A Search Model of Statistical Discrimination

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Motivation

- Much of the gender gap in wages is accounted for by occupational choice
- Common interpretation: women care more about amenities
- Another common interpretation: women accept low skill jobs because of under confidence and/or higher risk aversion

This Paper

- No fundamental differences between groups
- Construct equilibria in which women are over-represented in low paying jobs.
- Not explaining initial conditions. We think that is fine.
- Due to search externalities this works even when the underlying model without group identities has a unique equilibrium
- Being able to condition on group identities may destabilize symmetric equilibrium

This Paper

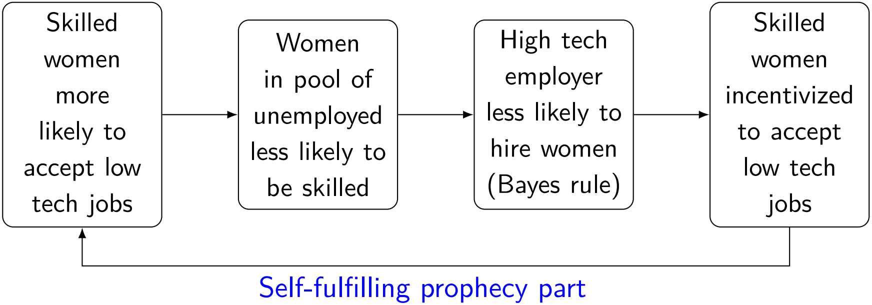
- *Statistical discrimination* (Arrow, 1973; Coate and Loury, 1993; Moro and Norman, 2004) meets *search*
- Standard statistical discrimination models driven by interactions between incentives to acquire human capital and firms' beliefs about human capital distributions.

  Problems

  - Lack of evidence on differences in incentives (Neal and Johnson, 1996; Neal, 2006)
  - Discriminated group is (in equilibrium) less productive, seemingly at odds with women being more educated
- Our model doesn't need skill differences. Skilled women take low end jobs.
- "As if" women are under confident

Basic Ingredients

- Self-confirming beliefs
- Cross-group spillovers via firm entry decisions



Model without Group Identities

Environment

- Infinite horizon, discrete time
- Unit mass of workers
- Fraction ψ of workers are *qualified*. Others referred to as *unqualified*
- Flow value of unemployment b

Environment

- *Low tech* firms: all workers equally productive
- *High tech* firms: only qualified workers are productive
- Vacancy in each sector costs K to maintain
- Wages exogenous
- Flow payoffs

	high tech	low tech
qualified	$w_h, y_h - w_h$	$w_l, y_l - w_l$
unqualified	$w_h, 0 - w_h$	$w_l, y_l - w_l$

- $w_l < y_l \Rightarrow$ low tech firm offers job to anybody
- w_h: $-w_h < 0 < y_h - w_h \Rightarrow$ high tech firm would hire qualified workers only if perfect information

Information Technology

- Firms cannot directly observe if worker is qualified
- Noisy signal (test, interview...) available
- Denote signal by θ. Without loss $\theta \in [0, 1]$
- Conditional distributions: $\theta \sim f_q$ if qualified, $\theta \sim f_u$ if unqualified
- By choice of labeling $\frac{f_q}{f_u}$ is strictly increasing (only actual assumptions: no mass-points and continuity)

Matching Technology

- Random search
- Simplest matching technology: short side of the market matches for sure
- Because of a fixed set of workers and entry of firms, workers match for sure and firms are rationed.
- An *endogenous* proportion p of firms is high tech. This is also the probability that worker matches with a high tech firm.

Separation

- All separations are exogenous
- For simplicity, these are the separations rates we consider

separation rate	high tech	low tech
qualified	ϕ	ϕ
unqualified	$\phi + (1 - \phi)r$	ϕ

- "Revelation" rate r not crucial, but useful to generate conditions ruling out uninteresting equilibria

Equilibrium Problem: Birds' Eye View

- Let π be *endogenous* proportion qualified in *pool of unemployed*
- Given π implies unique optimal hiring rule for high tech firm.
- Hiring threshold $s(\pi)$
- Low tech firms: make offers to anybody
- $(\pi, s(\pi))$ and fraction of high tech firms $p(\pi)$ determine optimal worker acceptance rule $\alpha(\pi)$.
- $\alpha(\pi)$ and π determines $p(\pi)$
- Thus, $(\alpha(\pi), p(\pi))$ determined as "static fixed point" $p(\pi)$
- Together $(\pi, s(\pi), \alpha(\pi), p(\pi))$ determine proportion qualified unemployed. "Dynamic" fixed point in π

Firm Value Functions

Only *high tech* firms have non-trivial decision problem beyond entry

- Continuation value of hiring a *qualified*:
  $W(q,\pi) = y_h - w_h + \beta\left[\phi W(0,\pi) + (1-\phi) W(q,\pi)\right]$
- Continuation value of hiring a *unqualified* is similar:
- With *free entry* $W(0,\pi) = 0$, so

$$
\begin{aligned}
W(u,\pi) &= \frac{-w_h}{1-\beta(1-\phi)(1-r)} \equiv W_u < 0 \\
W(q,\pi) &= \frac{y_h - w_h}{1-\beta(1-\phi)} \equiv W_q > 0
\end{aligned}
$$

Optimal Job Offers

- Given observed signal θ the posterior probability that worker is qualified perceived by firm is

$$\mu(\theta, \pi) = \frac{\pi f_q(\theta)}{\pi f_q(\theta) + (1 - \pi) f_u(\theta)}$$

- Hence, high tech firms hire the worker if and only if

$$\mu(\theta, \pi) W_q + (1 - \mu(\theta, \pi)) W_u \geq 0$$

- By MLR (without loss) there is unique $s(\pi)$ such that worker hired if and only if $\theta \geq s(\pi)$
- Notation: $A_q(\pi) = 1 - F_q(s(\pi))$ used for hiring probability for qualified conditional on match (relevant for worker optimization & firm entry). $A_u(\pi) = 1 - F_u(s(\pi))$ is hiring probability for unqualified worker.

Optimal Entry

- Value of being a matched low tech firm is

$$W_l = \frac{y_l - w_l}{1 - \beta(1-\phi)} > 0$$

- Free entry into high tech sector requires

$$\begin{aligned} K &= \beta p_f \underbrace{\left[\pi A_q(\pi) W_q + (1-\pi) A_u(\pi) W_u\right]}_{\text{profit of high tech sector}} \\ K &= \beta p_f \underbrace{\left[\pi \alpha(\pi) + (1-\pi)\right] W_l}_{\text{expected profit in low-tech sector}} \end{aligned}$$

where $p_f < 1$ is the inverse market tightness.

- Market tightness variable p_f endogenous, but irrelevant for other equilibrium conditions. Invisible in rest of talk.
- Two entry conditions collapsed into indifference condition

$$\pi A_q(\pi) W_q + (1-\pi) A_u(\pi) W_u = (\pi \alpha(\pi) + 1 - \pi) W_l$$

Summary of Optimal Entry

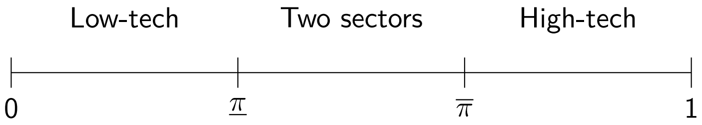
(1) If $\pi < \underline{\pi}$ firms enter in low tech sector only regardless of what workers do

(2) If $\pi > \overline{\pi}$ firms enter in high tech sector only regardless of what workers do

(3) If $\pi \in (\underline{\pi}, \overline{\pi})$, then is a unique $\alpha(\pi) \in (0, 1)$ such that the firms are willing to enter in each sector

(4) If $\pi = \overline{\pi}$, then firms enter in each sector *iff* $\alpha(\overline{\pi}) = 1$

(5) If $\pi = \underline{\pi}$, then firms enter in each sector *iff* $\alpha(\underline{\pi}) = 0$



Worker Problem (Qualified)

- Value of being unemployed

$$
\begin{aligned}
V_0(\pi) &= p(\pi)\left[A_q(\pi)V_h(\pi) + (1-A_q(\pi))(b+\beta V_0(\pi))\right] \\
&\quad + (1-p(\pi)) \max_{\alpha\in[0,1]} \{\alpha V_l(\pi) + (1-\alpha)(b+\beta V_0(\pi))\}
\end{aligned}
$$

- Value of being employed in low-tech sector

$$V_l(\pi) = w_l + \beta\left[\phi V_0(\pi) + (1-\phi)V_l(\pi)\right]$$

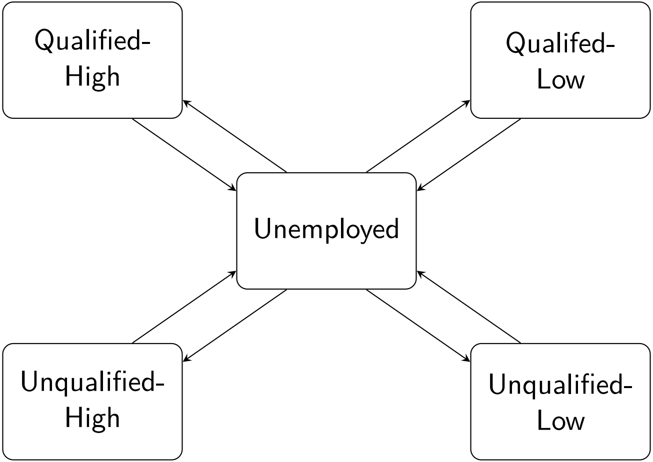
- Value of being employed in high-tech sector

$$V_h(\pi) = w_h + \beta\left[\phi V_0(\pi) + (1-\phi)V_h(\pi)\right]$$

Optimal Acceptance Rule

- Key to being able to solve model by hand is that all that matters for worker acceptance is the probability of a high tech offer in the next period.
- This is driven by exogenous wages and the constant matching probability for workers.
- Then, there exists $Q^* \in [0,1]$ such that qualified workers such that
  (1) Workers must accept for sure if $p(\pi) A_q(\pi) < Q^*$
  (2) Workers are indifferent if $p(\pi) A_q(\pi) = Q^*$
  (3) Workers must reject for sure if $p(\pi) A_q(\pi) > Q^*$

Steady State: Labor Flows



Steady State Conditions

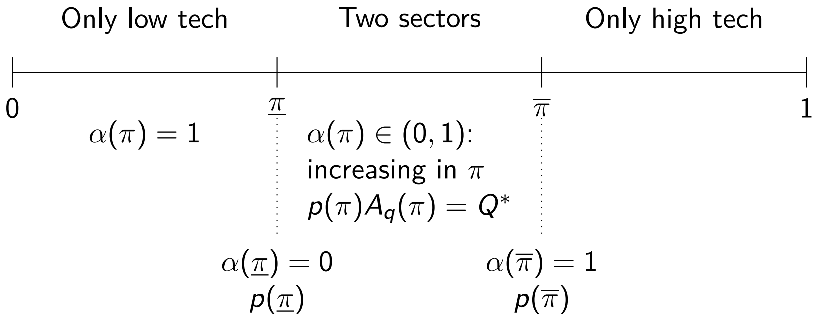
- Four inflow-outflow conditions (qualified & unqualified workers, high & low tech firms)
- Inflow-outflow equations combined with adding up identities on the two types of workers and optimal hiring threshold generate a single steady state condition

$$G(\pi, \alpha(\pi), p(\pi)) = 0$$

where

$$\begin{aligned} & G(\pi, \alpha, p) \\ = \quad & 1 + \frac{pA_u(\pi)}{\phi + (1-\phi)r} + \frac{(1-p)}{\phi} - \frac{1-\psi}{1-\pi}\frac{\pi}{\psi}\left[1 + \frac{pA_q(\pi)}{\phi} + \frac{(1-p)\alpha}{\phi}\right] \end{aligned}$$

Equilibrium Candidates



Introducing Observable Group Characteristic

Adding Observable Group Characteristic

- Workers are from group $j \in \{f, m\}$
- Group identity payoff irrelevant, but observable to firms
- In each group proportion ψ is qualified.
- Everything continuous. Small differences between groups OK.

Adding Observable Group Characteristic

- Proportion of workers from group j denoted λ^j
- Write $\boldsymbol{\pi} = (\pi^m, \pi^f)$
- Most of equilibrium characterization same as with a single group, but a key difference is that the probability of meeting a high tech firm is some $p(\boldsymbol{\pi})$ *for both groups*
- Every equilibrium in basic model corresponds with a *symmetric equilibrium* in model with groups
- Interested in asymmetric equilibria

Optimal Hiring Rule

- Conditional on a match, no change in firm offer rule
- Worker hired iff θ at least $s(\pi^j)$ where the threshold is identical to that in single group model with proportion of qualified unemployed π^j
- Hiring probability after a match are thus separable

$$
\begin{aligned}
A_q\left(\pi^j\right) &= 1 - F_q\left(s\left(\pi^j\right)\right) \\
A_u\left(\pi^j\right) &= 1 - F_u\left(s\left(\pi^j\right)\right)
\end{aligned}
$$

Indifference in Entry

- Interesting equilibria have both sectors active.
- Discriminatory equilibria must have both sectors active, requiring indifference in entry.
- Indifference in entry now reads:

$$\sum_{j=f,m} \lambda^j \left[\pi^j \alpha^j (\boldsymbol{\pi}) + (1-\pi^j)\right] W_l$$
$$= \sum_{j=f,m} \lambda^j \left[\pi^j A_q(\pi^j) W_q + (1-\pi^j) A_u(\pi^j) W_u\right]$$

- Key difference with single group model. One probability to meet a high tech firm determined by behavior in both groups.

Worker Acceptance Rule

- As in baseline model there is a critical probability of high tech offer making worker indifferent between accepting and rejecting low tech jobs
- Critical probability Q^* same as in single group model, so

$$\alpha^j(\boldsymbol{\pi}) = \begin{cases} 1 & \text{if } p(\boldsymbol{\pi}) A_q(\pi^j) < Q^* \\ [0,1] & \text{if } p(\boldsymbol{\pi}) A_q(\pi^j) = Q^* \\ 0 & \text{if } p(\boldsymbol{\pi}) A_q(\pi^j) > Q^* \end{cases}$$

- $p(\boldsymbol{\pi})$ **SAME** for both groups
  - If $\pi^m > \pi^f$ and group f randomizes group m rejects
  - If $\pi^m > \pi^f$ and group m randomizes group f accepts
- Arbitrarily small difference in pool of unemployed generate large difference in behavior. Hence, fully mixed symmetric equilibria non-robust.

- Arbitrarily small difference in pool of unemployed generate large difference in behavior. Hence, fully mixed symmetric equilibria non-robust.
- Have still not worked out the dynamics fully.

Equilibrium with Groups

Steady state conditions

$$
\begin{aligned}
G(\pi^f, \alpha^f(\boldsymbol{\pi}), p(\boldsymbol{\pi})) &= 0 \\
G(\pi^m, \alpha^m(\boldsymbol{\pi}), p(\boldsymbol{\pi})) &= 0
\end{aligned}
$$

- Discriminatory equilibria "pop up" even when symmetric equilibria unique.
- Analytic result
- Numerical results

A Theoretical Result

An informal statement if characterization in paper:

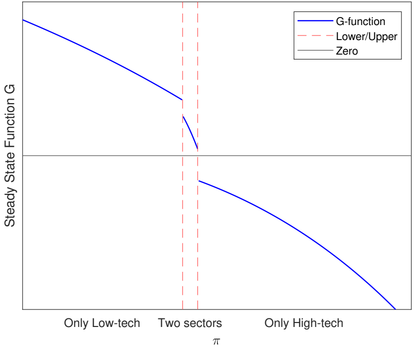
Proposition If there is a fully mixed symmetric equilibrium and the separation rate is small enough, then there exist a range of population proportions such that an equilibrium in which group m rejects and group f accepts low tech offers. In this equilibrium $\pi^m > \pi^f$

- Consistent with unique symmetric equilibrium.
- Group m is better off because they are pickier
- Idea: An interval of choices for p works for worker optimality.
- Each p in the interval corresponds with population proportions creating indifference in entry.
- Suspect similar arguments can be made starting from other types of symmetric equilibria. Numerically, presence of discriminatory equilibria seems very robust.

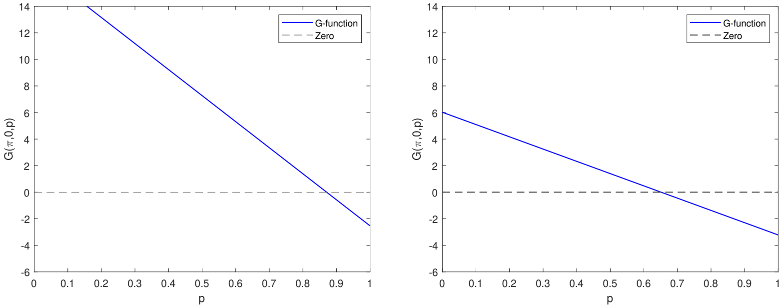
Numerical Example

Begin with baseline model

- Except at $\underline{\pi}$ and $\overline{\pi}$ proportion of high tech entrants and worker acceptance rule nailed by π. Steady state conditions thus determine π
- Hence, figure rules out equilibria with π not at the thresholds.



- At $\underline{\pi}$ workers must reject low tech jobs for sure
- and $\overline{\pi}$ workers must accept low tech jobs for sure
- In each case p adjusts to satisfy steady state condition

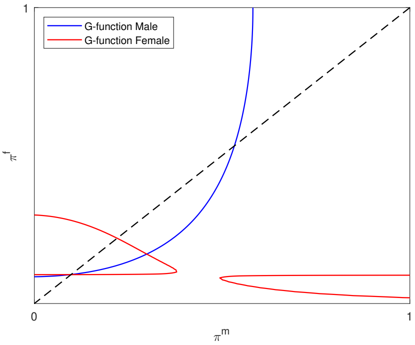


- (Not visible in figure) Left hand side value of p consistent with rejecting low tech jobs. Hence, an equilibrium.
- Right hand side value of p inconsistent with accepting low tech jobs. Hence, not an equilibrium.

Introducing Groups

- Parametrization has unique symmetric equilibrium
- Will now guess that group f is randomizing. This nails $p(\boldsymbol{\pi})$ to keep indifference as $Q^*/A_q\left(\pi^f\right)$
- We also guess group m rejects (so $\pi^m > \pi^f$). This nails $\alpha^m(\boldsymbol{\pi}) = 0$
- Female group $\alpha^m(\boldsymbol{\pi})$ pinned down by two active sectors condition
- Reduced form pairs of (π^m, π^f) consistent with steady state conditions can be plotted in the plane

Numerical Example: Discrimination



- Intersection between red and blue line with $\pi^f < \pi^m$ is an equilibrium
- Higher hiring standard and lower hiring probability for group f in high tech sector
- Group f more likely to accept low tech jobs

Affirmative Action

- Caveat: Regulating hiring probability requires "firms" with many positions. Ways to shoehorn that into model.
- Ignoring the issue above, a constraint that hiring probabilities consistent with population shares makes the group with smaller π more likely to get high tech job
- Hence, affirmative action eliminates asymmetric equilibria. No unintended incentive effects!

Conclusions/Next Steps

- Statistical discrimination driven by occupational choice/mismatch instead of human capital.
- Not subject to problems with human capital based models.
- Self-confirming prophecy, but NOT pure coordination. Other group getting pickier is bad.
- Empirical implication: Gender gaps associated with occupational choice differences may reflect discrimination.
- To do: work out dynamics.
- To do: endogenous wages.

The End