Statistical Discrimination in Ratings-Guided Markets

Yeon-Koo Che (Columbia) Teddy Kim (Emory)

Weijie Zhong (Stanford GSB)

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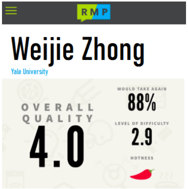
Ratings and Markets

- "Ratings" have become prevalent.



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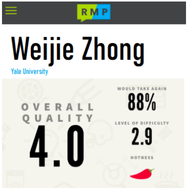

- Interaction (feedback) between ratings and markets
  - ($\Rightarrow$) Ratings guide market transactions.
  - ($\Leftarrow$) Market transactions produce (update) ratings.

This Paper

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   - Main mechanism
     - Ratings from transactions, so endogenous.
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   - Main mechanism
     - Ratings from transactions, so endogenous.
     - Feedback loop: Ratings less accurate for group 1 $\Rightarrow$ less transactions $\Rightarrow$ less accurate ratings $\Rightarrow$...
   - Different from Coate and Loury (1993)
     - Not due to multiplicity of symmetric equilibria, no coordination among (passive) sellers...

Road Map

1. Introduce the basic (color-blind) framework
    - Sellers: productive type $i = H, L$, rating $j = G, B$
    - Non-discriminatory (symmetric) equilibrium

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1. Introduce the basic (color-blind) framework
   - Sellers: productive type $i = H, L$, rating $j = G, B$
   - Non-discriminatory (symmetric) equilibrium
2. Illustrate the mechanism for statistical discrimination
   - Sellers: type $i = H, L$, rating $j = G, B$, group $\ell = 1, 2$
   - Discriminatory (asymmetric) equilibrium
   - Mechanism, properties

The Baseline (Color-Blind) Model

- Time is continuous, focus on steady state
- Sellers (workers): mass 1
  - *Type* $i = H, L$: productivity or quality
    - Unobservable to buyers, switching at rate δ
  - *Rating* $j = G, B$: observable
- Buyers (firms): mass Q, search for sellers
  - If matched, enjoy $u_i - p$ where $u_H > u_L$
  - For non-triviality, assume $(u_H + u_L)/2 > p$.

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- Ratings-guided search
  - Two submarkets, G and B
- Matching technology: in each submarket,
  - λ: the ratio of buyers to sellers (queue length)
  - $\psi(\lambda)$: seller's matching rate
  - $\phi(\lambda)$: buyer's matching rate
  - Focus on Cobb-Douglas matching function:

$$\psi(\lambda) = \lambda^k, \ \phi(\lambda) = \lambda^{k-1} \text{ for some } k \in (0, 1).$$

Rating Technology

- After each transaction, the seller's (wrong) rating gets corrected with prob. $\alpha \in (0, 1)$
  - $(i, j) = (H, B) \Rightarrow (i, j) = (H, G)$ with prob α
  - $(i, j) = (L, G) \Rightarrow (i, j) = (L, B)$ with prob α
- No change for correct ratings
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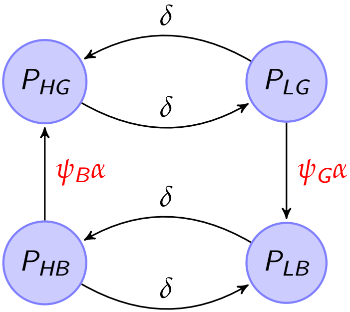
- Transition matrix

	HG	LG	HB	LB
HG	1	0	0	0
LG	0	$1-\alpha$	0	α
HB	α	0	$1-\alpha$	0
LB	0	0	0	1

Steady-State Distribution

- P_{ij}: total measure of sellers with $i = H, L$ and $j = G, B$.
- Let ψ_j denote a seller's trading rate with rating j.



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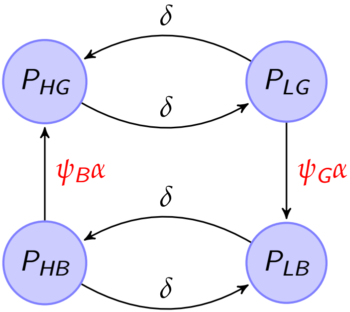

$$\Rightarrow \quad \begin{aligned} P_{HG} &= \frac{\psi_B(\delta + \psi_G\alpha)}{2\left(\delta(\psi_G + \psi_B) + \alpha\psi_G\psi_B\right)} \\ P_{LG} &= \frac{\psi_B\delta}{2\left(\delta(\psi_G + \psi_B) + \alpha\psi_G\psi_B\right)} \\ P_{HB} &= \frac{\psi_G\delta}{2\left(\delta(\psi_G + \psi_B) + \alpha\psi_G\psi_B\right)} \\ P_{LB} &= \frac{\psi_G(\delta + \psi_B\alpha)}{2\left(\delta(\psi_G + \psi_B) + \alpha\psi_G\psi_B\right)}. \end{aligned}$$

Expected Quality and Buyer Payoff in each Submarket

- Expected quality in submarket j:

$$\mu_G = 1 - \frac{\delta}{2\delta + \psi(\lambda_G)\alpha} \text{ and } \mu_B = \frac{\delta}{2\delta + \psi(\lambda_B)\alpha}.$$

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- Buyers' expected payoffs in each submarket

$$\begin{aligned} u_B(\lambda_B) &= \phi(\lambda_B)(\mu_B(\lambda_B)u_H + (1-\mu_B(\lambda_B))u_L) - p) \\ u_G(\lambda_G) &= \phi(\lambda_G)(\mu_G(\lambda_G)u_H + (1-\mu_G(\lambda_G))u_L) - p). \end{aligned}$$

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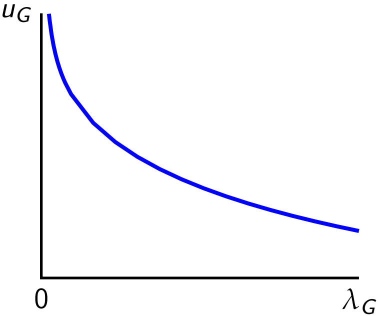
- u_B always decreasing in λ_B, but u_G may not.
  - Competition (congestion) effect: $u_B, u_G \downarrow$
  - Information effect: $u_B \downarrow$, while $u_G \uparrow$.

Buyers' Expected Payoffs

- For Cobb-Douglas matching $\psi(\lambda) = \lambda^k$, let

$$\underline{k} \equiv \frac{1 + \sqrt{1 - \frac{u_H - u_L}{2(u_H - p)}}}{2} (< 1).$$

- If $k \leq \underline{k}$, then u_G is monotone (left).



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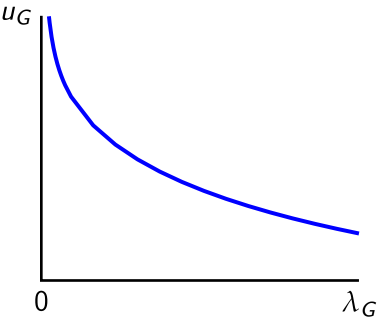
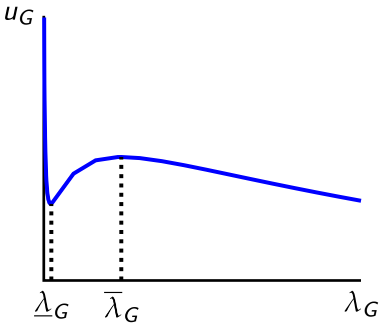
- If $k \in (\underline{k}, 1)$, then u_G decreases, increase, and decreases (right).





(Non-Discriminatory) Market Equilibrium

1. Buyers' indifference between G and B:

$$u_B(\lambda_B) = u_G(\lambda_G).$$

2. Market clearing:

$$\underbrace{Q}_{\text{buyer supply}} = \underbrace{\lambda_G(P_{HG} + P_{LG}) + \lambda_B(P_{HB} + P_{LB})}_{\text{buyer demand}}.$$

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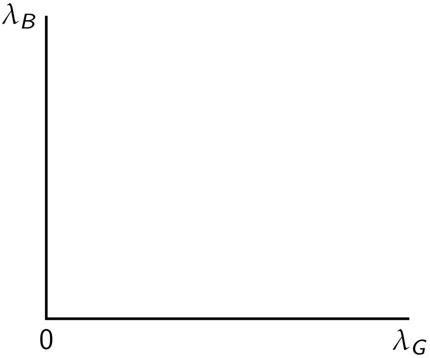

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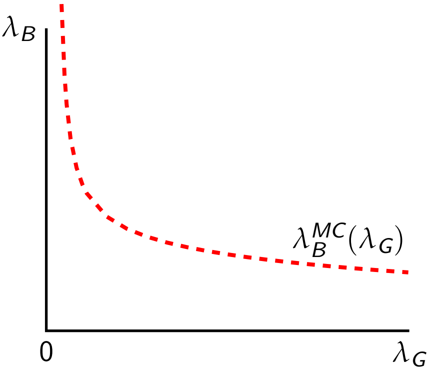

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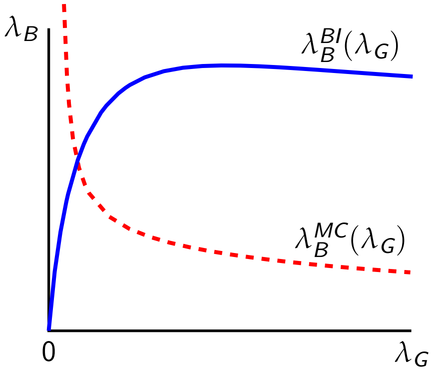

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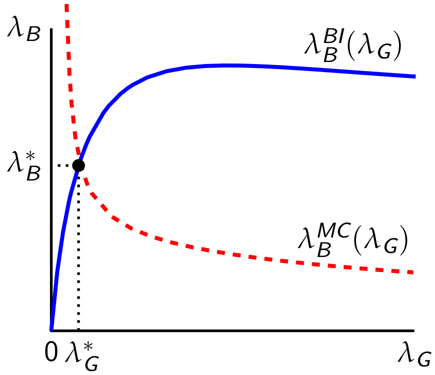
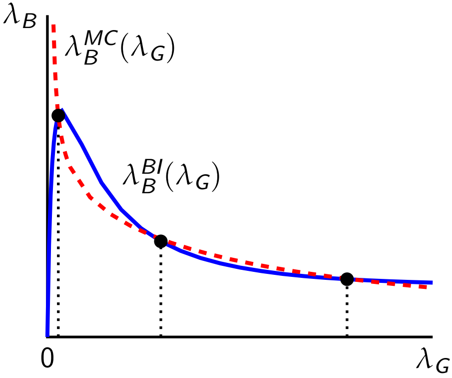


(Non-Discriminatory) Market Equilibrium

- If $k \leq \underline{k}$, then necessarily a unique equilibrium.
  - u_G monotone $\Rightarrow \lambda_B^{BI}$ monotone
- If $k \in (\underline{k}, 1)$, then unique or multiple.



Discriminatory Equilibrium

- Suppose there are two "groups" of sellers, $\ell = 1, 2$.
  - Payoff-irrelevant, persistent, *observable*
  - Gender, ethnicity, race
  - For strong symmetry, assume equal measure.
- Each seller now indexed by $i = H, L$, $j = G, B$, $\ell = 1, 2$.
- Submarket indexed by (j, ℓ)
  - $G1$ (G-rated group 1 workers), $B1$, $G2$, $B2$
  - λ_G^1, λ_B^1, λ_G^2, λ_B^2.

Discriminatory Equilibrium: Buyer Indifference

- In equilibrium, buyers should be indifferent over all 4 submarkets:

$$u_G(\lambda_G^1) = u_B(\lambda_B^1) = u_B(\lambda_B^2) = u_G(\lambda_G^2).$$

- Since $u_B(\cdot)$ is monotone,

$$u_B(\lambda_B^1) = u_B(\lambda_B^2) \Leftrightarrow \lambda_B^1 = \lambda_B^2 \Leftrightarrow \mu_B^1 = \mu_B^2.$$

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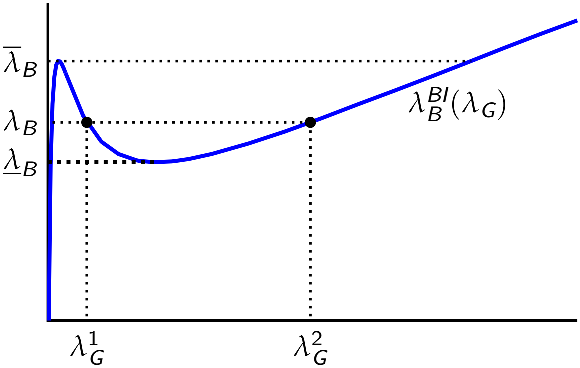
- For discrimination, $u_G(\lambda_G^1) = u_G(\lambda_G^2)$, but $\lambda_G^1 \neq \lambda_G^2$.
  - This is possible if and only if $u_G(\cdot)$ is non-monotone.
  - If $k \leq \underline{k}$, then $u_G(\cdot)$ is monotone, thus no discriminatory equilibrium.
  - If $k > \underline{k}$, then discriminatory equilibrium exists iff $Q \in [\underline{Q}, \overline{Q}]$

Discriminatory Equilibrium: Construction



- Let $\lambda_B^1 = \lambda_B^2 = \lambda_B$. Then, by construction,

$$u_G(\lambda_G^1) = u_B(\lambda_B^1) = u_B(\lambda_B^2) = u_G(\lambda_G^2).$$

- $(\lambda_G^1, \lambda_B^1, \lambda_G^2, \lambda_B^2)$ yields a discriminatory equilibrium if

$$Q = \sum_{j,\ell} \lambda_j^\ell (P_{Hj}^\ell + P_{Lj}^\ell).$$

Discriminatory Equilibrium: Properties

- For each $k \in (\underline{k}, 1)$, there exist $\underline{Q}$ and $\overline{Q}$ such that a discriminatory equilibrium exists if and only if $Q = [\underline{Q}, \overline{Q}]$.
- A discriminatory equilibrium exists even if there is a unique non-discriminatory (symmetric) equilibrium.
- Payoff inequality
  - If $\lambda_G^1 < \lambda_G^2$, then group 1's payoffs are strictly smaller than those of group 2.

Stability

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- If $k \leq \underline{k}$, then
  - Unique symmetric equilibrium, no asymmetric equilibrium.
  - The unique equilibrium is stable.
- Suppose $k > \underline{k}$.
  - Symmetric equil and asymmetric equil can coexist.
  - Symmetric may or may not be stable.
  - If an asymmetric equilibrium exists, then either it is stable or there exists another asymmetric equilibrium that is stable.

Rating Quality (Technology) and Discrimination

- Rating quality:

$$\beta \equiv \frac{\alpha}{\delta} = \text{relative updating frequency.}$$

Proposition

Fix $k \in (\underline{k}, 1)$ and Q. A discriminatory equilibrium exists if and only if $\beta \in (\underline{\beta}, \overline{\beta})$.

- Discrimination arises if and only if rating quality is neither too bad nor too good.

Conclusion

1. Present a simple conceptual framework for "ratings-guided markets"
2. Suggest the possibility of statistical discrimination induced by ratings.
   - Feedback loop between information and opportunities.
   - Discriminatory equilibrium can be stable.
3. Policies: ongoing
   - Traditional affirmative action: hiring quotas
   - Algorithm-based policy: what about if we make it easier (or harder) for group 1 to obtain and/or retain G rating?

Thank you!!

Stability

- Let $U(Q^\ell)$ denote buyers' expected payoffs with group $\ell = 1, 2$ and Q^ℓ measure of buyers targeting them.
- Whether discriminatory or not, in equilibrium,

$$U(Q^1) = U(Q^2) \Leftrightarrow u_G(\lambda_G^1) = u_B(\lambda_B^1) = u_B(\lambda_B^2) = u_G(\lambda_G^2).$$

Definition

An equilibrium is stable if $U'(Q^1) + U'(Q^2) \leq 0$ and unstable otherwise.

- Move some buyers from 1 to 2: $(Q^1, Q^2) \Rightarrow (Q^1 - \Delta, Q^2 + \Delta)$
- They want to go back to 1 only when

$$U(Q^1 - \Delta) \geq U(Q^2 + \Delta) \Leftrightarrow -\frac{U(Q^1) - U(Q^1 - \Delta)}{\Delta} \geq \frac{U(Q^2 + \Delta) - U(Q^2)}{\Delta},$$

which reduces to $U'(Q^1) + U'(Q^2) \leq 0$ as $\Delta \to 0$.

(Un)Stability of Non-Discriminatory Equilibrium

Proposition

A non-discriminatory equil. is stable iff $u_G(\cdot)$ is decreasing at λ_G.

Proof.

If non-discriminatory, then $Q^1 = Q^2 = Q/2$. Therefore,

$$U'(Q^1) + U'(Q^2) \leq 0 \Rightarrow U'(Q/2) \leq 0,$$

which is equivalent to $u_G(\cdot)$ decreasing. □

- If $k \leq \underline{k}$ (so no discriminatory equilibrium), then the unique (non-discriminatory) equilibrium is stable.
- If $k > \underline{k}$, then a non-discriminatory equilibrium may or may not be stable.

Stability of Discriminatory Equilibrium

Proposition

If there exists a discriminatory equilibrium, then there always exists a stable discriminatory equilibrium.

Proof.

Define

$$g(x) = U\left(\frac{Q}{2} + x\right) - U\left(\frac{Q}{2} - x\right).$$

- $g(x) < 0$ if x is close to $Q/2$.
- Since discriminatory exists, $g(x^*) = 0$ for some $x^* \in (0, Q/2)$.
- Hence, either $g'(x^*) < 0$ of $g'(x^{**}) < 0$ for some $x^{**} \in (x^*, Q/2)$.

□