies and demonstrate that such polices can eliminate the possibility of speculative equilibrium.

Optimal Policy with Contingent Policies. Consider next the unrestricted problem of choosing any feasible exchange rate policy to maximize ex ante expected utility of the traders. Clearly, in an equilibrium in which only foreign traders submit demand for foreign currency in period 0, the outcomes $\bar{e}_0$ and $\bar{e}_1$ from Proposition 1 are the same. The only difference with arbitrarily contingent policies is that the government is not permitted to change the period 0 exchange rate it offers when total demand in period 0 differs from μ_F.

For example, suppose that $e^f < R_0$ and consider the following policy[5]:

$$e_0(d_0) = \begin{cases} R_0 + \mu_C \lambda & \text{if } \int d_0^j dj = \mu_F \\ e^f & \text{if } \int d_0^j dj \neq \mu_F \end{cases}, e_1(d_1) = \begin{cases} \frac{1}{1-\lambda}\left(R_0 - \mu_F \lambda\right) & \text{if } \int d_1^j dj = \mu_C \\ e^f & \text{if } \int d_1^j dj \neq \mu_C \end{cases}, \tag{12}$$

and

$$\phi_0(d_0) = \begin{cases} 1 & \text{if } \int d_0^j dj = \mu_F \\ 0 & \text{if } \int d_0^j dj \neq \mu_F \end{cases}, \phi_1(d_0, d_1) = \begin{cases} 1 & \text{if } \int d_0^j dj = \mu_F \text{ and } \int d_1^j dj = (1-\lambda)\mu_C \\ 0 & \text{otherwise} \end{cases}. \tag{13}$$

Under this policy, if no crypto traders plan to speculate, Proposition 1 implies that each crypto trader prefers not to speculate. Alternatively, if any fraction of crypto traders are believed to speculate, then each crypto trader expects to receive $u((1-\lambda)e^f + \lambda)$ should she not speculate and $u([(1-\lambda)e^f + \lambda][1-t])$ should she choose to speculate. Hence, the policy trivially rules out alternative equilibria. More generally, there is a large class of policies which implement the efficient outcome in this economy while admitting a unique (no speculation) equilibrium. We state this result in the following lemma.

[5]Given our results in Section 3, the reader should view the assumption $e^f < R_0$ as an innocuous simplifying assumption for exposition that ensures the simple policy described in the text rules out speculative equilibrium.

Lemma 3: There exist exchange rate policies (with arbitrary contingencies) that implement the efficient outcome.

We view Lemma 3 as exchange rate policy analogue to the usefulness of suspension contracts in the literature on bank runs beginning with Diamond and Dybvig (1983). In that literature, the anticipation that the bank will suspend convertibility (of deposits to cash) deposits provides patient depositors with the knowledge that their deposits are safe; the anticipation of suspension, therefore, removes the incentives to run. In our model, the anticipation that the government will abandon its peg (before conversion into foreign currency) removes the incentives of crypto traders (those who do not truly require foreign currency) to speculate into foreign currency.

3 Exchange Rate Policy with Aggregate Risk

In this section, we show that even when the government faces aggregate uncertainty over fundamental demand for foreign currency, optimal exchange rate policies are immune to fluctuations driven purely by traders' speculative motives. We show that the government may eliminate the possibility of speculative equilibria even when its policies are required respect some constraints that naturally arise in this environment, such as a form of sequential service.

3.1 Model Environment with Aggregate Risk

The model environment is essentially the same as that in Section 2 except for two modifications. First, we modify the number of traders so that there are $J > 3$ traders (instead of a continuum of measure 1). As before, at the beginning of period 0, each trader learns whether their type is C or F and $\mu_C > 0$ and $\mu_F > 0$ represent the probability that a given trader is of each type.

This first modification implies that the model now features aggregate risk to the number of crypto (and foreign) traders. One natural interpretation of this risk in our context is that it reflects aggregate uncertainty in the fundamental demand for crypto-currencies. Critically, this source of aggregate risk implies that after observing a large volume of foreign currency demand (perhaps larger than expected), a government cannot determine whether this demand reflects speculative demand in anticipation of a depreciation or a fundamental shock to demand for (domestic) crypto-pesos. Our aim in the rest of this section is to analyze the implications of this uncertainty for the robustness of the government's optimally chosen exchange rate policies.

Our second modification changes the timing and information of actions in the game: we assume that each trader $j \in J$ chooses a strategy of foreign currency demand $d_0^j \in \{0, 1\}$ sequentially with the knowledge of the history of actions chosen by previous traders $i < j$. As before, a choice of $d_0^j = 0$ reflects a choice by the trader to not demand foreign currency—to not speculate—and a choice of $d_0^j = 1$ reflects a choice to demand foreign currency, or speculate. Equivalently, we treat a reported demand $d_0^j = 1$ as a report that the trader is a foreign trader and a reported demand $d_0^j = 0$ as a report that the trader is a crypto trader.

Given this timing and information modification, it is natural to examine how optimal exchange rate policies perform when the government has to convert crypto-pesos into foreign currency for traders sequentially. Below, we explicitly define policies that respect a sequential service constraint. We view this modification as critical to examining the robustness of our results given that the most straightforward policy to eliminate speculative attacks in the model without aggregate risk exploited full knowledge of total foreign currency demand.

3.2 Optimal Policy

Since the government serves traders sequentially given a remaining stock of foreign currency reserves and we will allow for history-contingent exchange rate policies, we may define an exchange rate policy simply as plans for period 0 and period 1 exchange rates. To define history-dependent exchange rate policies, we first define the history of foreign currency demands, $D_0^j = (d_0^1, \ldots, d_0^j)$ for all $j \in \{1, \ldots, J\}$. Then, an exchange rate policy is simply a set of functions $\{(e_0^j(D_0^J))_{j \in \{1,\ldots,J\}}, e_1(D_0^J)\}$.

Definition 3: An exchange rate policy satisfies *sequential service* if and only if for all j, $e_0^j(D_0^J)$ is measurable with respect to D_0^j.

In other words, exchange rate policies satisfy sequential service as long as for each trader j, the exchange rate they receive depends only on the trader's reported foreign currency demand and the reported demands of traders who have previously submitted demands to the government.

Given an initial stock of foreign currency reserves and an exchange rate policy, the foreign currency reserves remaining after the jth trader submits their demand in period 0, d_0^j, satisfies

$$R_0^j(D_0^j) = R_0^{j-1}(D_0^{j-1}) - d_0^j e_0^j(D_0^j). \tag{14}$$

Given a sequential service constraint, the government chooses an exchange rate policy to solve the following program.

$$\max \mathbb{E} \sum_{j=1}^{J} \left[d_0^j u(e_0^j(D_0^j)) + (1 - d_0^j) u \left((1-\lambda) e_1(D_0^J) + \lambda \right) \right] \tag{15}$$

subject to the reserve transition equations, (14), the feasibility constraints

$$\forall j \in \{1, \ldots, J\} \text{ and } D_0^{j-1}, \quad e_0^j(D_0^j) \leq R_0^{j-1}(D_0^{j-1}) \tag{16}$$

$$\forall D_0^J, \quad (1-\lambda) e_1(D_0^J) \sum_{j=1}^{J} (1 - d_0^j) \leq R_0 - \sum_{j=1}^{J} d_0^j e_0^j(D_0^j) \tag{17}$$

and the incentive constraints for crypto-traders,

$$\forall D_0^j, \quad \mathbb{E} \left[u \left((1-\lambda) e_1(D_0^J) + \lambda \right) \middle| D_0^j \right] \geq \mathbb{E} \left[u \left(\left[(1-\lambda) e_1(\hat{D}_0^J) + \lambda \right] \left[\frac{e_0^j(\hat{D}_0^j)}{e_1(\hat{D}_0^J)} - t \right] \right) \middle| D_0^j \right] \tag{18}$$

where the expectations in (18) are with respect to D_0^J or $\hat{D}_0^J$ and where $\hat{D}_0^j = (d_0^1, \ldots, d_0^{j-1}, 1)$ and $\hat{D}_0^j = (d_0^1, \ldots, d_0^{j-1}, 1, d_0^{j+1}, \ldots, d_0^J)$.

If we conjecture (and later verify) that the incentive constraints are slack, then we may solve for the optimal exchange rate policy by way of a straightforward backward induction argument. In particular, for any D_0^J, let $\Theta(D_0^J) = \sum_j (1 - d_0^j)$. Then, for any D_0^J and any given level of foreign currency reserves remaining at the beginning of period one, R, the optimal period one exchange rate $e_1(D_0^J)$ solves

$$W(\Theta(D_0^J); R) = \max \Theta(D_0^J) u \left((1-\lambda) e + \lambda \right) \tag{19}$$

subject to the feasibility constraint, $\Theta(D_0^J)(1-\lambda)e \leq R$. Since there is no reason to retain reserves beyond period 1, it is immediate that for all D_0^J, $e(D_0^J) = R/[\Theta(D_0^J)(1-\lambda)]$ and

$$W(\Theta(D_0^J); R) = \Theta(D_0^J) u \left(\frac{R}{\Theta(D_0^J)} + \lambda \right). \tag{20}$$

We proceed in similar fashion to define the government's value function in period 0 for each possible trader's position, j. This value function depends on the remaining reserves of the government, R, and the total number of previous reports where traders reported that they are crypto traders, $\Theta(D_0^{j-1})$. We omit the dependence of Θ on D_0^{j-1} to simplify notation. The government's value function in period 0 for trader j then satisfies

$$V_0^j(\Theta; R) = \max_{e \leq R} \mu_F \left[u(e) + V_0^{j+1}(\Theta; R - e) \right] + \mu_C V_0^{j+1}(\Theta + 1; R) \tag{21}$$

with the convention $V_0^{J+1}(\Theta; R) = W(\Theta; R)$.

The program described by (19)-(21) is straightforward to solve given functional forms for preferences and a given J either analytically or computationally. Next, we focus our analysis to an economy featuring exactly three traders to highlight theoretically certain features of the optimal exchange rate policy and traders' incentives to speculate (or not). Below, we examine more general economies computationally.

3.3 The Three Trader Economy

Consider a sample economy where $J = 3$. Analysis, detailed in Appendix A reveals the following optimal policy.

Proposition 4: Suppose $J = 3$ and $u(c) = -\exp(-\alpha c)$. Then,

$$\begin{aligned}
e_0^1(R) &= \frac{R}{3} - \frac{2}{3\alpha} \log D \\
e_0^2(\Theta; R) &= \frac{R}{2+\Theta} - \frac{1+\Theta}{\alpha(2+\Theta)} \log \left[\mu_F \exp\left(-\alpha\lambda \frac{\Theta}{1+\Theta} \right) + \mu_C \exp(-\alpha\lambda) \right] \\
e_0^3(\Theta; R) &= \frac{R}{1+\Theta} + \frac{\Theta}{1+\Theta}\lambda \\
e_1(\Theta; R) &= \mathbb{1}_{[\Theta \geq 1]} \frac{R}{\Theta(1-\lambda)} + \mathbb{1}_{[\Theta = 0]} e^f
\end{aligned}$$

where D is a function of the fundamental parameters μ_C, μ_F, λ and α that satisfies $D \leq 1$.

It is straightforward to show that as $\mu_C \to 1$, the period 0 exchange rates satisfy $e_0^j \to (R + 2\lambda)/3$. For a relatively high fraction of crypto-traders, then, the government's optimal policy "pegs" the exchange rate for traders in period 0. Note also that $e_0^3(0; R) = R$ so that in the case where traders 1 and 2 both report they are foreign, the government is prepared to exhaust its remaining reserves for the final trader in case she also reports she is a foreign trader. In this case, the government is prepared to abandon its policy for period 1 and allow the exchange rate to float.

We now demonstrate that this policy is immune to purely speculative equilibria. Our approach mirrors that used by Green and Lin (2003) to demonstrate that bank runs do not occur under the optimal liquidity insurance arrangement with finitely many traders in the canonical bank-run model of Diamond and Dybvig (1983). Formally, we will show by way of a backward induction argument that independent of the actions of the first two traders, trader 3 always prefers to report her type (foreign or crypto trader) truthfully and therefore does not speculate. This result implies that when trader 2 is a crypto trader and must decide whether to report truthfully or not, she knows that independent of her report, the probability trader 3 reports she is a crypto trader is equal to the objective probability that trader 3 is a crypto-trader which is equal to μ_C. Given these objective probabilities, we show that independent of the first trader's report, trader 2 always prefers to report truthfully and therefore does not speculate when she is a crypo-trader. A similar result holds for trader 1.

This backward induction argument implies that when the government implements the optimal policy described in Proposition 4, the game (among traders) has a unique equilibrium in which only foreign traders demand foreign currency in period 0. In this sense, the optimal policy is immune to speculative equilibria.

To show these results, we begin by considering the incentives of trader 3 when she is a crypo-trader (given our preference structure, a foreign trader will always report she is foreign and submit a demand for foreign currency in period 0). Truth-telling, or choosing $d_0^3 = 1$ for this trader is a dominant strategy if and only if for all Θ and R

$$(1-\lambda)e_1(\Theta+1;R)+\lambda \geq \underbrace{\left[(1-\lambda)e_1(\Theta;R-e_0^3(\Theta;R))+\lambda\right]}_{\text{Consump. per Crypto-Peso}}\underbrace{\left[\frac{e_0^3(\Theta;R)}{e_1(\Theta;R-e_0^3(\Theta;R))}-t\right]}_{\text{Spec. Profit}}. \quad (22)$$

Since trader 3 is last, there is no residual uncertainty about the total number of foreign and crypto-traders. As a result, this trader simply compares the payoff she will receive from reporting truthfully to that of lying or speculating. The left-hand side of (22) represents the consumption (of foreign and crypo-goods) of the last trader if she does not speculate where she understands that the period 1 exchange rate will be given by $e_1(\Theta+1;R)$. The right-hand side represents her period 1 consumption payoff per unit of crypto-pesos multiplied by her speculative profit. Notice that this trader recognizes that by mis-representing her type, she influences the period 1 exchange rates. In particular, if $\Theta = 0$ so that the first two traders have reported that they are foreign traders, trader 3 recognizes that by mis-reporting her type, the government simply reduces the exchange rate.

Using the optimal policies from Proposition 4, one may show that (22) holds when $\Theta \geq 1$

if and only if

$$\frac{R}{1+\Theta} + 1 \geq -t\left[\frac{R}{1+\Theta} + 1 - \frac{1}{1+\Theta}\right] \tag{23}$$

and holds when $\Theta = 0$ if and only if

$$R + \lambda \geq \left[(1-\lambda)e^f + \lambda\right]\left[\frac{R}{e^f} - t\right]. \tag{24}$$

Notice that as $\lambda \to 1$, (23) requires $(R+\Theta)(1+t)+1 \geq 0$ which always holds while (24) requires

$$1 + t \geq R\left[\frac{1}{e^f} - 1\right]. \tag{25}$$

The inequality (25) places a lower bound on the floating rate that we will impose at $R = R_0$ to guarantee the third trader always prefers to report truthfully. Notice that the lower bound in (25) necessarily lies below the upper bound on e^f implied by the assumption in Lemma 2.

Consider next the incentives of trader 2 to speculate or not when she is a crypto-trader. These incentives depend critically on the reported type of trader 1 along with the remaining reserves available to the government. When trader 1 reports she is a crypto-trader, trader 2 will prefer to report she is a crypto-trader when

$$\begin{aligned}
&\mu_F u\left[(1-\lambda)e_1(2; R_0 - e_0^3(2; R_0)) + \lambda\right] + \mu_C u\left[(1-\lambda)e_1(3; R_0) + \lambda\right] \\
&\geq \mu_F u\left(\left[(1-\lambda)e_1(1; R_0 - e_0^2(1; R_0) - e_0^3(1; R_0 - e_0^2(1; R_0))) + \lambda\right]\left[\frac{e_0^2(1; R_0)}{e_1(1; R_0 - e_0^2(1; R_0) - e_0^2(1; R_0 - e_0^2(1; R_0)))} - t\right]\right) \\
&\quad + \mu_C u\left(\left[(1-\lambda)e_1(2; R_0 - e_0^2(1; R_0)) + \lambda\right]\left[\frac{e_0^2(1; R_0)}{e_1(2; R_0 - e_0^2(1; R_0))} - t\right]\right).
\end{aligned} \tag{26}$$

On both the left-hand and right-hand side of (26), the probabilities represent the objective probability that trader 3 is a foreign or crypto-trader. If trader 2 reports truthfully and does not demand foreign currency, then she faces risk in the period 1 exchange rate she will receive arising from the possible reports of trader 3. The same is true when trader 2 speculates although in this case the trader also bears risk in her speculative profits. Substituting for the optimal policy rules, we may compute this incentive constraint as a function of fundamentals and the reserves of the government R_0:[6]

$$\mu_F u\left(\frac{1}{3}R_0+\frac{2}{3}\lambda\right)+\mu_C u\left(\frac{1}{3}R_0+\lambda\right)$$
$$\geq \mu_F u\left(\left[\frac{1}{3}R_0+\frac{1}{3}\frac{1}{\alpha}\log B+\frac{1}{2}\lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0-\frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0+\frac{1}{3}\frac{1}{\alpha}\log B-\frac{1}{2}\lambda}-t\right]\right)$$
$$+\mu_C u\left(\left[\frac{1}{3}R_0+\frac{1}{3}\frac{1}{\alpha}\log B+\lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0-\frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0+\frac{1}{3}\frac{1}{\alpha}\log B}-t\right]\right) \tag{27}$$

where $B=\mu_F\exp(-\alpha\lambda/2)+\mu_C\exp(-\alpha\lambda)$. Notice that by the concavity of $u(\cdot)$, to show (27) holds, it suffices to show that $u\left(\frac{1}{3}R_0+\frac{2}{3}\lambda\right)$ is larger than the utility the trader receives from the probability weighted average consumption the trader receives from speculating. In Appendix A, we show that as $\lambda\to 1$, this sufficient condition is equivalent to the requirement that

$$\frac{1}{3}R_0+\frac{2}{3}\geq -t\left[\frac{1}{3}R_0+\frac{1}{3}\frac{1}{\alpha}\log B\right]-t\left[\frac{\mu_F}{2}+\mu_C\right]. \tag{28}$$

In Appendix A, we use similar logic to show that, a sufficient condition to ensure that truth-telling is a dominant strategy for trader 2 when trader 1 has reported she is a foreign trader is

$$\frac{1}{2}R_1+\frac{1}{2}\lambda \;\geq\; \mu_F\left(\left[(1-\lambda)e^f+\lambda\right]\left[\frac{\frac{R_1}{2}-\frac{1}{2\alpha}\log D}{e^f}-t\right]\right)$$
$$+\mu_C\left(\left[\frac{R_1}{2}+\frac{1}{2\alpha}\log D+\lambda\right]\left[\frac{(1-\lambda)\left[\frac{R_1}{2}-\frac{1}{2\alpha}\log D\right]}{\frac{R_1}{2}+\frac{1}{2\alpha}\log D}-t\right]\right) \tag{29}$$

where again D is the same positive constant from Proposition 4.

If indeed the sufficient conditions (23), (24), (28), and (29) hold, then we may determine the incentive constraint for the first trader. Towards this end, the expected utility associated with truthful reporting for Trader 1 is given by

$$\begin{aligned} U^{tt} \;=\; & \mu_C^2 u\left((1-\lambda)e_1(3;R_0)+\lambda\right) \\ & +\mu_F\mu_C u\left((1-\lambda)e_1\left(2;R_0-e_0^2(1;R_0)\right)+\lambda\right) \\ & +\mu_C\mu_F u\left((1-\lambda)e_1\left(2;R_0-e_0^3(2;R_0)\right)+\lambda\right) \\ & +\mu_F^2 u\left((1-\lambda)e_1\left(1;R_0-e_0^2(1;R_0)-e_0^3(1;R_0-e_0^2(1;R_0))\right)+\lambda\right). \end{aligned} \tag{30}$$

[6]Recall that in this case we assume the first trader reports she is a crypto-trader and therefore the government has not used up any reserves for the first trader.

The expected utility associated with speculation is given by

$$
\begin{aligned}
U^{spec} = {} & \mu_C^2 u\left(\left[(1-\lambda)e_1(2; R_0 - e_0^1(R_0)) + \lambda\right]\left[\frac{e_0^1(R_0)}{e_1(2; R_0 - e_0^1(R_0))} - t\right]\right) \\
& + \mu_F \mu_C u\left(\left[(1-\lambda)e_1(1; R_0 - e_0^1(R_0) - e_0^2(0; R_0 - e_0^1(R_0))) + \lambda\right]\left[\frac{e_0^1(R_0)}{e_1(1; R_0 - e_0^1(R_0) - e_0^2(0; R_0 - e_0^1(R_0)))} - t\right]\right) \\
& + \mu_C \mu_F u\left(\left[(1-\lambda)e_1(1; R_0 - e_0^1(R_0) - e_0^3(1; R_0 - e_0^1(R_0))) + \lambda\right]\left[\frac{e_0^1(R_0)}{e_1(1; R_0 - e_0^1(R_0) - e_0^1(1; R_0 - e_0^1(R_0)))} - t\right]\right) \\
& + \mu_F^2 u\left(\left[(1-\lambda)e^f + \lambda\right]\left[\frac{e_0^1(R_0)}{e^f} - t\right]\right)
\end{aligned}
\tag{31}
$$

In Appendix A, we show that as $\lambda \to 1$, a sufficient condition for $U^{tt} \geq U^{spec}$ is given by

$$
\begin{aligned}
\frac{R_0}{3} + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2} \geq & - tp^2\left[\frac{R_0}{3} + 1 + \frac{1}{3}\frac{1}{\alpha}\log D\right] - t\mu_F p\left[\frac{R_0}{3} + 1 + \frac{1}{3}\frac{1}{\alpha}\log D + \frac{1}{2}\frac{1}{\alpha}\log C\right] \\
& - tp\mu_F\left[\frac{R_0}{3} + \frac{1}{2} + \frac{1}{3}\frac{1}{\alpha}\log D\right] + \mu_F^2\left[\frac{\frac{R_0}{3} - \frac{2}{3}\frac{1}{\alpha}\log D}{e^f} - t\right].
\end{aligned}
\tag{32}
$$

With these conditions in hand, we then have the following result.

Theorem 5: For λ and μ_C in a neighborhood of $\lambda = \mu_C = 1$ and $1 + t \geq R_0[\frac{1}{e^f} - 1]$, the optimal policy in Proposition 4 admits a unique equilibrium with no speculative trade.

The proof is immediate given the sufficiency conditions, (23), (24), (28), (29), and (32). The two limits play independent, but complementary roles in the proof of Theorem 5. Notice from Proposition 4 that as $\lambda \to 1$, $e_1(\Theta; R) \to \infty$ as long as $R > 0$. Whenever $\Theta > 0$, the government's optimal policy retains reserves into period 1 so that in any such state crypto traders expect the exchange rate to actually *appreciate* from their opportunity to trade in period 0 until period 1. This expected appreciation reduces their incentives to speculate since they expect to end up with a negative holding of crypto-pesos after bearing the transaction costs of speculating.

This logic fails, however, in the state where all traders report they are foreign traders—an event which occurs with strictly positive probability in the finite trader economy. As a result, a crypto-trader in an early position, such as the first trader, may expect to earn speculative profits from a depreciation in the event that traders 2 and 3 happen to be foreign traders. As $\mu_C \to 1$, the likelihood of this profitable event tends to 0, though, and the losses the trader earns in other states of the economy dominate leading the trader to prefer not to speculate.

As we show next, by of numerical simulations, however, these limiting results are not particularly special. That is, we show that in many cases, the optimal exchange rate policy is dominant strategy incentive compatible even if on average, the optimal policy features no appreciation between periods 0 and period 1.

4 Large, Finite Economies

The model presented so far has focussed on the period-zero exchange rate of sequential trader j, e_0^j, relative to the period one exchange rate e_1. The optimal policy considering the trades that have happened prior to j's arrival sets e_0^j such that speculative trade across e_0^j and e_1 is an iteratively dominated strategy. We are aiming towards a strategy we can implement on a blockchain. So the real-time dynamics of rates throughout period zero are interesting. In particular, does the stochastic path of rates within period zero, $e_0,^1, ..., e_0^j, e_0^{j+1}, ..., e_0^J$ and period one rate e_1 allow for speculative trade. Here we explore this with a numerical example.

To solve for the optimal exchange rate policy, we solve recursively the dynamic program of the policy maker specified in equations (20) and (21). As above, this does not include any no-speculation constraints – we can check to see if they are slack in the optimal policy. There are three state variables relevant to the optimal policy. The environment here has a finite horizon, so the policy depends on trader order, $j \in \{1, ..., J\}$. Next, at trader j, the policy depends on the behavior of the people who have arrived before j. It turns out, and this is easy to see by inspecting equations (20) and (21), that the behavior prior to j's arrival is captured by the number of prior traders that are type C, $\Theta_j \in \{0, 1, ..., j-1\}$, and the level of reserves available, $R \in [0, R_0]$. Solving this numerically is straightforward. Of the state variables, only reserves needs to be approximated numerically with a grid and linear interpolation. Finally, we also assume the floating rate is sufficiently low that the optimal policy exchange rate is higher. This implies that reserves are allocated to the traders and, in this setting, no additional reserves are incoming.

The parameters of the simulation are arbitrary – this is too stylized an example to calibrate. Here, traders have CRRA preferences with (mild) constant relative risk aversion of 0.75. We consider $J = 100$ traders. Consistent with the spirit of our setting, most are are expected to be type C with $\mu_C = 0.85$ and $\mu_F = 1 - \mu_C = 0.15$. To make the example interesting, we need the currency board's initial level of reserves to be sensible. If reserves are too low, particularly with respect to date one demand, then it is not feasible to support the exchange rate. Given the specific preferences in equation (2), this translates to initial reserves that are above the expected intrinsic demand from the type C, i.e., $R > J\mu_C(1-\lambda)$. The $J\mu_C$ is the expected number of C people and $1-\lambda$ is the strength of their preference for the foreign currency. For our choice of R, here we happen to use $R = 2$, this implies a λ close to one. Here we use $\lambda = 0.97$. Similarly, if the reserves are too high, akin to one-for-one backing, then there is no worry of speculative trade. Again, roughly, we need reserves not to be large with respect to total expected demand $J\mu_F + \mu_C(1-\lambda)$. This leads to a calibration where e_1 is not mechanically large relative to e_0^j.

Figure 2 summarize the simulations. The vertical axis is the exchange rate (e). The horizontal axis is the position of the trader arrivals (j and with j=J+1 as e_1). With a finite number of traders, our setting has aggregate risk as in Section 3, so the exchange rate offered to j is ex ante stochastic. Here we simulate paths the economy (100 paths) with draws that assigns each trader j to C with probability μ_C or F with probability μ_F. The solid and thick line is the mean across simulations. The shaded regions show the $25-75$ percentile range (darker red) and the $10-90$ percentile range (lighter red). As a boundary, the top line is the exchange rate policy that would result in the (unlikely) event all traders were C. Similarly, the bottom line is the case all traders are F.

Figure 2 highlights a few elements of the model. First note the exchange rate is, on average, flat. There is no trend that will tempt a type C trader to sell at e_0^j in hopes of repurchasing later at an expected lower value of e_0^{j+n} or e_1. In fact, in this example we do not need a particularly large value for the transaction cost to rule out speculation. Here, the expected value of the speculative trade is close to zero. Given the variance of such a trade is not zero, risk aversion, in addition to any transaction cost, dissuades speculation. The red bands in Figure 2 highlight the variance of the exchange rate policy. The exchange rate reacts to traders reporting C or F. Notice that the reaction of the optimal exchange rate to a trader j's type is stronger when j is larger. That is, as the uncertainty resolves about the number of C traders, Θ, the optimal policy allocates reserves more aggressively between the two types.

To see how the backward induction approach rules out a speculative attack equilibrium, see Figure 3. Here, across all four panels, the draws of all the simulations are identical. The figure highlights how the optimal policy reacts to trader reports. In Figure 3, panel (a), the reports of traders $j = 10, 11, 12$ are set to C (top line) or type F (bottom line). For the bottom line where three type F has been reported, the optimal policy has depreciated the exchange rate in response to this unusual (given $\mu_F = 0.15$) event. Note that path of the exchange rate from $j = 13$ and forward is flat and will not support profitable speculation. Panels (b), (c), and (d) are analogous but move the position of the traders we set. For panel (d), when the traders $j = 80, 81, 82$ are set to F the exchange rate devaluation is stronger (compare with panel (a)). Starting at $j = 80$, the sting of three F types is a much larger fraction of the remaining traders to come. Hence the larger reaction.

5 Implementation Discussion

The model environment we have presented is not specific to the currencies involved. However, the conditional exchange rate policy is well suited to some blockchain settings. In particular, some blockchains facilitate "smart contracts." The smart contracts are not legal contracts that require court enforcement. Instead, the smart contracts are scripts and associated data (or states) that are stored and executed on a distributed platform. They are contracts in that they are credible enforced by an irreversible distributed-ledger blockchain technology. Since our conditional peg policy depends on real-time currency demand, it may be hard to communicate, make credible, and monitor. So implementing the policy via a smart contract would help.

The Ethereum Network is the largest (measured by market capitalization) blockchain setting with a smart contracting environment. Ethereum is also Turing-complete meaning it offers a rich set of possible contracts. The coding environment for Ethereum, "Solidity," is an object-oriented language that is designed for the distributed setting – executed code has to yield the same result for everyone on the distributed network who might update the blockchain. One important standardized object is the ERC20 tokens.[7] ERC20 tokens have standard properties of an asset in that they can be owned and transferred. Of course, whether or not these assets serve as a currency is separate issue. ERC20 tokens are typically the "coins" that are created and sold when a new blockchain-related business on the Ethereum Network does a "Initial Coin Offering" (ICO). Often these tokens of the ICO are like tickets that give access to the soon-to-be-created service or application. Filecoin, as an example, is used to rent file storage space.[8] The standardization means these tokens can span contracts that otherwise have different authors and distinct businesses. That is, we can write a contract/script that exchanges one ERC20 token for another (or for Ether, the primary cryptocurrency on Ethereum's network) even if we are unrelated to the contracts that created the coins.

Creating and deploying a smart contract to implement our conditional peg policy is, in

[7]The term "ERC20" stands for Ethereum Request For Comments with 20 to distinguish it from other standards discussions.

[8]It is a separate and interesting question as to why these businesses are choosing to use a separate token for access as opposed to just pricing in terms of, say, Ethereum's Ether. Presumably, some motive for the "pre-sale" of tokens is capital structure (Davydiuk, Gupta, and Rosen (2019)). Other motives are coordination and commitment in the industrial organization of the business (Lee and Parlour (2019), Li and Mann (2018), and Goldstein, Gupta, and Sverchkov (2019)).

principle, straightforward. Imagine we were using a stock of Ether reserves, R, to support an ERC20 coin using a policy function calculated as in the Section 4. Our smart contract script would allow users to exchange their ERC20 coin for Ether at the rate $e_0^j(\Theta_j)$ with j as an index of the trader and Θ_j tracking the state variable of how many ERC20 trades have occurred prior to j.[9] The Ethereum protocol facilitates transparency. The underlying Solidity code can be published and it is straightforward to check that the published source code matches the compiled code deployed on the blockchain.[10] Similarly, the protocol facilitates commitment by making contracts immutable.[11]

The key to eliminating speculative attacks is that traders are in a known queue. In $e_0^j(\Theta_j)$, the j is the trader's known place in the queue and condition their decision on their location. One standard solution for ordering is to code the "ordering" as a separate function in the contract. Traders first transaction is to "join" the queue. The order the transactions appear on the blockchain creates the observable queue prior to trading.[12] From Figure 2, note that traders earlier in the queue face less uncertainty about the exchange rate. So a worry with letting a "join" transactions create the queue is that traders have an incentive to influence their position in the queue. On Ethereum, they can do this using the transaction cost.[13] Alternatively, assigning each trader to a (pseudo)random place in the queue, as happens in the simulations in Section 4, is not straightforward. The distributed nature of Ethereum requires scripts to be deterministic – any computer can run/re-run the smart contract code will yield the same result. Generating pseudo-random numbers in smart contracts is a challenge in many settings – gambling, for example – and there are a variety of possible solutions.[14] We leave the exact implementation to future work but it is not an insurmountable task.

ERC20 tokens and Ether currency are both intrinsic to the Ethereum Network. Ownership

[9]We omit the many "user interface" details like functionality to quote a price prior to trade.

[10]This is done via a "hash" of the code. See `https://tokenmint.io/blog/how-to-verify-ethereum-smart-contracts-source-code.html`.

[11]A "self-destruct" function that would remove the contract would need to be explicitly incorporated into the published source code.

[12]Ordering transactions in a blockchain is a key outcome of the consensus protocol. Transactions are ordered in blocks and blocks are ordered in a chain. A node processing transactions (a "miner") orders transactions when constructing a block. Different miners can receive transactions in differing orders. Usually this does not matter as only a single minder will create a valid block, update the blockchain, and miners work on new blocks. When (nearly) simultaneous blocks occur, this lack of consensus about ordering is called a fork. Ethereum has a sophisticated set of rules for resolving these classes. For more on consensus in blockchains see: Biais, Bisiere, Bouvard, and Casamatta (2019) and Ebrahimi, Routledge, and Zetlin-Jones (2019).

[13]Users submitting transactions and calls to smart contracts on Ethereum pay a transaction cost ("gas"). Setting a high "gas limit" can increase the priority of the transaction.

[14]See Ahmed (2019) for one example.

and control is entirely contained on the Ethereum blockchain. In this setting, our conditional peg policy is implementable on the same blockchain (with the complications we noted above). With assets outside the blockchain – dollars, pesos, gold – this is obviously not the case. Even with cryptocurrencies like Bitcoin and Ethereum that use unconnected separate blockchains, implementing a conditional peg policy cannot be done entirely with a smart contract. The extra step that is needed is a legal framework that links the dollars to the blockchain. In the Ethereum setting, for example, this would consist of issuing an ERC20 token that is a legal claim to the off-chain dollars. This transaction is the blockchain analog of creating an asset-backed security. Once created, the ERC20 token serves as the reserve currency in a smart contract. Interestingly, much of the active development of blockchain technology is at the boundary between assets and the blockchain. For example, the Australian Stock Exchange is developing blockchain for securities settlement and. Ripple and Inter-Ledger-Protocol standards look to link payments across blockchains. An interesting policy question for central banks creating new digital currencies is how interoperable they should be with existing blockchains.

6 Conclusion

We have shown that the classical problem of speculative attacks against an under-collateralized currency peg arises from an hoc restriction that exchange rate policy be unconditional. We have shown that the optimal conditional policy that considers traders in sequence adjusts the conversion rate based on demand-to-date. The optimal conditional policy eliminates the speculative attack since traders late in the sequence have a dominant strategy not to speculate.

A significant concern with the optimal conditional policy we derive is the required degree of trust that it requires. Specifically, one must believe that policymakers will abide by the specified, complicated policy (ex post). Implementing this policy using blockchain-based smart contracts removes the scope for moral hazard by the policymaker and therefore persuades individuals that the specified conditional policy will actually be implemented. In this context, the key value of blockchain is in its ability to generate trust that policies will be implemented as specified by policymakers.

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A Analysis of the Three Trader Economy

A.1 Proof of Proposition 4

In this section, we solve the finite trader model when there are 3 traders with utility function $u(c) = -\exp(-\alpha c)$. We solve the model using backward induction in closed form. The value of the currency board of entering period 1 with reserves R and θ crypto-traders to be paid is

$$W(\theta, R) = \theta u\left(\frac{R}{\theta} + \lambda\right).$$

where we have used the fact in period 1, the government always uses up all of its resources, or$e_1(\theta; R) = \frac{R}{\theta(1-\lambda)}$.

Period 0 exchange rates for the 3rd Trader. Suppose 2 traders have already arrived, θ of them have reported they are crypto-traders, and the government has R reserves outstanding. If the trader reports she is a crypto-trader, then the currency board pays nothing out and obtains utility $W(\theta + 1, R)$. If the trader reports she is foreign, then the currency board chooses $e_0^3(\theta; R)$ to solve

$$\max_{e \leq R} u(e) + W(\theta, R - e).$$

Assuming the resource constraint ($e \leq R$) is slack, this maximization requires $e_0^3(\theta; R)$ to satisfy

$$u'(e) = u'\left(\frac{R-e}{\theta} + \lambda\right),$$

or

$$e_0^3(\theta; R) = \frac{R}{1+\theta} + \frac{\theta}{1+\theta}\lambda.$$

Note, $e_0^3(\theta; R) \leq R$ as long as $\lambda \leq R$. We may define a value function for the government as

$$V_3(\theta, R) = \mu_F[u(e_0^3(\theta; R)) + W(\theta, R - e_0^3(\theta; R))] + \mu_C W(\theta + 1, R).$$

Using the optimal policy, $e_0^3(\theta; R)$, we find

$$V_3(\theta, R) = \mu_F(1+\theta)u\left(\frac{R + \theta\lambda}{1+\theta}\right) + \mu_C(1+\theta)u\left(\frac{R + (1+\theta)\lambda}{1+\theta}\right).$$

Period 0 exchange rates for the 2nd trader. We find $e_0^2(\theta; R)$ as the solution to

$$\max_{e \leq R} u(e) + V_3(\theta, R - e).$$

Using the solution to $V_3(\theta, R)$, we may find $e_0^2(\theta; R)$ as the exchange rate that satisfies

$$u'(e) = \mu_F u'\left(\frac{R - e + \theta\lambda}{1+\theta}\right) + \mu_C u'\left(\frac{R - e + (1+\theta)\lambda}{1+\theta}\right).$$

Given CARA utility, this implies

$$\exp(-\alpha e) = \mu_F \exp\left(-\alpha\left[\frac{R+\lambda\theta}{1+\theta} - \frac{e}{1+\theta}\right]\right) + \mu_C \exp\left(-\alpha\left[\frac{R+(1+\theta)\lambda}{1+\theta} - \frac{e}{1+\theta}\right]\right).$$

Solving for e, we conclude

$$e_0^2(\theta; R) = \frac{R}{2+\theta} - \frac{1+\theta}{\alpha(2+\theta)} \log\left[\mu_F \exp\left(-\alpha\lambda\frac{\theta}{1+\theta}\right) + \mu_C \exp\left(-\alpha\lambda\right)\right].$$

As above, we may define a value function for the government facing the 2nd trader in period 0:

$$V_2(\theta, R) = \mu_F[u(e_0^2(\theta; R)) + V_3(\theta, R - e_0^2(\theta; R))] + \mu_C V_3(\theta+1, R).$$

Period 0 exchange rates for the 1st trader. We find e_0^1 as the solution to

$$\max_{e \le R} u(e) + V_2(0, R-e).$$

Note that

$$e_0^2(0; R) = \frac{R}{2} - \frac{1}{2\alpha}\log\left[\mu_F + \mu_C \exp(-\alpha\lambda)\right].$$

If we let C denote the constant,

$$C = \mu_F + \mu_C \exp(-\alpha\lambda)$$

then $e_0^2(0; R) = R/2 - \frac{1}{2\alpha}\log C$ and $R - e_0^2(0; R) = \frac{R}{2} + \frac{1}{2\alpha}\log C$. Then

$$V_2(0, R) = \mu_F\left[u\left(\frac{R}{2} - \frac{1}{2\alpha}\log C\right) + V_3\left(0, \frac{R}{2} + \frac{1}{2\alpha}\log C\right)\right] + \mu_C V_3(1, R).$$

Using the closed form for $V_3(\theta, R)$, V_2 satisfies

$$\begin{aligned} V_2(0, R) =& \mu_F\left[u\left(\frac{R}{2} - \frac{1}{2\alpha}\log C\right) + \mu_F u\left(\frac{R}{2} + \frac{1}{2\alpha}\log C\right) + \mu_C u\left(\frac{R}{2} + \frac{1}{2\alpha}\log C + \lambda\right)\right] \\ &+ \mu_C\left[2\mu_F u\left(\frac{R+\lambda}{2}\right) + 2\mu_C u\left(\frac{R}{2} + \lambda\right)\right]. \end{aligned}$$

The optimal exchange rate policy satisfies

$$u'(e_0^1) = \frac{d}{dR}\hat{V}_2(0, R - e_0^1),$$

or

$$\begin{aligned} u'(e_0^1) =& \mu_F\left[\frac{1}{2}u'\left(\frac{R-e_0^1}{2} - \frac{1}{2\alpha}\log C\right) + \frac{1}{2}\mu_F u'\left(\frac{R-e_0^1}{2} + \frac{1}{2\alpha}\log C\right) + \frac{1}{2}\mu_C u'\left(\frac{R-e_0^1}{2} + \frac{1}{2\alpha}\log C + \lambda\right)\right] \\ &+ \mu_C\left[\mu_F u'\left(\frac{R-e_0^1+\lambda}{2}\right) + \mu_C u'\left(\frac{R-e_0^1}{2} + \lambda\right)\right]. \end{aligned}$$

Using the functional form of $u(\cdot)$, the optimal policy satisfies

$$\exp(-\alpha e_0^1) = \exp\left(-\alpha\left[\frac{R-e_0^1}{2}\right]\right)\left\{\mu_F\left[\frac{1}{2}C^{\frac{1}{2}} + \frac{1}{2}\mu_F C^{\frac{-1}{2}} + \frac{1}{2}\mu_C C^{\frac{-1}{2}}\exp(-\alpha\lambda)\right]\right.$$
$$\left. + \mu_C\left[\mu_F \exp\left(-\alpha\frac{\lambda}{2}\right) + \mu_C \exp(-\alpha\lambda)\right]\right\}.$$

Let D denote the constant,

$$D = \mu_F\left[\frac{1}{2}C^{\frac{1}{2}} + \frac{1}{2}\mu_F C^{\frac{-1}{2}} + \frac{1}{2}\mu_C C^{\frac{-1}{2}}\exp(-\alpha\lambda)\right] + \mu_C\left[\mu_F\exp\left(-\alpha\frac{\lambda}{2}\right) + \mu_C\exp(-\alpha\lambda)\right].$$

Since $C = \mu_F + \mu_C \exp(-\alpha\lambda)$, it follows that

$$D = \mu_F B^{\frac{1}{2}} + \mu_C\left[\mu_F \exp\left(-\frac{\alpha\lambda}{2}\right) + \mu_C\exp(-\alpha\lambda)\right].$$

Then,

$$-\alpha e_0^1 = -\frac{\alpha R}{2} + \frac{\alpha e_0^1}{2} + \log D$$

or

$$e_0^1 = \frac{R}{3} - \frac{2}{3\alpha}\log D.$$

We have shown

$$\begin{aligned}
e_0^1(R) &= \frac{R}{3} - \frac{2}{3\alpha}\log D \\
e_0^2(\theta;R) &= \frac{R}{2+\theta} - \frac{1+\theta}{\alpha(2+\theta)}\log\left[\mu_F\exp\left(-\alpha\lambda\frac{\theta}{1+\theta}\right) + \mu_C\exp(-\alpha\lambda)\right] \\
e_0^3(\theta;R) &= \frac{R}{1+\theta} + \frac{\theta}{1+\theta}\lambda \\
e_1(\theta;R) &= \frac{R}{\theta(1-\lambda)}.
\end{aligned}$$

We also have the transition function for reserves. Let $R_i^0(\theta)$ denote the reserves left to the government when the ith trader arrives in period 0 and θ previous traders have declared they are crypto-traders. Then,

$$\begin{aligned}
R_1^0 &= R_0 \\
R_2^0(\theta) &= R_0 - (1-\theta)e_0^1
\end{aligned}$$

and, remaining reserves at the third person depend on the specific history since it is possible that $e_0^1(R_0) \neq e_0^2(1;R_0)$. We have

$$R_3^0(\theta) = \begin{cases} R_0 & \text{if } \theta = 2 \\ R_0 - e_0^1(R_0) & \text{if } \theta_1 = 0 \text{ and } \theta_2 = 1 \\ R_0 - e_0^2(1;R_0) & \text{if } \theta_1 = 1 \text{ and } \theta_2 = 0 \\ R_0 - e_0^1(R_0) - e_0^2(0;R_0 - e_0^1(R_0)) & \text{if } \theta = 0 \end{cases}$$

Notice, as $\mu_C \to 1$ (so that all agents are crypto with high probability), we find

$$e_0^1(R) \to \frac{R+2\lambda}{3}, e_0^2(1;R) \to \frac{R+2\lambda}{3}, e_0^3(2;R) = \frac{R+2\lambda}{3}$$

and in this sense the exchange rate is "pegged."

A.2 Proof of Theorem 5

Incentives for the 3rd Trader. The incentive constraint when $\theta \geq 1$ requires

$$(1-\lambda)e_1(\theta+1;R) + \lambda \geq \left[(1-\lambda)e_1(\theta;R-e_0^3(\theta;R)) + \lambda\right]\left[\frac{e_0^3(\theta;R)}{e_1(\theta;R-e_0^3(\theta;R))} - t\right]$$

where

$$e_1(\theta;R) = \frac{R}{\theta(1-\lambda)} \quad \text{and } e_0^3(\theta;R) = \frac{R+\theta\lambda}{1+\theta}.$$

Hence,

$$R - e_0^3(\theta;R) = R - \frac{R+\theta\lambda}{1+\theta} = \frac{\theta(R-\lambda)}{1+\theta}$$

and

$$e_1(\theta;R-e_0^3(\theta;R)) = \frac{R-\lambda}{(1+\theta)(1-\lambda)}.$$

These results imply the incentive constraint may be re-written as

$$\frac{R}{1+\theta} + \lambda \geq \left[\frac{R-\lambda}{1+\theta} + \lambda\right]\left[\frac{(R+\theta\lambda)(1-\lambda)}{R-\lambda} - t\right].$$

We show that as $\lambda \to 1$ this incentive constraint necessarily holds. Taking limits of both sides (assuming $R \neq 1$, we have

$$\frac{R}{1+\theta} + 1 \geq -t\left[\frac{R}{1+\theta} + 1 - \frac{1}{1+\theta}\right].$$

Multiplying by $1+\theta$ and simplifing, we have

$$(R+\theta)(1+t) + 1 \geq 0$$

which holds for any $R \geq 0$, $\theta \geq 1$ and $t \geq 0$. When $\theta = 0$, we require

$$(1-\lambda)e_1(1;R) + \lambda \geq \left[(1-\lambda)e_0^3(0;R) + \lambda\frac{e_0^3(0;R)}{e^f}\right] - t\left[(1-\lambda)e^f + \lambda\right].$$

Since $e_1(1;R) = R/(1-\lambda)$ and $e_0^3(0;R) = R$, this constraint requires

$$R\lambda + \lambda + t[(1-\lambda)e^f + \lambda] \geq \lambda\frac{R}{e^f}$$

As $\lambda \to 1$, this condition requires

$$1 + t \geq R\left[\frac{1}{e^f} - 1\right].$$

Since this should hold for all R and R is necessarily (weakly) decreasing in the sequence of traders, is suffices to impose

$$1 + t \geq R_0 \left[\frac{1}{e^f} - 1 \right].$$

We then have for all R and θ that for λ sufficiently close to 1, the last trader has a dominant strategy to report truthfully.

Incentives for the 2nd Trader. Suppose first that trader 1 is a crypto-trader so that $\theta = 1$. The incentive constraint is

$$\begin{aligned} &\mu_F u \left[(1-\lambda) e_1(2; R_0 - e_0^3(2; R_0)) + \lambda \right] + \mu_C u \left[(1-\lambda) e_1(3; R_0) + \lambda \right] \\ &\geq \mu_F u \left(\left[(1-\lambda) e_1(1; R_0 - e_0^2(1; R_0) - e_0^3(1; R_0 - e_0^2(1; R_0))) + \lambda \right] \left[\frac{e_0^2(1; R_0)}{e_1(1; R_0 - e_0^2(1; R_0) - e_0^3(1; R_0 - e_0^2(1; R_0)))} - t \right] \right) \\ &\quad + \mu_C u \left(\left[(1-\lambda) e_1(2; R_0 - e_0^2(1; R_0)) + \lambda \right] \left[\frac{e_0^2(1; R_0)}{e_1(2; R_0 - e_0^2(1; R_0))} - t \right] \right). \end{aligned}$$

To simplify this constraint, note first that one may show

$$\begin{aligned} e_1(2; R_0 - e_0^3(2; R_0)) &= \frac{1}{3(1-\lambda)} \left[R_0 - \lambda \right], \\ e_1(3; R_0) &= \frac{1}{3(1-\lambda)} R_0. \end{aligned}$$

Hence, the left-hand side of the incentive constraint is simply

$$\mu_F u \left(\frac{1}{3} R_0 + \frac{2}{3} \lambda \right) + \mu_C u \left(\frac{1}{3} R_0 + \lambda \right).$$

Second, note that

$$e_0^2(1; R_0) = \frac{1}{3} R_0 - \frac{2}{3} \frac{1}{\alpha} \log B$$

where B is a constant that satisifes

$$B = \mu_F \exp\left(-\frac{1}{2} \alpha \lambda \right) + \mu_C \exp(-\alpha \lambda).$$

Then, one may show

$$\begin{aligned} e_0^3(1; R_0 - e_0^2(1; R_0)) &= \frac{1}{3} R_0 + \frac{1}{3} \frac{1}{\alpha} \log B + \frac{1}{2} \lambda, \\ e_1(1; R_0 - e_0^2(1; R_0) - e_0^3(1; R_0 - e_0^2(1; R_0))) &= \frac{1}{(1-\lambda)} \left[\frac{1}{3} R_0 + \frac{1}{3} \frac{1}{\alpha} \log B - \frac{1}{2} \lambda \right], \\ e_1(2; R_0 - e_0^2(1; R_0)) &= \frac{1}{(1-\lambda)} \left[\frac{1}{3} R_0 + \frac{1}{3} \frac{1}{\alpha} \log B \right]. \end{aligned}$$

Using these results, we may re-write the right-hand side of the incentive constraint as

$$\mu_F u\left(\frac{1}{3}R_0 + \frac{2}{3}\lambda\right) + \mu_C u\left(\frac{1}{3}R_0 + \lambda\right)$$
$$\geq \mu_F u\left(\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2}\lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B - \frac{1}{2}\lambda} - t\right]\right)$$
$$+ \mu_C u\left(\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B} - t\right]\right)$$

Using the concavity of $u(\cdot)$, it suffices (to prove the incentive constraint holds) to show

$$\frac{1}{3}R_0 + \frac{2}{3}\lambda \geq \mu_F\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2}\lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B - \frac{1}{2}\lambda} - t\right]$$
$$+ \mu_C\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B + \lambda\right]\left[\frac{(1-\lambda)\left[\frac{1}{3}R_0 - \frac{2}{3}\frac{1}{\alpha}\log B\right]}{\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B} - t\right]$$

Using tedious, but straightforward algebra, one may show that as $\lambda \to 1$, the right hand side tends to

$$-t\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B(1)\right] - t\frac{1+\mu_C}{2}.$$

Hence, the necessary inequality condition (as $\lambda \to 1$) requires

$$\frac{1}{3}R_0 + \frac{2}{3} \geq -t\left[\frac{1}{3}R_0 + \frac{1}{3}\frac{1}{\alpha}\log B(1)\right] - t\frac{1+\mu_C}{2}$$

or

$$\frac{1}{3}(1+t)R_0 + t\frac{1+\mu_C}{2} \geq -\frac{t}{3}\frac{1}{\alpha}\log B.$$

Since $B \to \mu_F \exp(-\alpha/2) + \mu_C \exp(-\alpha)$ as $\lambda \to 1$, $\lim_{\lambda\to 1} B \geq \exp(-\alpha)$. It follows that $-t\log B/3\alpha \leq t/3$. Since $t/2 \geq t/3$, for all R_0,

$$\frac{1}{3}(1+t)R_0 + t\frac{1}{2} + \frac{t\mu_C}{2} \geq t\frac{1}{3} \geq -\frac{t}{3}\frac{1}{\alpha}\log B$$

so that the required incentive constraint holds for all R_0, α.

Suppose next that Trader 1 is foreign so that $\theta = 0$. The incentive constraint is

$$\mu_F u\left[(1-\lambda)e_1(1; R_1 - e_0^3(1; R_1)) + \lambda\right] + \mu_C u\left[(1-\lambda)e_1(2; R_1) + \lambda\right]$$
$$\geq \mu_F u\left(\left[(1-\lambda)e^f + \lambda\right]\left[\frac{e_0^2(0; R_1)}{e^f} - t\right]\right)$$
$$+ \mu_C u\left(\left[(1-\lambda)e_1(1; R_1 - e_0^2(0; R_1)) + \lambda\right]\left[\frac{e_0^2(0; R_1)}{e_1(1; R_1 - e_0^2(0; R_1))} - t\right]\right)$$

As above, we use the optimal exchange rate policies to express the incentive constraint in terms of reserves and fundamentals. Note that

$$e_1(1; R_1 - e_0^3(1; R_1)) \frac{1}{2(1-\lambda)} \left[R_1 - \lambda \right],$$
$$e_1(2; R_1) = \frac{1}{2(1-\lambda)} R_1,$$

so that the left-hand side of the incentive constraint is

$$\mu_F u \left(\frac{1}{2} R_1 + \frac{1}{2} \lambda \right) + \mu_C u \left(\frac{1}{2} R_1 + \lambda \right).$$

Similarly, letting the constant C satisfy $C = \mu_F + \mu_C \exp(-\alpha\lambda)$,

$$e^2(0; R_1) \frac{R_1}{2} - \frac{1}{2\alpha} \log C,$$
$$e_1(1; R_1 - e_0^2(0; R_1)) = \frac{1}{1-\lambda} \left[\frac{R_1}{2} + \frac{1}{2\alpha} \log C \right].$$

Then the right-hand side of the incentive constraint is

$$\mu_F u \left(\left[(1-\lambda) e^f + \lambda \right] \left[\frac{\frac{R_1}{2} - \frac{1}{2\alpha} \log C}{e^f} - t \right] \right) + \mu_C u \left(\left[\frac{R_1}{2} + \frac{1}{2\alpha} \log C + \lambda \right] \left[\frac{(1-\lambda) \left[\frac{R_1}{2} - \frac{1}{2\alpha} \log C \right]}{\frac{R_1}{2} + \frac{1}{2\alpha} \log C} - t \right] \right)$$

We proceed as in the previous case and prove

$$\frac{1}{2} R_1 + \frac{1}{2} \lambda$$
$$\geq \mu_F \left(\left[(1-\lambda) e^f + \lambda \right] \left[\frac{\frac{R_1}{2} - \frac{1}{2\alpha} \log C}{e^f} - t \right] \right) + \mu_C \left(\left[\frac{R_1}{2} + \frac{1}{2\alpha} \log C + \lambda \right] \left[\frac{(1-\lambda) \left[\frac{R_1}{2} - \frac{1}{2\alpha} \log C \right]}{\frac{R_1}{2} + \frac{1}{2\alpha} \log C} - t \right] \right)$$

As $\lambda \to 1$, straightforward algebra reveals that this inequality holds if

$$\frac{1}{2} R_1 + \frac{1}{2} \geq \mu_F \left[\frac{\frac{R_1}{2} - \frac{1}{2\alpha} \log C}{e^f} - t \right] + \mu_C \left[-t \left(\frac{1}{2} R_1 + \frac{1}{2} \frac{1}{\alpha} \log C \right) - t \right].$$

Noting that $-\log C \leq \alpha$ (as $\lambda \to 1$), we have

$$RHS \leq \mu_F \left[\frac{\frac{R_1}{2} + \frac{1}{2}}{e^f} - t \right] + \mu_C \left[-t \left(\frac{1}{2} R_1 - \frac{1}{2} \right) - t \right].$$

The incentive constraint holds, therefore, as long as

$$\frac{R_1}{2} \left[1 - \frac{\mu_F}{e^f} + \mu_C t \right] + \frac{1}{2} - \frac{1}{2} \frac{\mu_F}{e^f} + t \left[1 - \frac{1}{2} \mu_C \right] \geq 0.$$

Note that if

$$1 - \frac{\mu_F}{e^f} + \mu_C t \geq 0,$$

then

$$\frac{1}{2}-\frac{1}{2}\frac{\mu_F}{e^f}+t\left[1-\frac{1}{2}\mu_C\right]\geq 0.$$

Hence, suppose the first inequality holds (which is a restriction on μ_C given t, e^f that holds whenever $\mu_C \to 1$). Then, at $R_1 = 0$, the inequality holds and raising R_1 relaxes the incentive constraint. Hence, for all $R_1 \in [0, R_0]$ the inequality must also hold.

Incentives for the 1st Trader. Given truthful reporting is dominant for traders 2 and 3, the expected utility associated with truthful reporting for Trader 1 is given by

$$\begin{aligned}U^{tt} &= \mu_C^2 u\left((1-\lambda)e_1(3;R_0)+\lambda\right)+\mu_F\mu_C u\left((1-\lambda)e_1\left(2;R_0-e_0^2(1;R_0)\right)+\lambda\right)\\&+\mu_C\mu_F u\left((1-\lambda)e_1\left(2;R_0-e_0^3(2;R_0)\right)+\lambda\right)+\mu_F^2 u\left((1-\lambda)e_1\left(1;R_0-e_0^2(1;R_0)-e_0^3(1;R_0-e_0^2(1;R_0))\right)+\lambda\right).\end{aligned}$$

The expected utility associated with speculation is given by

$$\begin{aligned}U^{spec} &= \mu_C^2 u\left(\left[(1-\lambda)e_1(2;R_0-e_0^1(R_0))+\lambda\right]\left[\frac{e_0^1(R_0)}{e_1(2;R_0-e_0^1(R_0))}-t\right]\right)\\&+\mu_F\mu_C u\left(\left[(1-\lambda)e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))+\lambda\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))}-t\right]\right)\\&+\mu_C\mu_F u\left(\left[(1-\lambda)e_1(1;R_0-e_0^1(R_0)-e_0^3(1;R_0-e_0^1(R_0)))+\lambda\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^1(1;R_0-e_0^1(R_0)))}-t\right]\right)\\&+\mu_F^2 u\left(\left[(1-\lambda)e^f+\lambda\right]\left[\frac{e_0^1(R_0)}{e^f}-t\right]\right)\end{aligned}$$

Tedious algebra reveals that

$$\begin{aligned}U^{tt} &= \mu_C^2 u\left(\frac{R_0}{3}+\lambda\right)+\mu_F\mu_C u\left(\frac{R_0}{3}+\frac{1}{3}\frac{1}{\alpha}\log B+\lambda\right)\\&+\mu_C\mu_F u\left(\frac{R_0}{3}+\frac{2}{3}\lambda\right)+\mu_F^2 u\left(\frac{R}{3}+\frac{1}{3}\frac{1}{\alpha}\log B+\frac{1}{2}\lambda\right)\end{aligned}$$

and

$$\begin{aligned}U^{spec} &= \mu_C^2 u\left(\left[\frac{R_0}{3}+\lambda+\frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(2;R_0-e_0^1(R_0))}-t\right]\right)\\&+\mu_F\mu_C u\left(\left[\frac{R_0}{3}+\lambda+\frac{1}{3}\frac{1}{\alpha}\log D+\frac{1}{2}\frac{1}{\alpha}\log C\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^2(0;R_0-e_0^1(R_0)))}-t\right]\right)\\&+\mu_C\mu_F u\left(\left[\frac{R_0}{3}+\frac{1}{2}\lambda+\frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(1;R_0-e_0^1(R_0)-e_0^1(1;R_0-e_0^1(R_0)))}-t\right]\right)\\&+\mu_F^2 u\left(\left[(1-\lambda)e^f+\lambda\right]\left[\frac{e_0^1(R_0)}{e^f}-t\right]\right)\end{aligned}$$

where

$$
\begin{aligned}
B &= \mu_F \exp\left(-\frac{\alpha\lambda}{2}\right) + \mu_C \exp(-\alpha\lambda) \\
C &= \mu_F + \mu_C \exp(-\alpha\lambda) \\
D &= \mu_F C^{\frac{1}{2}} + \mu_C B
\end{aligned}
$$

We know that $U^{tt} \geq u\left(\frac{R}{3} + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2}\lambda\right)$ and that U^{spec} is bounded above by the utility of the convex combination of speculative consumption. Hence, it suffices to prove that

$$
\begin{aligned}
&\frac{R}{3} + \frac{1}{3}\log B + \frac{1}{2}\lambda \\
&\geq \mu_C^2 \left[\frac{R_0}{3} + \lambda + \frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(2; R_0 - e_0^1(R_0))} - t\right] \\
&\quad + \mu_F\mu_C \left[\frac{R_0}{3} + \lambda + \frac{1}{3}\frac{1}{\alpha}\log D + \frac{1}{2}\frac{1}{\alpha}\log C\right]\left[\frac{e_0^1(R_0)}{e_1(1; R_0 - e_0^1(R_0) - e_0^2(0; R_0 - e_0^1(R_0)))} - t\right] \\
&\quad + \mu_C\mu_F \left[\frac{R_0}{3} + \frac{1}{2}\lambda + \frac{1}{3}\frac{1}{\alpha}\log D\right]\left[\frac{e_0^1(R_0)}{e_1(1; R_0 - e_0^1(R_0) - e_0^1(1; R_0 - e_0^1(R_0)))} - t\right] \\
&\quad + \mu_F^2 \left[(1-\lambda)e^f + \lambda\right]\left[\frac{e_0^1(R_0)}{e^f} - t\right].
\end{aligned}
$$

As $\lambda \to 1$, this inequality tends towards

$$
\begin{aligned}
&\frac{R_0}{3} + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2} \\
&\geq -t\mu_C^2 \left[\frac{R_0}{3} + 1 + \frac{1}{3}\frac{1}{\alpha}\log D\right] - t\mu_F\mu_C \left[\frac{R_0}{3} + 1 + \frac{1}{3}\frac{1}{\alpha}\log D + \frac{1}{2}\frac{1}{\alpha}\log C\right] \\
&\quad - t\mu_C\mu_F \left[\frac{R_0}{3} + \frac{1}{2} + \frac{1}{3}\frac{1}{\alpha}\log D\right] + \mu_F^2 \left[\frac{\frac{R_0}{3} - \frac{2}{3}\frac{1}{\alpha}\log D}{e^f} - t\right].
\end{aligned}
$$

or

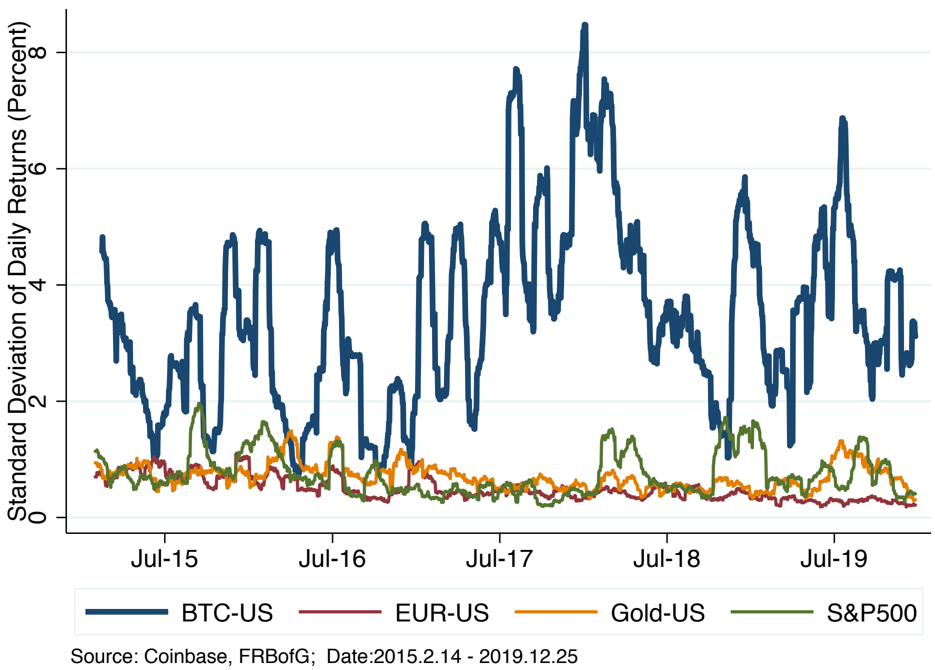
$$
\begin{aligned}
&\frac{R_0}{3}\left[1 - \frac{\mu_F^2}{e^f} + t\left(1 - \mu_F^2\right)\right] + \frac{1}{3}\frac{1}{\alpha}\log B + \frac{1}{2} + \frac{\mu_F^2}{e^f}\frac{2}{3}\frac{1}{\alpha}\log D \\
&\quad + t\left[\mu_C^2\left(1 + \frac{1}{3}\frac{1}{\alpha}\log D\right) + \mu_C\mu_F\left(\frac{3}{2} + \frac{2}{3}\frac{1}{\alpha}\log D + \frac{1}{2}\frac{1}{\alpha}\log C\right) + \mu_F^2\right] \geq 0
\end{aligned}
$$

Using the fact that $C^{\frac{1}{2}} \geq D \geq B \geq \exp(-\alpha)$, the above inequality holds as long as

$$
\frac{R_0}{3}\left[1 - \frac{\mu_F^2}{e^f} + t\left(1 - \mu_F^2\right)\right] + \frac{1}{6} - \frac{2}{3}\frac{\mu_F^2}{e^f} + t\left[\frac{2}{3}\mu_C^2 + \frac{1}{3}\mu_C\mu_F + \mu_F^2\right] \geq 0.
$$

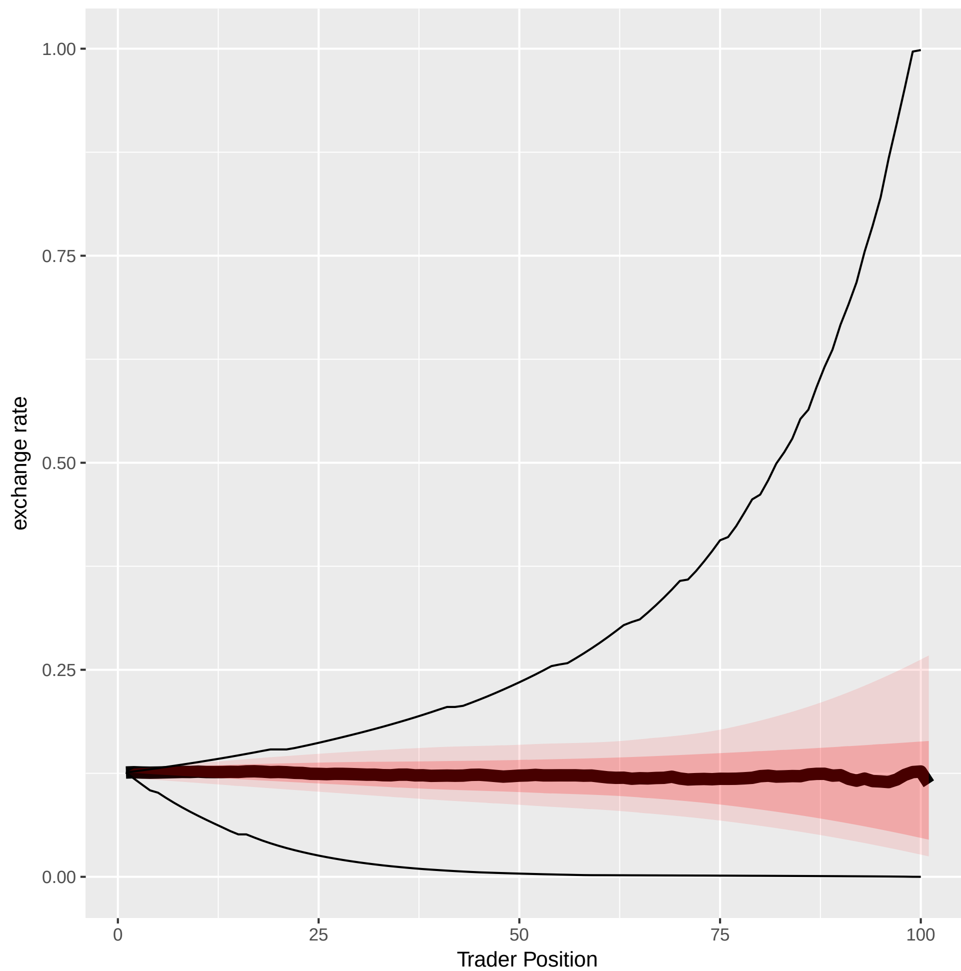
Notice, this inequality is necessarily satisfied as $\mu_C \to 1$.

Figure 1: Daily Volatility



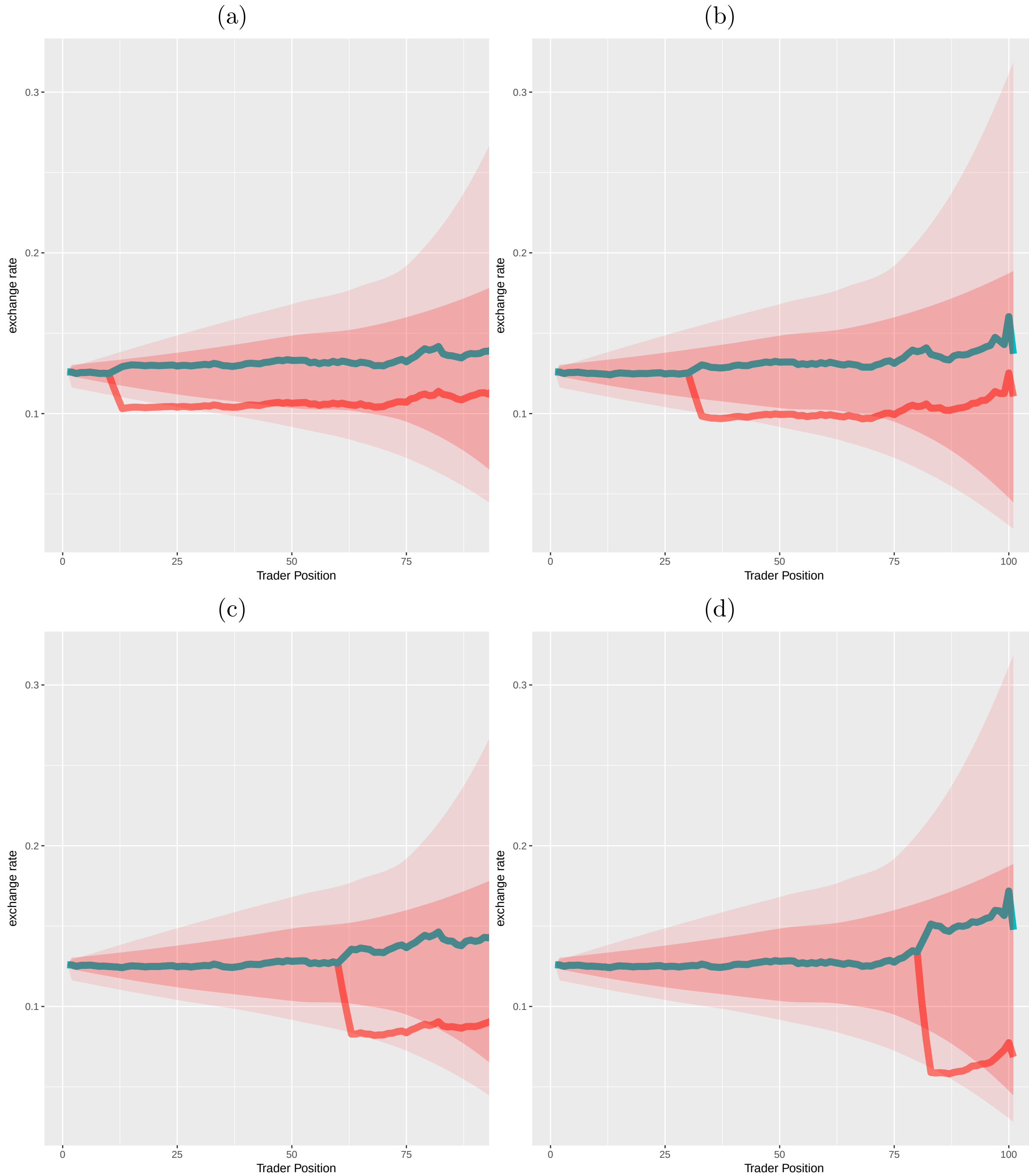
The 30-day (rolling) standard deviation of daily USD price changes of Bitcoin, Euro, S&P500 stock, Gold.

Figure 2: Policy Simulation



Simulated path of exchange rate through $J = 100$ traders in period 0 and period 1 (J=101). The solid black line is the mean path. The 25 – 75 and 10 – 90 percentile range is shown in red. The upper line is the (unlikely) realization of all C-type traders (who do not demand foreign reserves in date 0). The lower line is the path of all F-type traders (who demand foreign reserves).

Figure 3: Policy Simulation



Simulated path of exchange rate through $J = 100$ traders in period 0 and period 1 (J=101). The solid black line is the mean path. The 25 – 75 and 10 – 90 percentile range is shown in red. The simulations in all four panels have identical draws of the C and F types except as follows: In Panel (a), traders $j = 10, 11, 12$ are modified to be either type C (top line) or type F (bottom line). The other panels modify traders similarly but at different points, (b) $j = 30, 21, 32$, (c) $j = 60, 61, 62$, and (d) $j = 80, 81, 82$.